3

Fundamentals of spread-spectrum techniques

In this chapter we consider the spread-spectrum transmission schemes that demand channel bandwidth much greater than is required by the Nyquist sampling theorem. You will recall from Chapter 2 that the minimum bandpass bandwidth required for data transmission through an ideal channel is equal to the data symbol rate. You will also recall that wideband reception allows a large amount of input noise power to the detector and thus degrades the quality of the detected data. Therefore, the receivers for spread-spectrum schemes have to convert the received wideband signals back to their original narrowband waveforms before detection. This process generates a certain amount of processing gain that can be used to combat radio jamming and interference. We will describe and discuss in detail the properties and methods of generation of the functions used in creating wide spectrum signals. Finally, we consider the multiple access properties of the spread-spectrum systems and outline the analytical model for evaluating the system performance.

3.1 Historical background

There was intensive use of communications warfare during World War II. This technique outlined the ability to intercept and interfere with hostile communications. Consequently, this procedure stimulated a great deal of interest which led to the development of secure communications systems and work in this field was carried out on two fronts. Firstly, development in communication theory initiated encryption schemes (Shannon, 1949) to provide certain information protection. Secondly, work was initiated to harness the development of a new technology. This technology is called the Spread-Spectrum techniques (Scholtz, 1982), which exchanges bandwidth expansion for communications security and targets ranging for military applications.

By the end of the war, the theory of spread-spectrum techniques had developed and its anti-jamming capability had been recognized. Communication systems were developed by the military establishments during the 1960s, using frequency hopping and pseudo-noise spread-spectrum schemes. During this period, a multiple users' pseudo-noise spread-spectrum system was constructed, providing a 16 dB processing gain (Corneretto, 1961). An interesting system was also developed which combines pseudo-noise spread spectrum with Fourier transform (Goldberg, 1981). This is conceptually similar to the contemporary multicarrier spread-spectrum schemes.

Work on spread spectrum during the 1970s prompted commercial use of the spread-spectrum techniques, and theoretical work on spread-spectrum systems revealed the new system's ability to offer multiple access communications at an increased capacity compared to the time division or frequency division schemes of that time (Yue, 1983). The RAKE receiver concept (Price and Green, 1958) was developed to further accelerate the implementation of the systems. By the end of the decade, commercial applications of spread spectrum had become a reality.

The 1980s witnessed the development of the Global System of Mobile Telecommunications (commercially known as GSM) system, and a slow frequency hopping concept from spread-spectrum technique was implemented in the GSM systems to randomize the affects of interference from multiple users accessing the GSM network. The first trial of commercial spread-spectrum system with multiple access capabilities was carried out by Qualcom in the USA in 1993 (Gilhouse et al., 1991). The Qualcom's system was built according to the interim standard IS-95. The first commercial cellular radiophone service based on spread spectrum was inaugurated in Hong Kong in 1995. Korea and the USA soon introduced similar services.

During the 1990s, the spread-spectrum technique was further developed into 'multicarrier techniques' (Fazel, 1993) providing a higher diversity gain against deep fade than a single carrier spread-spectrum system could provide. The spread-spectrum multicarrier technique is based upon low rate data transmission over orthogonal frequency division multiplexing. This scheme generates multiple copies of the conventional spread spectrum; each copy is transmitted on a separate carrier. At the time of writing, many billions of dollars have already been invested in spread-spectrum development for the provision of high data rate for the next generation of communication networks.

3.2 Benefits of spread-spectrum technology

3.2.1 Avoiding interception

In military communications, interception of hostile communications is commonly used for various operations such as identification, jamming, surveillance or reconnaissance. The successful interceptor usually measures the transmitted power in the allocated frequency band. Thus, spreading the transmitted power over a wider band undoubtedly lowers the power spectral density, and thus hides the transmitted information within the background noise. The intended receiver recovers the information with the help of system processing gain generated in the spread process. However, the unintended receiver does not get the advantage of the processing gain and consequently will not be able to recover the information. Because of its low power level, the spread spectrum transmitted signal is said to be a low probability of interception (LPI) signal.

3.2.2 Privacy of transmission

The transmitted information over the spread-spectrum system cannot be recovered without knowledge of the spreading code sequence. Thus, the privacy of individual user communications is protected in the presence of other users. Furthermore, the fact that spreading is independent of the modulation process gives the system some flexibility in choosing from a variety of modulation schemes.

3.2.3 Resistance to fading

In a multipath propagation environment, the receiver acquires frequent copies of the transmitted signal. These signal components often interfere with each other causing what is commonly described as signal fading. The resistance of the spread-spectrum signals to multipath fading is brought about by the fact that multipath components are assumed to be independent. This means that if fading attenuates one component, the other components may not be affected, so that unfaded components can be used to recover the information.

3.2.4 Accurate low power position finding

The distance (range) between two points can be determined by measuring the time in seconds, taken by a signal to move from one point to the other and back. This technique is exploited in the Global Positioning System (GPS). Since the signal travels at the speed of light (3×10^8 metres/sec),

$$\text{Range in metres} = 3 \times 10^8 \, \frac{\text{transit time}}{2}$$

It is clear from the above expression that the accuracy of the transit time measurement determines the ultimate range accuracy.

In practice, the transit time is determined by monitoring the correlation between transmitted and received code sequences. The transit time can be computed by multiplying the code duration by the number of code bits needed to align the two sequences. Clearly, higher resolution requires code symbols to be narrow which means high code rates. Thus, the sequences are selected to provide the required resolution so that if the code sequence has N chips, each with duration T_c seconds, then:

$$\text{Maximum range} = 1.5NT_c \cdot 10^8 \text{ metres}$$

The range resolution requires the chip duration T_c to be small so that sequence chip rate is as high as possible. On the other hand, maximum range requires a long sequence (i.e. N is large) so that many chips are transmitted in a single sequence period.

The GPS system consists of twenty-four satellites orbiting the earth along six orbital planes, spaced 60 degrees apart with nominally four satellites in each orbit. These clusters of satellites provide any user with visibility of five to eight satellites from any point on earth. The position, in 3-D, of a moving receiver and its speed can be measured using signals received from at least four satellites.

GPS provides two services. The precise positioning service uses very long code sequence at a code rate of 10.23 MHz. The standard positioning service, on the other hand, uses a shorter code (1023 bits) at a rate of 1.023 MHz. Each satellite is identified by a different phase of the short code.

3.2.5 Improved multiple access scheme

Multiple access schemes are designed to facilitate the efficient use of a given network resource by a group of users. Conventionally, there are two schemes in use: the Frequency Division Multiple Access (FDMA), and the Time Division Multiple Access (TDMA). In FDMA, the radio spectrum is shared between the users such that a fraction of the channel is allocated to each user at a time. On the other hand, in TDMA, each user is able to access the whole of the spectrum at a unique time slot.

The spread spectrum offers a new network access scheme due to the use of unique code sequences. Users transmit and receive signals with access interference that can be controlled or even minimized. This technique is called Code Division Multiple Access (CDMA) and is considered in more detail in Chapter 6.

3.3 Principles of spread-spectrum communications (Scholtz, 1977)

Digital transmission schemes which provide satisfactory performance and an adequate bit rate can be arranged into two categories.

- In applications like satellite communications, these schemes provide efficient usage of the limited power available.
- In applications such as mobile wireless, where the schemes achieve efficient usage of the limited bandwidth available for the service in demand.

However, both schemes are narrowband and vulnerable to hostile jamming and radio interference. The novelty of the spread-spectrum concept is that it provides protection against such attacks. This concept is based upon exchanging bandwidth expansion for anti-jamming capability.

The bandwidth expansion in spread spectrum is acquired through a coding process that is independent of the message being sent or the modulation being used. The spread spectrum, unlike FM, does not combat interference originated from thermal noise. The trade-off between signal-to-noise ratio (SNR) and data bit rate (or bandwidth) in the spread-spectrum scheme can be demonstrated by the following.

Consider a digital signal transmission over a Gaussian channel occupying a bandwidth B with SNR = 10 dB. A channel coding scheme can be used to receive data with as small an error probability as desired if transmission is carried out at a data bit rate less or equal to the channel capacity (C) defined by the Shannon equation:

$$C = B \cdot \log_2(1 + SNR) \tag{3.1}$$

Substituting for the SNR = 10 dB in equation (3.1) gives the ratio of bit rate to bandwidth:

$$\frac{C}{B} = \log_2(11) = 3.46$$

Now if we reduce the channel SNR to 5 dB (i.e. to 3.16 in ratio), then referring to the bandwidth-efficiency diagram shown in Figure 3.1, the reliable transmission is still possible at the same bit rate but with expanded bandwidth B' given by:

$$\frac{C}{B'} = \log_2(4.16) = 2.06$$

Now consider $B' = \frac{C}{2.06}$ and $B = \frac{C}{3.46}$ so that the expansion in the bandwidth is given by

$$\frac{B'}{B} = \frac{3.46}{2.06} = 1.73 \quad \text{Thus } B' = 1.73B$$

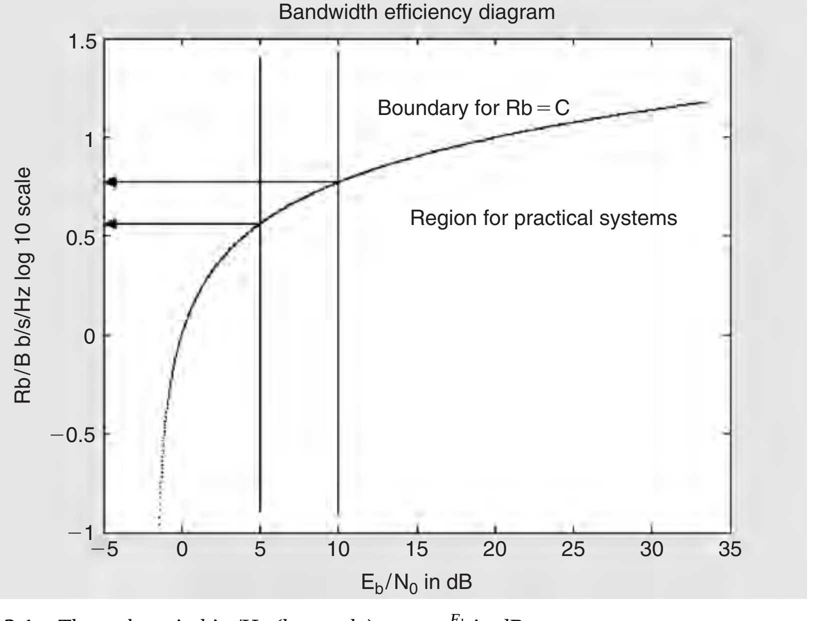


Figure 3.1 *Throughput in bits/Hz (log scale) versus $\frac{E_b}{N_0}$ in dB.*

The original bandwidth has to be expanded by a factor of about 1.73 to compensate for the reduction in the channel SNR.

It is worth noting that increasing the transmission bandwidth will undoubtedly increase the amount of the input noise power in a wideband receiver. But, as we will see, we commonly use a narrowband receiver to limit the amount of the input noise.

Example 3.1

Binary data is transmitted through an additive white Gaussian noise (AWGN) channel with SNR = 3.5 dB and bandwidth B. Channel coding is used to ensure reliable communications. Then:

i. What is the maximum bit rate that can be transmitted?
ii. If the bit rate is increased to 3B, how much must the channel SNR be increased to ensure reliable transmission?

Solution

SNR = 3.5 dB (=2.24 in ratio)

i. Channel capacity is given by Shannon equation (3.1):

$$\begin{aligned} C &= B \cdot \log_2(1 + 2.24) = B \cdot \log_2(3.24) \\ &= B \cdot \frac{\log_{10}(3.24)}{\log_2(2)} = 1.7B \end{aligned}$$

Note the maximum bit rate for binary transmission that can be achieved with no errors in an ideal channel (no noise) is 2B. In this example the bit rate is about 1.7B.

ii. $C = 3B = B\log_2(1 + SNR)$ where SNR represents the channel's new signal-to-noise ratio.

Thus $(1 + SNR) = 2^3 = 8$, therefore, SNR = 7 = 8.45 dB

The increase in the channel SNR = 8.45 − 3.5 = 4.95 dB.

Note in this case, the bit rate is greater than 2B and the transmission of the data over the channel is multi-level but the symbol rate is still 2B.

Example 3.2

Binary data is transmitted at the rate of R_b bits/sec over a channel occupying a bandwidth B and the channel SNR = 3 dB. If the data bit rate is increased to $2.65R_b$ and the bandwidth is increased to 1.75B:

i. What would be the channel SNR for the new system?
ii. What channel bandwidth is required to keep the same channel signal-to-noise ratio?

Solution

i. Substitute the SNR in equation (3.1):

$$\text{SNR} = 3\,\text{dB} = 2 \text{ in ratio}$$

So that for the first case: $R_b = B\log_2(1+2)$ and for the second case:

$$2.65 \cdot R_b = 1.75B\log_2(1 + \text{SNR}) \text{ where SNR is the channel SNR for the new system.}$$

$$\frac{1}{2.65} = \frac{B \cdot \log_2(3)}{1.75B \cdot \log_2(1 + \text{SNR})} = \frac{\log_{10}(3)}{1.75\log_{10}(1 + \text{SNR})}$$

Therefore, $1.75\ \log_{10}(1+\text{SNR}) = 2.65\ \log_{10}(3)$. This gives

$$\text{SNR} = 4.28 (=6.3\,\text{dB}).$$

ii. If the channel signal-to-noise ratio is kept at 3 dB, the expanded bandwidth (B') is computed from $\frac{1}{2.65} = \frac{B}{B'}$.

Thus $B' = 2.65B$ compared with $1.75B$ in the first case.

The spread-spectrum concept has developed from the principle of Shannon theorem. If data is transmitted at a rate of R_b over a channel occupying a bandwidth much greater than R_b, Shannon theorem indicates that reliable communications can be achieved at a reduced SNR. However, if the transmitted power is kept fixed, even though the power density is substantially reduced, a surplus in the SNR is generated and can be used to combat interference and jamming. This surplus in SNR is called processing gain.

The spreading of the energy is achieved by phase modulating the input data with the user code sequence. The modulation reduces the high power density of the original data to a low level shown in Figure 3.2(a).

A simple Matlab code is written to compare the power spectral density of 6 data symbols with power spectral density of the same data symbols spread using Gold sequence number 7 of length 31 and is shown in Figure 3.2(b). The spreading process generates enough processing gain to protect the transmission from hypothetical jammer employing a narrow band tone as shown in Figure 3.2(c).

The received signal has to be converted into the original narrowband to limit the amount of input noise accompanying the wideband reception. The conversion is performed at the receiver with the aid of a locally generated code sequence causing the spread spectrum to collapse. Moreover, the de-spreading process is accompanied with spreading of the jamming power into background noise as shown in Figure 3.2(d). Thus, de-spreading the wanted signal is accompanied by reduction of the impact of jamming attack on the data transmission.

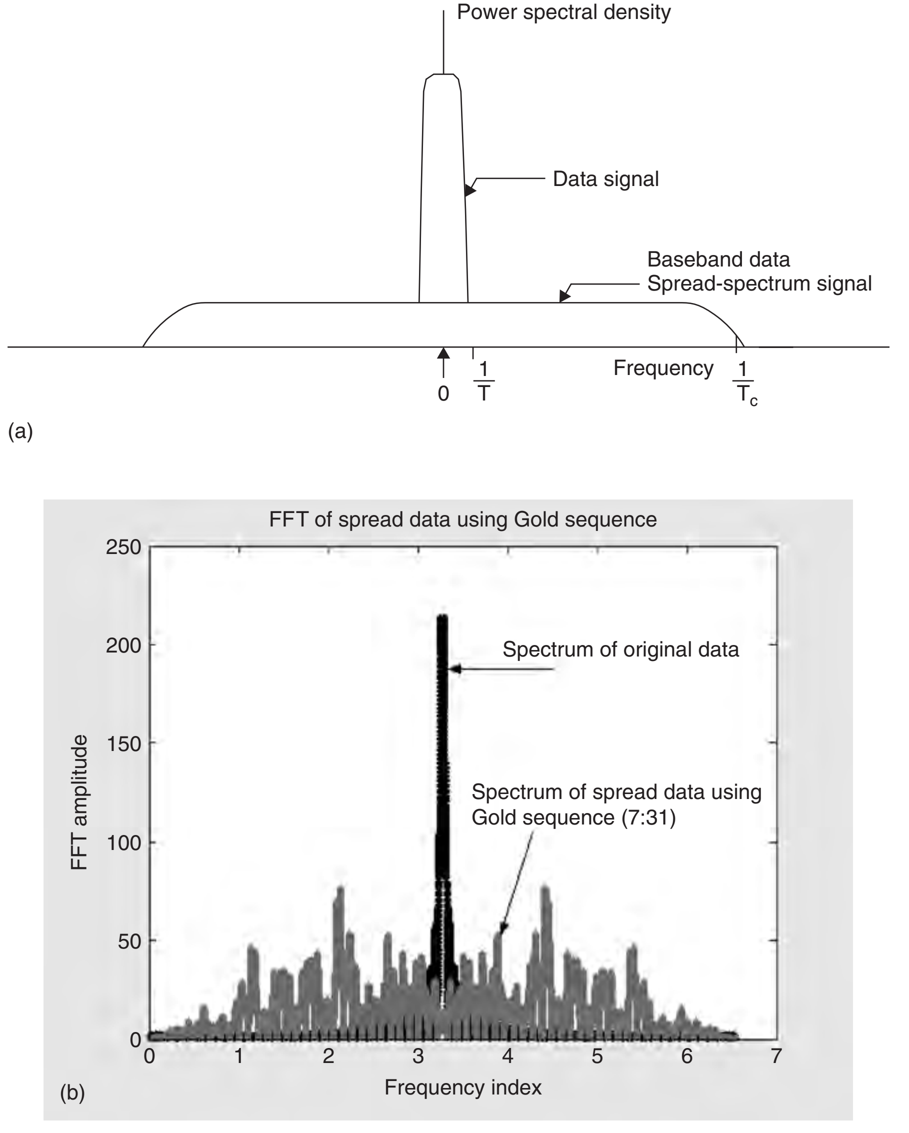
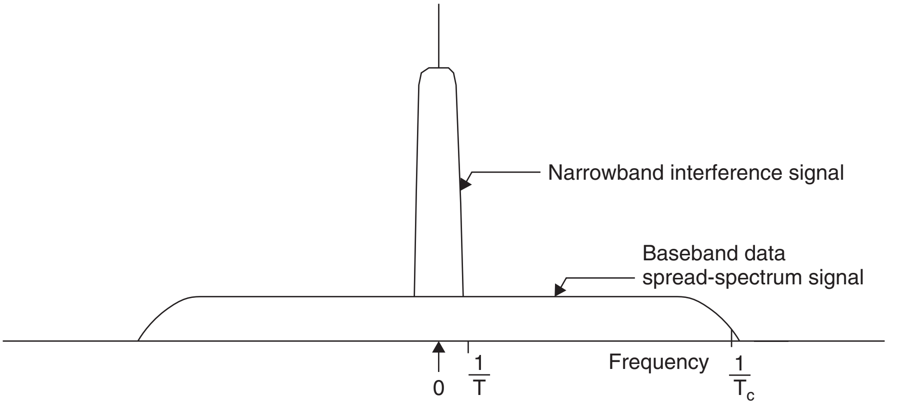


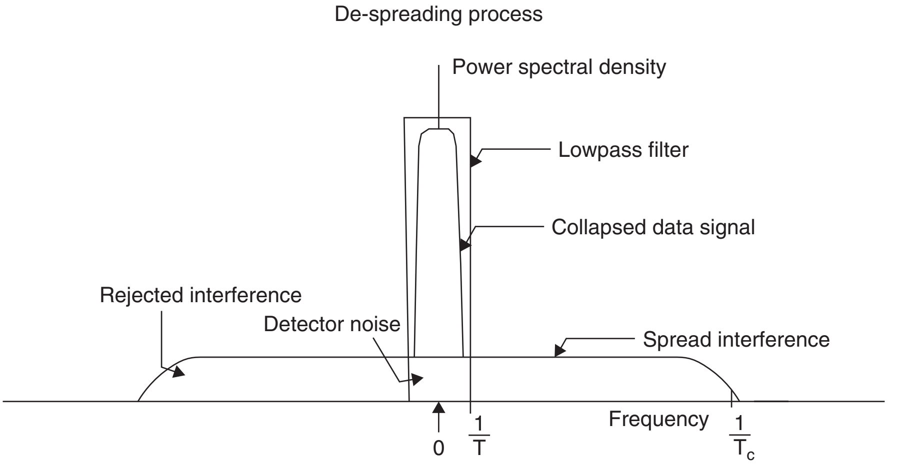
Figure 3.2 *(a) Power spectral density of data signal before and after spreading; (b) Power spectral density of spread-spectrum signal using Gold code sequence (7:31) generated by Matlab.*

3.4 Most common types of spread-spectrum systems

Two spread-spectrum systems are widely employed in the provision of reliable communications: the direct sequence spread spectrum (DS-SS), and the frequency hopped spread spectrum (FH-SS) systems. The DS-SS system executes the spreading of the data energy in real time by phase modulating the data with a high rate code sequence. On the other hand, the FH-SS scheme performs the energy spreading in the frequency domain. The



(c)



(d)

Figure 3.2 *Continued (c) Power spectral density of spread-spectrum signal with narrowband jamming signal; (d) Power spectral density of signal and interference after the despreading process.*

latter is accomplished by forcing the narrowband carrier to jump pseudo-randomly from one frequency slot to the next according to the state of the code sequence in use.

Furthermore, a hybrid of both schemes can be developed to improve the processing gain compared to what is obtainable from a single scheme. The emphasis in this textbook is on the DS-SS systems and their applications in wireless communications.

3.4.1 DS-SS systems

A block diagram of the modulator that generates DS-SS signals is shown in Figure 3.3.

The binary data m(t) is first multiplied by the high rate code sequence to acquire the energy spreading. The baseband signal $S_n(t)$ is filtered to confine energy within the bandwidth

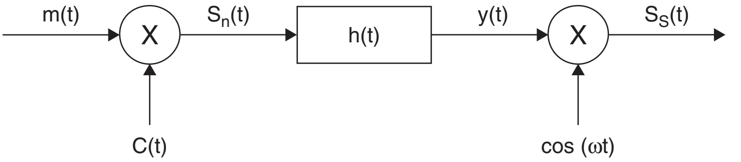


Figure 3.3 *Direct sequence spread-spectrum modulation system.*

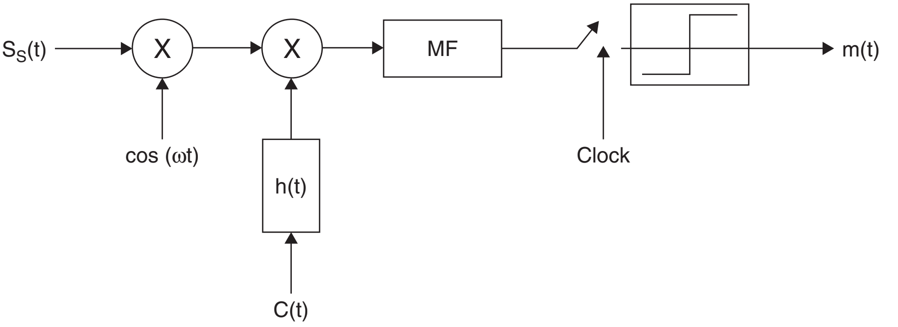


Figure 3.4 *Matched filter spread-spectrum receiver.*

defined by the code rate. The carrier modulation commonly used in spread spectrum is phase shift keying. Considering Figure 3.3, we get:

$$S_n(t) = m(t) \cdot C(t) \tag{3.2}$$

The baseband signal $S_n(t)$ is convoluted with the impulse response of the spectrum-shaping filter to yield y(t):

$$y(t) = S_n(t) * h(t) \quad \text{where * denotes convolution} \tag{3.3}$$

The bandpass signal $$S_S(t) = [S_n(t) * h(t)] \cdot \cos \omega_C t \tag{3.4}$$

A basic block diagram of the matched filter receiver is shown in Figure 3.4. The received bandpass signal $S_S(t)$ is converted to an equivalent complex lowpass signal A(t) by mixing with a locally generated coherent carrier. The lowpass spread spectrum is caused to collapse by multiplying by a locally generated in-phase copy of the transmitted code sequence. The de-spread signal B is matched filtered and sampled.

The complex lowpass signal $$A(t) = S_S(t) \cdot \cos \omega_C t \tag{3.5}$$

The de-spread signal $$B(t) = A(t) \cdot [C(t) * h(t)] \tag{3.6}$$

The output of the matched filter $$D(T) = \int_{(K-1)T}^{KT} B(t) \cdot dt \tag{3.7}$$

The receiver decodes the data according to the following rule:

$$D(T) > 0 \text{ decode binary '1' otherwise decode binary '0'.}$$

Example 3.3

A binary data stream of 4 digits [1011] is spread using an 8-chip code sequence $C(t) = [01\ 10\ 10\ 01]$. The spread data phase modulates a carrier using binary phase shift keying. The transmitted spread-spectrum signal is exposed to interference from a tone at the carrier frequency but with 30 degrees phase shift. The receiver generates an in-phase copy of the code sequence and a coherent carrier from a local oscillator.

i. Determine the baseband transmitted signal.
ii. Express the signal received. Ignore the background noise.
iii. Assuming negligible noise, determine the detected signal at the output of the receiver.

Solution

i. Let the data stream be denoted as $m(t)$. The baseband spread-spectrum data $m_S(t)$ can be represented as:

$$m_S(t) = m(t) \cdot C(t) = [01\ 10\ 10\ 01, 10\ 01\ 01\ 10, 01\ 10\ 10\ 01, 01\ 10\ 10\ 01]$$

Since the data is transmitted as binary PSK, we map $0 \to 1$ and $1 \to -1$.

ii. The baseband spread-spectrum signal, $m_S(t)$, now modulates a carrier at frequency ω_C and the transmitted signal, $m_t(t)$, is given by:

$$m_t(t) = m_S(t) \cdot \cos \omega_C t.$$

The received signal $m_r(t)$ comprised the baseband signal $m_t(t)$, the interfering tone $I(t)$, and additive white noise $n(t)$. However in this example we ignore the noise so that signal plus interference is:

$$m_r(t) = m_t(t) + I(t)$$

The interfering signal is a sinusoidal waveform at frequency ω_c with 30 degrees phase shift:

$$I(t) = \cos(\omega_C t + 30)$$

Thus, the received signal $m_r(t) = m_t(t) + \cos(\omega_C t + 30)$

iii. The front end stage of the receiver mixes the received signal $m_r(t)$ with the local oscillator by multiplying $m_r(t)$ by the reference carrier, $(\cos \omega_C t)$ to compose the baseband signal, $m_b(t)$. Therefore:

$$\begin{aligned} m_b(t) &= m_t(t) \cdot \cos \omega_C t + \cos(\omega_C t + 30) \cdot \cos \omega_C t \\ &= 0.5 m_S(t)[1 + \cos 2\omega_C t] + 0.5[\cos 30 + \cos(2\omega_C t + 30)] \end{aligned}$$

Assume that $2\omega_C$ is removed by filtering and the signal level adjusted to unit by amplification then:

$$m_b(t) = m_S(t) + \cos 30$$

The next stage in the detection provokes the collapse of the spread spectrum into its original narrowband data. The de-spread signal $m_d(t)$ is given by multiplying $m_b(t)$ by the locally generated code sequence, that is:

$$\begin{aligned} m_d(t) &= m_b(t) \cdot C(t) = [m_S(t) + \cos 30] \cdot C(t) \\ &= m(t) \cdot C(t) \cdot C(t) + 0.866C(t) \end{aligned}$$

Now $C(t) \cdot C(t)$ is a constant which can be normalized to one. The detector samples the de-spread signal at the code sequence rate and adds the samples to be compared with a threshold level. The summation of the sample of C(t) when sampled at the code rate is

$$\sum_{k=0}^{7} C(kT_c) = -1 + 1 + 1 - 1 + 1 - 1 - 1 + 1 = 0$$

Therefore $m_d(t) = m(t)$
The output of the receiver is [10 11].

The quadrature spread-spectrum modulator, shown in Figure 3.5(a), comprises two orthogonal binary modulators similar to the one just described. The input data is demultiplexed into two parallel streams. Data transported on the in-phase channel is spread by the code sequence $C_i(t)$ and data on the quadrature channel is spread by the code sequence $C_q(t)$. The two parallel channels are combined to modulate a main RF carrier.

The quadrature spread-spectrum receiver consists of two binary matched filter receivers as shown in Figure 3.5(b). The detection of the data is carried out by each channel separately in a method identical to the one described for the binary channel.

3.4.2 Frequency hopping spread-spectrum system

Frequency hopping entails the transmission carrier frequency hopping between available channels within the spread-spectrum band. A narrow spectral band and an individual carrier frequency at the centre of the band define each transmitted channel. Successive carrier frequencies are chosen in accordance with the pseudo-random phases of the spreading code sequence. There are two widely used FH schemes: (1) Fast frequency hopping where one complete, or a fraction of the data symbol, is transmitted within the duration between carrier hops. Consequently, for a binary system, the frequency hopping rate may exceed the data bit rate. (2) On the other hand, in a slow frequency hopping system, more than one symbol is transmitted in the interim time between frequency hops.

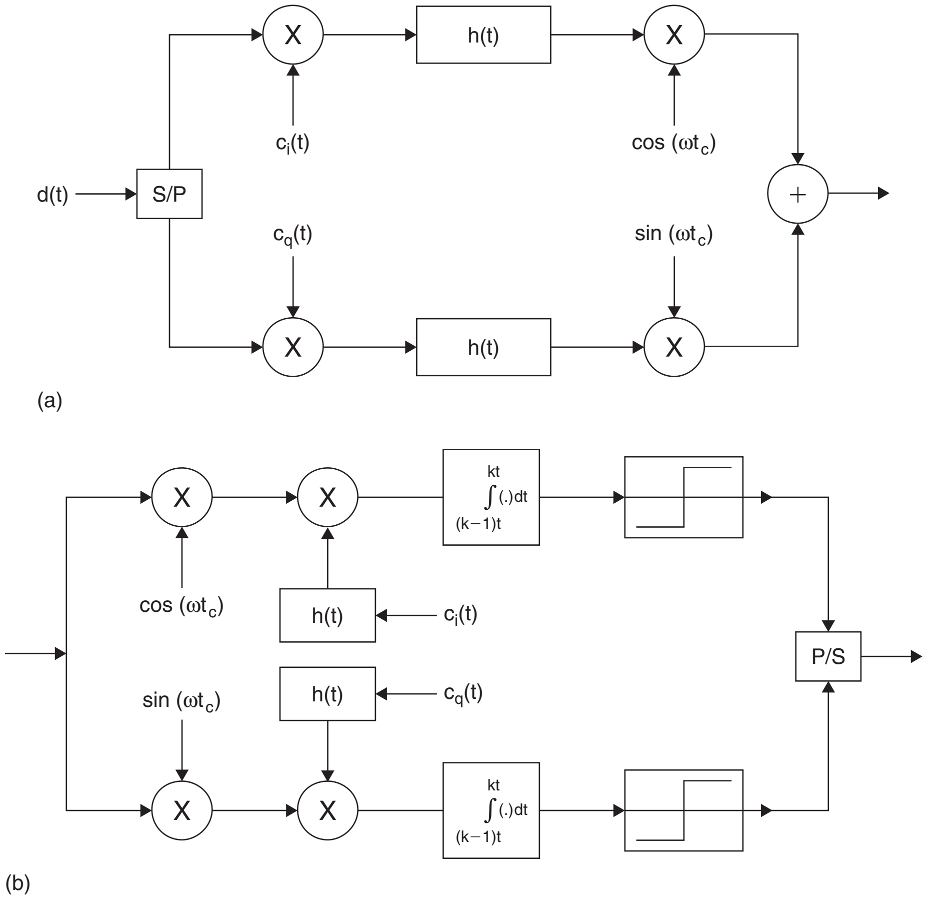


Figure 3.5 *(a) Quadrature spread-spectrum modulator; (b) Quadrature spread-spectrum receiver.*

Figure 3.6 illustrates how the carrier frequency hops with time. Let time duration between hops be T_h and data bit duration be denoted by T_b, then:

$$T_h \leq T_b \quad \text{for fast hopping} \tag{3.8}$$

$$T_h > T_b \quad \text{for slow hopping} \tag{3.9}$$

The basic FH modulation system, depicted in Figure 3.7(a), comprises a digital phase or frequency shift keying modulator and a frequency synthesizer. The latter generates carrier frequencies according to the pseudo-random phases of the spreading code sequence that is then mixed with the data carrier to originate the FH signal.

In the basic FH receiver, shown in Figure 3.7(b), the received FH signal is first filtered using a wideband bandpass filter and then mixed with a replica of the FH carrier. The mixer output is applied to the appropriate demodulator. A coherent demodulator may be used when a PSK carrier is received.

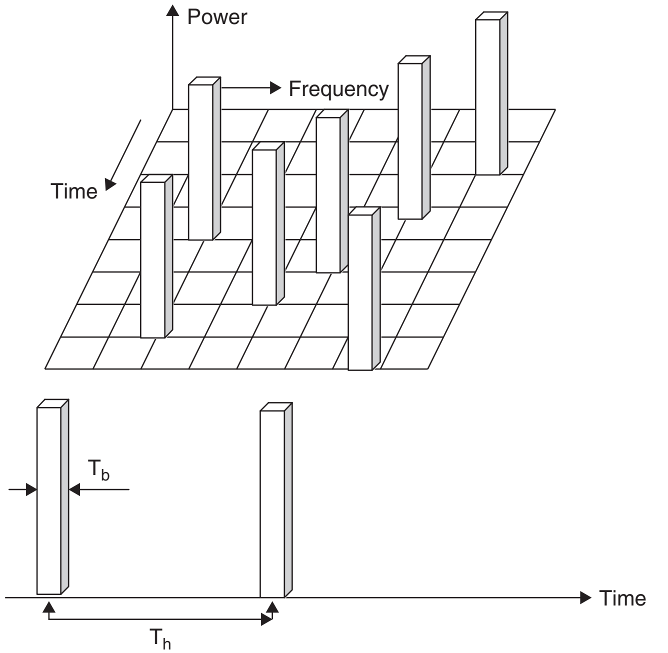


Figure 3.6 *Carrier frequency hopping from one frequency to another.*

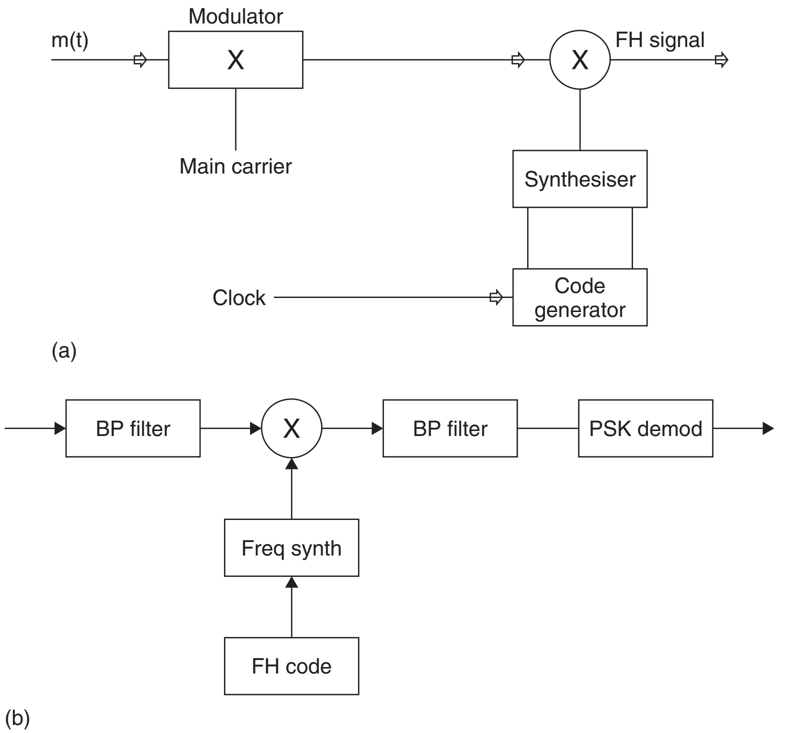


Figure 3.7 *(a) Basic FH modulator; (b) Basic FH receiver.*

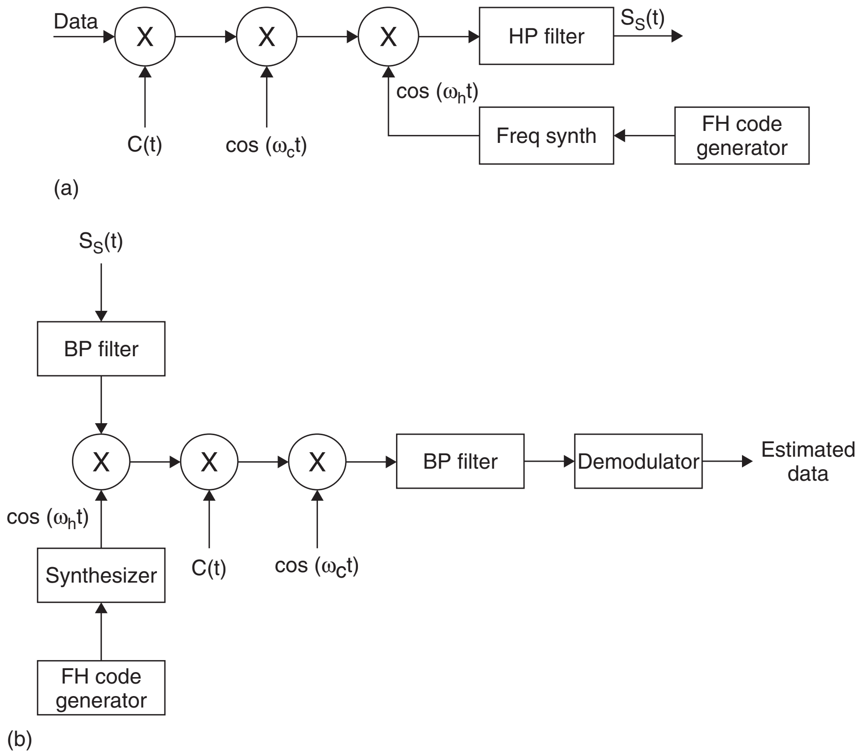


Figure 3.8 *(a) Direct sequence/Frequency hopping spread-spectrum transmitting system; (b) DS/FH spread-spectrum hybrid receiving system.*

3.4.3 Hybrid DS/FH systems

In special applications such as anti-jamming work, there may be a need for a hybrid system using both the DS and FH spread-spectrum schemes. A hybrid system is shown conceptually in Figure 3.8. Two code sequences are employed in this system. The DS/FH hybrid modulation system is shown in Figure 3.8(a); first code sequence is used to generate the DS-SS signal as described previously. The resulting signal is linearly modulated on a hopping carrier frequency generated by a frequency synthesizer according to the second code sequence. A replica of the hopping carrier is generated locally at the receiver using a coherent hopping code sequence.

The DS/FH hybrid receiver is shown in Figure 3.8(b) where the received signal is filtered and mixed with the hopping frequency and the output of the mixer is de-spread using the DS code.

3.5 Processing gain

Digital signal transmission is normally preceded by signal processing such as filtering, modulation and coding. At the receiver, processing like matched filtering and detection is used to recover the data.

In each of these processing methods, certain characteristics of the input signal are being modified or amplified. The effectiveness of the processor is measured with a factor called the processing gain G_p defined as:

$$\frac{\text{Modified signal parameter at processor output}}{\text{Signal parameter at input}}$$

In spread-spectrum systems, the parameter of interest is the signal spectrum at the input (B_b) and the spectrum of the output (B_s). Thus:

$$G_p = \frac{B_s}{B_b} \tag{3.10}$$

Thus the processing gain (G_p) expresses the bandwidth expansion factor. For a DS-SS system:

$$G_p = \frac{R_c}{R_b} \tag{3.11}$$

where R_c is the code sequence rate and R_b is the data bit rate. The processing gain generated by FH-SS system is:

$$G_p = \text{number of available channels} = N \tag{3.12}$$

Example 3.4

A speech conversation is transmitted by a DS-SS system. The speech is converted to PCM using an anti-aliasing filter with a cut-off frequency of 3.4 kHz and using 256 quantization levels. It is anticipated that the processing gain should not be less that 23 dB.

i. Find the required chip rate.
ii. If the speech was transmitted by an FH-SS system, what would be the number of hopping channels?

Solution

i. Sampling the speech at the Nyquist frequency generates $2 \times 3.4 = 6.8$ k samples/sec. We encode these samples using 256 quantization levels. Thus each sample is represented by n bits where

$$256 = 2^n$$

Thus $n = 8$

The PCM bit rate $= R_b = n \times 6.8 = 54.4$ k bits/sec

Processing gain $= 23\text{dB} = 199.53 = G_p = \dfrac{R_c}{R_b}$

Substituting for R_b gives $R_c = 10854.2$ k chip/sec.

ii. Applying the definition of processing gain to transmission over an FH-SS system, we get:

$$G_p = \frac{B_s}{B_b} = N$$

Therefore the number of FH channels $= N \approx 200$.

3.6 Correlation functions (Sarwate and Pursley, 1980)

The interaction and the interdependence between two time (or frequency) varying signals are defined by the correlation function derived from the comparison of the two signals. The comparison of a signal with itself is described as the autocorrelation function. On the other hand, the cross-correlation is a measure of similarity between two autonomous signals. The correlation processing forms the basis upon which optimum detection algorithms in digital communication systems are derived.

Consider two binary sequences $\{a\}$ and $\{b\}$ with elements $\widehat{a}_n$ and $\widehat{b}_n$ that can be real or complex such that:

$$\{a\} = \{\widehat{a}_0, \widehat{a}_1, \widehat{a}_2, \ldots\ldots, \widehat{a}_{N-1}\} \tag{3.13}$$

$$\{b\} = \{\widehat{b}_0, \widehat{b}_1, \widehat{b}_2, \ldots\ldots, \widehat{b}_{N-1}\} \tag{3.14}$$

In the analysis, we assume the two sequences to be periodic with long period N and $0 \leq n \leq N-1$. The reason behind this assumption is that, while code sequences in practical CDMA systems have long period N, they are in essence considered pseudo-random.

Two correlation functions of interest when considering spread-spectrum communications: periodic correlation function and aperiodic correlation function. Each is designed for a specific application.

3.6.1 Periodic correlation function

The periodic correlation function $R_{a,b}(\tau)$ of N-element sequences $\{a\}$ and $\{b\}$ is defined by:

$$R_{a,b}(\tau) = \sum_{n=0}^{N-1} \widehat{a}_n \cdot \widehat{b}^{*}_{n+\tau} \tag{3.15}$$

where $\widehat{b}_i^*(\tau)$ denotes the complex conjugate of $\widehat{b}_i(\tau)$. When $\widehat{a}_n = \widehat{b}_n$, $R_a(\tau)$ represents the periodic autocorrelation function [PACF] and with $\widehat{a}_n \neq \widehat{b}_n$, $R_{a,b}(\tau)$ describes the periodic cross-correlation function [PCCF]. The normalized correlation function is given by:

$$R_{a,b}(\tau)\big|_{norm} = \frac{R_{a,b}(\tau)}{N} \tag{3.16}$$

Now we consider the equation (3.15) for the periodic correlation $R_{a,b}(\tau)$ in more detail by expanding the summation:

$$\begin{aligned} R_{a,b}(\tau) = {} & \widehat{a}_0 \cdot \widehat{b}_\tau^* + \widehat{a}_1 \cdot \widehat{b}_{\tau+1}^* + \widehat{a}_2 \cdot \widehat{b}_{\tau+2}^* + \cdots\cdots\cdots\cdots + \widehat{a}_{N-1-\tau} \cdot \widehat{b}_{N-1}^* + \widehat{a}_{N-\tau} \cdot \widehat{b}_N^* \\ & + \widehat{a}_{N-\tau+1} \cdot \widehat{b}_{N+1}^* + \widehat{a}_{N-\tau+2} \cdot \widehat{b}_{N+2}^* + \cdots\cdots\cdots\cdots + \widehat{a}_{N-1} \cdot \widehat{b}_{N-1+\tau}^* \end{aligned} \tag{3.17}$$

$$\begin{aligned} = {} & \widehat{a}_0 \cdot \widehat{b}_\tau^* + \widehat{a}_1 \cdot \widehat{b}_{\tau+1}^* + \widehat{a}_2 \cdot \widehat{b}_{\tau+2}^* + \cdots\cdots\cdots\cdots + \widehat{a}_{N-1-\tau} \cdot \widehat{b}_{N-1}^* \\ & + \widehat{a}_{N-\tau} \cdot \widehat{b}_{N \cdot \text{mod}\, N}^* + \widehat{a}_{N-\tau+1} \cdot \widehat{b}_{(N+1)\cdot \text{mod}\, N}^* + \widehat{a}_{N-\tau+2} \cdot \widehat{b}_{(N+2)\cdot \text{mod}\, N}^* \\ & + \cdots\cdots\cdots\cdots + \widehat{a}_{N-1} \cdot \widehat{b}_{(N-1+\tau)\cdot \text{mod}\, N}^* \end{aligned} \tag{3.18}$$

Since the code sequences are assumed periodic with period = N then:

$$\widehat{b}_{(N+i)\cdot \text{mod}\, N}^* = \widehat{b}_i^*$$

Thus $R_{a,b}(\tau)$ in equation (3.18) is simplified to:

$$\begin{aligned} R_{a,b}(\tau) = {} & \widehat{a}_0 \cdot \widehat{b}_\tau^* + \widehat{a}_1 \cdot \widehat{b}_{\tau+1}^* + \widehat{a}_2 \cdot \widehat{b}_{\tau+2}^* + \cdots\cdots\cdots\cdots + \widehat{a}_{N-1-\tau} \cdot \widehat{b}_{N-1}^* \\ & + \widehat{a}_{N-\tau} \cdot \widehat{b}_0^* + \widehat{a}_{N-\tau+1} \cdot \widehat{b}_1^* + \widehat{a}_{N-\tau+2} \cdot \widehat{b}_2^* + \cdots\cdots\cdots\cdots + \widehat{a}_{N-1} \cdot \widehat{b}_{\tau-1}^* \end{aligned} \tag{3.19}$$

Now let us define the functions $R'_{a,b}(\tau)$ and $R''_{a,b}(\tau)$ as:

$$R'_{a,b}(\tau) = \widehat{a}_0 \cdot \widehat{b}_\tau^* + \widehat{a}_1 \cdot \widehat{b}_{\tau+1}^* + \widehat{a}_2 \cdot \widehat{b}_{\tau+2}^* + \cdots\cdots\cdots\cdots + \widehat{a}_{N-1-\tau} \cdot \widehat{b}_{N-1}^* \tag{3.20}$$

$$R''_{a,b}(\tau) = \widehat{a}_{N-\tau} \cdot \widehat{b}_0^* + \widehat{a}_{N-\tau+1} \cdot \widehat{b}_1^* + \widehat{a}_{N-\tau+2} \cdot \widehat{b}_2^* + \cdots\cdots\cdots\cdots + \widehat{a}_{N-1} \cdot \widehat{b}_{\tau-1}^* \tag{3.21}$$

Comparing equations (3.20) and (3.21) with (3.19), we have:

$$R_{a,b}(\tau) = R'_{a,b}(\tau) + R''_{a,b}(\tau) \tag{3.22}$$

Computation of $R'_{a,b}(\tau)$ and $R''_{a,b}(\tau)$ is carried out in the method explained in the following example. We start by sketching sequence {a} and {b} when aligned with zero time shifts $(\tau = 0)$.

$\widehat{b}^*_0$	$\widehat{b}^*_1$	$\widehat{b}^*_2$	...	...	...	...	...	...	...	...	...	...	...	$\widehat{b}^*_{N-2}$	$\widehat{b}^*_{N-1}$
$\widehat{a}_0$	$\widehat{a}_1$	$\widehat{a}_2$	...	...	...	...	...	...	...	...	...	...	...	$\widehat{a}_{N-2}$	$\widehat{a}_{N-1}$

Now we delay sequence {a} by $\tau \neq 0$ relative to sequence {b} and only integer values of τ are considered. In practice, the shift τ can take on any real value. The sequences would look as in the following sketch.

			τ						$N-1$	
$\widehat{b}^*_0$	$\widehat{b}^*_1$	$\widehat{b}^*_2$	......	$\widehat{b}^*_\tau$	$\widehat{b}^*_{\tau+1}$				$\widehat{b}^*_{N-2}$	$\widehat{b}^*_{N-1}$
$\widehat{a}_{N-\tau}$			$\widehat{a}_{N-1}$	$\widehat{a}_0$	$\widehat{a}_1$	$\widehat{a}_2$	......	......		$\widehat{a}_{N-\tau-1}$

The term $R'_{a,b}(\tau)$ is given by equation (3.20) as the summation of the product of the corresponding elements of the two sequences. Elements of sequence {b} have indices limited to $\tau \leq n \leq N-1$ and sequence {a} elements have indices limited to $0 \leq n \leq N-\tau-1$ as shown in the following sketch:

τ				$N-1$	
$\widehat{b}^*_\tau$	$\widehat{b}^*_{\tau+1}$		......	$\widehat{b}^*_{N-2}$	$\widehat{b}^*_{N-1}$
$\widehat{a}_0$	$\widehat{a}_1$	$\widehat{a}_2$	......	$\widehat{a}_{N-\tau-2}$	$\widehat{a}_{N-\tau-2}$
0				$N-\tau-1$	

The term $R''_{a,b}(\tau)$ is given by equation (3.21) as the summation of the product of the sequence elements shown in following sketch:

0					$\tau-1$	
$\widehat{b}^*_0$	$\widehat{b}^*_1$			......	$\widehat{b}^*_{\tau-2}$	$\widehat{b}^*_{\tau-1}$
$\widehat{a}_{N-\tau}$	$\widehat{a}_{N-\tau+1}$	...	......	......	$\widehat{a}_{N-2}$	$\widehat{a}_{N-1}$

The periodic autocorrelation of the spreading code sequences plays an important role in the time tracking of the spread-spectrum system code sequence, as we will see in Chapter 5.

Example 3.5

Sequences {a} and {b}, each with period $N = 15$, are given by:

$$\{a\} = \{1, 1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, -1, -1, -1\}$$
$$\{b\} = \{1, -1, -1, -1, -1, 1, -1, -1, -1, -1, 1, -1, -1, -1, -1\}$$

Find the periodic autocorrelation and cross-correlation functions of the sequences.

Solution

Periodic autocorrelation functions with shift right is shown in the following sketches:

Sequence {a}

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$R_a(\tau)$
1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1	15
−1	1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1
−1	−1	1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1
−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1
−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	−1	1	−1
1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	−1	−1
−1	1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	−1
1	−1	1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	−1
−1	1	−1	1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1
−1	−1	1	−1	1	−1	−1	−1	−1	1	1	1	−1	1	1	−1
1	−1	−1	1	−1	1	−1	−1	−1	−1	1	1	1	−1	1	−1
1	1	−1	−1	1	−1	1	−1	−1	−1	−1	1	1	1	−1	−1
−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1	1	1	1	−1
1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1	1	1	−1
1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1	1	−1
1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1	15
−1	1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1

Sequence {b}

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$R_b(\tau)$
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	15
−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	3
−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	3
−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	3
−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	3
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	3
−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	3
−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	3
−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	3
−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	3
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	15
−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	3
−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	3
−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	3
−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	3
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	3
−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	3

Periodic cross-correlation function

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$R_{a,b}(\tau)$
1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1	
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	7
−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1
−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1
−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1
−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	7
−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1
−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1
−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1
−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	7
−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1
−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1
−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1
−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	7

3.6.2 Aperiodic correlation function

The aperiodic correlation function between sequence {a} and {b} is defined by $C_{a,b}(\tau)$ where:

$$C_{a,b}(\tau) = \sum_{n=0}^{N-1-\tau} \widehat{a}_n \cdot \widehat{b}^*_{n+\tau} \quad 0 \le \tau \le N-1 \tag{3.23}$$

$$\begin{aligned} &= \sum_{n=0}^{N-1+\tau} \widehat{a}_{n-\tau} \cdot \widehat{b}^*_n \quad 1-N \le \tau \le 0 \\ &= 0 \qquad\qquad |\tau| \ge N \end{aligned} \tag{3.24}$$

Again if $\widehat{a}_n = \widehat{b}_n$, the expression $C_{a,b}(\tau)$ represents the aperiodic autocorrelation function [AACF]. When $\widehat{a}_n \ne \widehat{b}_n$, the expression defines the aperiodic cross-correlation function [ACCF]. The significance of [ACCF] becomes evident when we consider the access interference in multi-user spread-spectrum system in Chapter 6.

Let us focus our attention for now on the aperiodic cross-correlation, $C_{a,b}(\tau - N)$, between {a} and {b} when {a} is time shifted to the left by $(\tau - N)$ with respect to {b} such that:

$$C_{a,b}(\tau - N) = \sum_{n=0}^{N-1+(\tau-N)} \widehat{a}_{n-(\tau-N)} \cdot \widehat{b}^*_n \quad 1-N \le \tau \le 0 \tag{3.25}$$

$$= \sum_{n=0}^{\tau-1} \widehat{a}_{n-(\tau-N)} \cdot \widehat{b}_n^*$$

$$= \widehat{a}_{N-\tau} \cdot \widehat{b}_0^* + \widehat{a}_{N-\tau+1} \cdot \widehat{b}_1^* + \widehat{a}_{N-\tau+2} \cdot \widehat{b}_2^* + \cdots\cdots + \widehat{a}_{N-1} \cdot \widehat{b}_{\tau-1}^* \tag{3.26}$$

Now compare equation (3.26) with equation (3.21), where we have just proved that:

$$C_{a,b}(\tau - N) = R''_{a,b}(\tau) \tag{3.27}$$

Similarly, we can show that the aperiodic cross-correlation $C_{a,b}(\tau)$ is given as:

$$C_{a,b}(\tau) = R'_{a,b}(\tau) \tag{3.28}$$

Thus, the periodic cross-correlation that has been defined in the previous section can be expressed in terms of the aperiodic cross-correlation as:

$$R_{a,b}(\tau) = C_{a,b}(\tau - N) + C_{a,b}(\tau) \tag{3.29}$$

Example 3.6

Consider the sequences given in Example 3.5. Calculate the aperiodic correlation functions.

Solution

The AACF for sequence {a}

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$C_a(\tau)$
1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1	15
	1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	0
		1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	1
			1	1	1	−1	1	1	−1	−1	1	−1	1	−1	2
				1	1	1	−1	1	1	−1	−1	1	−1	1	1
					1	1	1	−1	1	1	−1	−1	1	−1	0
						1	1	1	−1	1	1	−1	−1	1	1
							1	1	1	−1	1	1	−1	−1	−2
								1	1	1	−1	1	1	−1	1
									1	1	1	−1	1	1	−2
										1	1	1	−1	1	−1
											1	1	1	−1	−2
												1	1	1	−3
													1	1	−2
														1	−1
													1	1	−2
												1	1	1	−3
											1	1	1	−1	−2
										1	1	1	−1	1	−1
									1	1	1	−1	1	1	−2

The AACF for sequence {b}

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$C_b(\tau)$
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	15
	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	4
		1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	3
			1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	2
				1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	1
					1	−1	−1	−1	−1	1	−1	−1	−1	−1	10
						1	−1	−1	−1	−1	1	−1	−1	−1	3
							1	−1	−1	−1	−1	1	−1	−1	2
								1	−1	−1	−1	−1	1	−1	1
									1	−1	−1	−1	−1	1	0
										1	−1	−1	−1	−1	5
											1	−1	−1	−1	2
												1	−1	−1	1
													1	−1	0
														1	−1
													1	−1	0
												1	−1	−1	1
											1	−1	−1	−1	2
										1	−1	−1	−1	−1	5
									1	−1	−1	−1	−1	1	0
								1	−1	−1	−1	−1	1	−1	1
							1	−1	−1	−1	−1	1	−1	−1	2
						1	−1	−1	−1	−1	1	−1	−1	−1	3
					1	−1	−1	−1	−1	1	−1	−1	−1	−1	10
				1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	1
			1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	2
		1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	3
	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	4

The ACCF between sequences {a} and {b}

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$C_{a,b}(\tau)$
1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1	
1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	7
	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	0
		1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	1
			1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	2

(*Continued*)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$C_{a,b}(\tau)$
				1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	1
					1	−1	−1	−1	−1	1	−1	−1	−1	−1	8
						1	−1	−1	−1	−1	1	−1	−1	−1	1
							1	−1	−1	−1	−1	1	−1	−1	0
								1	−1	−1	−1	−1	1	−1	3
									1	−1	−1	−1	−1	1	0
										1	−1	−1	−1	−1	5
											1	−1	−1	−1	2
												1	−1	−1	1
													1	−1	0
														1	−1

3.6.3 Even and odd cross-correlation functions

Another classification of the correlation functions frequently used is the even and odd correlation functions. These functions can be defined in terms of the periodic and aperiodic functions as we shall show in this section. Consider data $m_a(t)$ that is spread using code sequence a(t) and data $m_b(t)$ that is spread using code sequence b(t). Transmission of $m_b(t)$ is delayed by (τ) relative to transmission of data $m_a(t)$. The receiver is synchronized to code sequence a(t) so that the received signal r(t) can be expressed as:

$$r(t) = m_a(t) \cdot a(t) + m_b(t - \tau) \cdot b(t - \tau). \tag{3.30}$$

The correlation of r(t) with code sequence a(t) during the kth symbol of data $m_a(t)$ is given by:

$$\begin{aligned} y(kT) &= \frac{1}{T}\int_{kT}^{(k+1)T} m_a(t) \cdot a(t) \cdot a(t) \cdot dt + \frac{1}{T}\int_{kT}^{(k+1)T} m_b(t-\tau) \cdot b(t-\tau) \cdot a(t) \cdot dt \\ &= \frac{1}{T} m_a(kT) \int_{kT}^{(k+1)T} a(t) \cdot a(t) \cdot dt + \frac{1}{T}\int_{kT}^{kT+\tau} m_b(t-\tau) \cdot b(t-\tau) \cdot a(t) \cdot dt \\ &\quad + \frac{1}{T}\int_{kT+\tau}^{(k+1)T} m_b(t-\tau) \cdot b(t-\tau) \cdot a(t) \cdot dt \end{aligned} \tag{3.31}$$

Now $\frac{1}{T}\int_{kT}^{(k+1)T} a(t) \cdot a(t) \cdot dt =$ autocorrelation function of code sequence a(t) at zero timeshift $= R_a(0)$. The 2nd and 3rd terms in the expression (3.31) are illustrated in Figure 3.9.

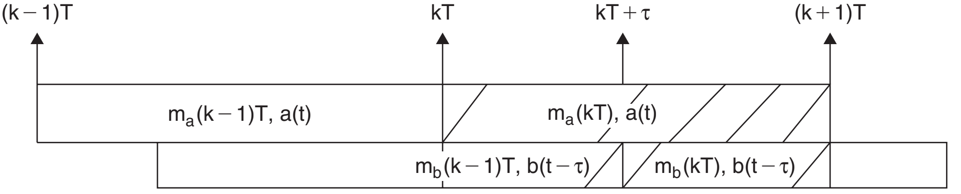


Figure 3.9 *Second and third terms for equation (3.31).*

Now considering the sketch in Figure 3.9, we have:

$$\frac{1}{T}\cdot\int_{kT}^{kT+\tau} m_b((k-1)T)\cdot b(t-\tau)\cdot a(t)\cdot dt+\tfrac{1}{T}\cdot\int_{kT+\tau}^{(k+1)T} m_b(kT)\cdot b(t-\tau)\cdot a(t)\cdot dt$$
$$= m_b(k-1)T\cdot C_{a,b}(\tau-N)+m_b(kT)\cdot C_{a,b}(\tau)$$

Thus

$$y(kT)=m_a(kT)\cdot R_a(0)+m_b(k-1)T\cdot C_{a,b}(\tau-N)+m_b(kT)\cdot C_{a,b}(\tau) \tag{3.32}$$

If $m_b(kT)=m_b(k-1)T$, then

$$y(kT)=m_a(kT)\cdot R_a(0)+m_b(kT)[C_{a,b}(\tau-N)+C_{a,b}(\tau)] \tag{3.33}$$

But when $m_b(kT)=-m_b(k-1)T$,

$$y(kT)=m_a(kT)\cdot R_a(0)+m_b(kT)\left[C_{a,b}(\tau-N)-C_{a,b}(\tau)\right] \tag{3.34}$$

Thus, the even cross-correlation $R_{a,b}(\tau)$ and the periodic cross-correlation are the same, that is:

$$R_{a,b}(\tau)=C_{a,b}(\tau-N)+C_{a,b}(\tau) \tag{3.35a}$$

The odd cross-correlation $\hat{R}_{a,b}(\tau)$ is defined as:

$$\widehat{R}_{a,b}(\tau)=C_{a,b}(\tau-N)-C_{a,b}(\tau) \tag{3.35b}$$

Similarly, the even and odd autocorrelation functions can be expressed in terms of the aperiodic autocorrelation function as follows:

$$R_a(\tau)=C_a(\tau-N)+C_a(\tau) \tag{3.36a}$$

$$\widehat{R}_a(\tau)=C_a(\tau-N)-C_a(\tau) \tag{3.36b}$$

Let the discrete Fourier transform (DFT) of the periodic code sequence {a} and {b} be sequence {A} and {B}, respectively, such that:

$$A_k=\frac{1}{N}\sum_{i=0}^{N-1} a_i\, e^{-j\cdot 2\pi\cdot\frac{ki}{N}} \tag{3.37}$$

$$B_k = \frac{1}{N} \sum_{i=0}^{N-1} b_i \, e^{-j \cdot 2\pi \cdot \frac{ki}{N}} \tag{3.38}$$

Now we are in a position to consider the even and odd correlation functions in the frequency domain. Let DFT $(R_{a,b}(\cdot))$ denote the DFT of the periodic cross-correlation $R_{a,b}(\tau)$. It is shown in Sarwate and Pursley (1980) that:

$$\text{DFT}\,(R_{a,b}(k)) = N \cdot A_{-k} \cdot (B_{-k})^* \tag{3.39}$$

$$\text{DFT}\,(R_a(k)) = N \cdot |A_{-k}|^2 \tag{3.40}$$

Example 3.7

Consider the following two Walsh-Hadamard sequences, each with period $N = 8$, where sequences {a} and {b} are given by:

{a} = {010 1001}
{b} = {0000 1111}

i. Find the even and odd cross-correlation.
ii. Find the DFT of the periodic cross-correlation and express it in terms of the DFT of both sequences.

Solution

i. The even cross-correlation $R_{a,b}(\tau)$ is given by equation (3.35a):

$$R_{a,b}(\tau) = C_{a,b}(\tau - N) + C_{a,b}(\tau)$$

The odd cross-correlation $\widehat{R}_{a,b}(\tau)$ is given by equation (3.35b):

$$\widehat{R}_{a,b}(\tau) = C_{a,b}(\tau - N) - C_{a,b}(\tau)$$

The periodic cross-correlation is given by:

$$R_{a,b}(\tau) = \sum_{n=0}^{N-1} \hat{a}_n \cdot \hat{b}^*_{n+\tau}$$

We use the convention: binary '1' = +1 and '0' = −1.

For $\tau = 0$

−1	1	1	−1	1	−1	−1	1
−1	−1	−1	−1	1	1	1	1

1 + (−1) + (−1) + 1 + 1 + (−1) + (−1) + 1

$R_{a,b}(0) = 0$

For $\tau = 1$

−1	1	1	−1	1	−1	−1	1
1	−1	−1	−1	−1	1	1	1

$(-1) + (-1) + (-1) + 1 + (-1) + (-1) + (-1) + 1$

$R_{a,b}(1) = -4$

For $\tau = 2$

−1	1	1	−1	1	−1	−1	1
1	*1*	−1	−1	−1	−1	1	1

$(-1) + 1 + (-1) + 1 + (-1) + 1 + (-1) + 1$

$R_{a,b}(2) = 0$

It can be shown that:

τ	0	1	2	3	4	5	6	7
$R_{a,b}(\tau)$	0	−4	0	4	0	4	0	−4

The aperiodic cross-correlation, $C_{a,b}(\tau)$, is computed as follows:

For $\tau = 0$

−1	1	1	−1	1	−1	−1	1
−1	−1	−1	−1	1	1	1	1

$1 + (-1) + (-1) + 1 + 1 + (-1) + (-1) + 1$

$C_{a,b}(0) = 0$

For $\tau = 1$

−1	1	1	−1	1	−1	−1	1
	−1	−1	−1	−1	1	1	1

$(-1) + (-1) + 1 + (-1) + (-1) + (-1) + 1$

$C_{a,b}(1) = -3$

For $\tau = 2$

−1	1	1	−1	1	−1	−1	1
		−1	−1	−1	−1	1	1

$(-1) + 1 + (-1) + 1 + (-1) + 1$

$C_{a,b}\ (2) = 0$

It can be shown that:

τ	0	1	2	3	4	5	6	7
$C_{a,b}(\tau)$	0	−3	0	3	0	1	0	−1

The odd cross-correlation is calculated as follows:

τ	0	1	2	3	4	5	6	7
$R_{a,b}(\tau)$	0	−4	0	4	0	4	0	−4
$C_{a,b}(\tau)$	0	−3	0	3	0	1	0	−1
$C_{a,b}(\tau - N)$	0	−1	0	1	0	3	0	−3
$\hat{R}_{a,b}(\tau)$	0	2	0	−2	0	2	0	−2

ii. The Fast Fourier Transform (FFT) of $R_{a,b}(\tau)$ is:

τ	0	1	2	3	4
$FFT(R_{a,b})$	0	−1.414	0	1.414	0

The FFT of sequence {a} is:

τ	0	1	2	3	4
FFT(a)	0	$0.104 - j0.25$	0	$-0.604 + j0.25$	0

and the FFT of sequence {b} is:

τ	0	1	2	3	4
FFT(b)	0	$-0.25 + j0.604$	0	$-0.25 + j0.104$	0

Therefore, $N.A_{-k} \cdot (B_{-k})^* = 8$.

$$\begin{array}{llllll} 0 & (0.104 + j0.25)(-0.25 + j0.604) & 0 & (-0.604 - j0.25)(-0.25 + j0.104) & 0 \\ = & & & & \\ 0 & -1.416 + j0.002528 & 0 & 1.416 - j0.002528 & 0 \end{array}$$

Therefore, (FFT) of $R_{a,b}(\tau) \approx N \cdot A_{-k} \cdot (B_{-k})^*$

3.6.4 The Merit Factor (Golay, 1982)

The Merit Factor (M_F) is defined by the ratio of the energy of the in-phase autocorrelation (Ca(0)) to the total energy of the out-of-phase autoscorrelation ($C_a(\tau)$); that is:

$$M_F = \frac{|C_a(0)|^2}{2 \sum_{\tau=1}^{N-1} |C_a(\tau)|^2} \tag{3.41}$$

The Merit Factor provides an insight into the behaviour of the sequence autocorrelation function, such that we can use M_F as an indicator to improve the design of code sequences. Ideally, sequences used in spread spectrum should exhibit large in-phase autocorrelation and zero (or very small) out-of-phase autocorrelation components. Consequently, such sequences enjoy very large Merit Factor. However, in practice such sequences do not necessarily have acceptable cross-correlation properties. Thus, the design of code sequences is based upon a compromise between providing low cross-correlation and a large Merit Factor.

Example 3.8

Compute the Merit Factor of sequence {a} where:

$\{a\} = \{001100000101011\}$

Solution

The Merit Factor (M_F) can be computed using equation (3.41) as follows:

For $\tau = 0$

−1	−1	1	1	−1	−1	−1	−1	−1	1	−1	1	−1	1	1
−1	−1	1	1	−1	−1	−1	−1	−1	1	−1	1	−1	1	1

$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$

$C_a(0) = 15$

For $\tau = 1$

−1	−1	1	1	−1	−1	1	−1	−1	1	−1	1	−1	1	1
	−1	−1	1	1	−1	−1	−1	−1	−1	1	−1	1	−1	1

$1 + (-1) + 1 + (-1) + 1 + 1 + 1 + 1 + (-1) + (-1) + (-1) + (-1) + (-1) + 1$

$C_a(1) = 0$

For $\tau = 2$

−1	−1	1	1	−1	−1	−1	−1	−1	1	−1	1	−1	1	1
		−1	−1	1	1	−1	−1	−1	−1	−1	1	−1	1	−1

$(-1) + (-1) + (-1) + (-1) + 1 + 1 + 1 + (-1) + 1 + 1 + 1 + 1 + (-1)$

$C_a(2) = 1$

Therefore, the aperiodic autocorrelation is:

τ	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$C_a(\tau)$	15	0	1	−2	1	0	1	0	−1	−2	−1	2	1	−2

Thus, Merit Factor is given by:

$$M_F = \frac{|15|^2}{2 \cdot [0+1+4+1+0+1+0+1+4+1+4+1+4+1]} = 4.89$$

3.6.5 *Interference rejection capability*

Interference can be caused by an external transmitter tuned to a frequency within the passband of the intended receiving equipment, possibly with the same modulation and with enough power to override any signal at the intended receiver. There are many other types of signal interferences such as interference from random noise, random radio pulse, sweep-through, and stepped tones. Radio interference limits the effectiveness of the communication equipments.

Consider a spread-spectrum system transmitting information signal m(t) between two fixed points. Further, assume that the transmission is being exposed to a jamming signal, j(t). The channel noise and the interfering signal are assumed to be uncorrelated. The received signal r(t) can be expressed as:

$$r(t) = s_s(t) + j(t) + n(t) \tag{3.42}$$

where signal $s_s(t)$ is given by:

$$s_s(t) = m(t) \cdot C(t) \cdot \cos \omega_c t \tag{3.43}$$

where C(t) is the spreading code sequence. The reference signal used by the matching filter receiver is given by:

$$r_{ref} = C(t - \tau) \cdot \cos(\omega_c t + \theta) \tag{3.44}$$

where τ represents the phase delay between transmitted and locally generated sequences and θ is the carrier phase shift. The *signal component* at the matched filter output is:

$$s_0(t) = \int_0^{T_b} s_s(t) \cdot r_{ref}(t) \cdot dt \tag{3.45}$$

$$\begin{aligned}
&= \int_0^{T_b} m(t) \cdot C(t) \cdot \cos(\omega_c t) \cdot C(t - \tau) \cdot \cos(\omega_c t + \theta) \cdot dt \\
&= \int_0^{T_b} m(t) \cdot C(t) \cdot C(t - \tau) \cdot \left[\frac{\cos\theta + \cos(2\omega_c t + \theta)}{2}\right] \cdot dt \\
&= \int_0^{T_b} m(t) \cdot C(t) \cdot C(t - \tau) \cdot \left[\frac{\cos\theta}{2}\right] \cdot dt + \int_0^{T_b} m(t) \cdot C(t) \cdot C(t - \tau) \cdot \frac{\cos(2\omega_c t + \theta)}{2} \cdot dt
\end{aligned} \tag{3.46}$$

To simplify the analysis, choose ω_c to be integer multiple of the data rate $\left(\frac{1}{T_b}\right)$ and τ is an integer number of chips so that:

$$\int_0^{T_b} m(t) \cdot C(t) \cdot C(t - \tau) \cdot \frac{\cos(2\omega_c t + \theta)}{2} \cdot dt = 0 \tag{3.47}$$

Substituting equation (3.47) in equation (3.46), we get:

$$s_0(t) = \int_0^{T_b} m(t) \cdot C(t) \cdot C(t-\tau) \cdot \left[\frac{\cos\theta}{2}\right] \cdot dt \tag{3.48}$$

Now during each symbol interval, the input data m(t) is either +1 or −1 so that equation (3.48) simplifies to:

$$s_0(t) = \pm\cos\theta \cdot R_C(\tau) \tag{3.49}$$

where $R_C(\tau)$ is the autocorrelation function of the code sequence C(t) at time shift τ and defined by:

$$R_C(\tau) = \int_0^{T_b} C(t) \cdot C(t-\tau) \cdot dt \tag{3.50}$$

The output *noise component* is given by:

$$n_0(t) = \int_0^{T_b} n(t) \cdot r_{ref}(t) \cdot dt \tag{3.51}$$

The *interference component* at matched filter receiver output is:

$$j_0(t) = \int_0^{T_b} j(t) \cdot r_{ref}(t) \cdot dt \tag{3.52}$$

It is always useful to make the analysis more generic and so we will now consider power analysis of the receiver output rather than proceed with time domain analysis of the matched filter outputs as given by equations (3.49), (3.51) and (3.52) since further time domain analysis of these expressions requires specifications of the signal, interference and reference used.

The noise considered has white spectral density and zero mean value. Let the one-sided noise power density at the input of the receiver be N_0 in W/Hz. Clearly, the noise power at the output of the matched filter depends only on the noise spectral density at its input and the receiver bandwidth. The noise power is normally independent of the code chip rate. Let the bandwidth of the narrowband receiver be B_b, so the noise power output is:

$$\frac{N_0}{2} B_b \tag{3.53}$$

Let the interference power at the input of the matched filter be J, and assume it is uniformly distributed across the spread-spectrum bandwidth B_S. Consequently, we can assume the average interference power spectral density to be $\frac{J}{B_s}$. It follows that the interference power at the receiver output is $\frac{J}{B_s}B_b$. Since we assume the noise to be independent of the interference, the total noise output power is the addition of output noise power and output interference power.

Let the received signal power be P_r with the receiver providing unit power gain. The ratio of output signal power to noise power, $(SNR)_0$ is expressed as:

$$(SNR)_0 = \frac{P_r}{\frac{N_0}{2}B_b + \frac{J}{B_s}B_b} \tag{3.54}$$

Substituting for the processing gain $G_p = \frac{B_s}{B_b}$ gives the output signal-to-noise ratio as:

$$(SNR)_0 = \frac{G_p \cdot P_r}{\frac{N_0}{2}B_s + J} \tag{3.55}$$

The signal power to noise power ratio at the input of the receiver is:

$$(SNR)_i = \frac{P_r}{\frac{N_0}{2}B_s + J} \tag{3.56}$$

Therefore, combining equation (3.55) with (3.56) we get:

$$(SNR)_0 = G_p \cdot (SNR)_i \tag{3.57}$$

The interference rejection capability of the spread-spectrum system can be evaluated in terms of the jamming margin, M_j, which is defined as the level of interference (jamming) that the system is able to tolerate and still maintain a specified level of performance such as specified bit error rate even though the signal-to-noise ratio is <1.

Let Loss be system losses between transmitter and receiver in dB, then jamming margin, M_j, is defined by:

$$M_j(dB) = -(SNR)_i(dB) - L(dB) \tag{3.58}$$

But
$$(SNR)_0(dB) = G_p(dB) + (SNR)_i(dB) \tag{3.59}$$

Combining equation (3.58) with (3.59) we get:

$$M_j(dB) = G_p(dB) - (SNR)_0(dB) - L(dB) \tag{3.60}$$

The above equation (3.60) indicates that, under ideal conditions, the desired signal can be recovered with minimum distortion provided that there is enough processing gain to eradicate the effects of the jamming signal.

The most effective form of jamming against a DS spread-spectrum system is tone jamming at the centre of the spread-spectrum band (Dixon, 1994). We will not delve into details of various schemes available in the literature to combat jamming, but it is important to inspire the reader with two efficient schemes that can be employed when the processing gain is not sufficient to remove the effects of jamming: a closed loop scheme to cancel the effects of interference described in Mowbray et al. (1992) and a phase lock loop proposed to acquire and subtract the jamming tone is described in Abu-Rgheff et al. (1989).

Example 3.9

A message is transmitted at bit rate of 9.6 kb/s using a direct sequence spread-spectrum system. The clock rate of the code sequence is 512 Mb/s. The receiver reference carrier is off by 12 degrees and the code synchronization error is 30%. The one-sided noise density (N_0) at the receiver input is 10^{-18} W/Hz. The received signal power is 2.3×10^{-14} W.

i. Find the signal-to-noise ratio at receiver output $(SNR)_0$ in dB
ii. What is the maximum obtainable $(SNR)_0$ in dB?
iii. Find the synchronization error that would reduce $(SNR)_0$ by 1.5 dB when carrier phase coherence has been achieved.

Solution

i. Recall from previous analysis, the output signal $s_0(t)$ is given by equation (3.49) as:

$$s_0(t) = m(t) \cdot \cos\theta \cdot R_c(\tau)$$

Thus, received power $P_r' = P_r \cdot \cos^2\theta \cdot R_c^2(\tau)$. In the ideal case, the autocorrelation, $R_c(\tau)$, is shown in Figure 3.10.

In the region between 0 and $\pm T_c$, the autocorrelation function $R_c(\tau)$ can be expressed in terms of τ as:

$$R_c(\tau) = \left[1 - \frac{|\tau|}{T_c}\right]$$

Given $\frac{|\tau|}{T_c} = 30\%$, then $R_c(\tau) = 0.70$

Therefore, $P_r' = 2.3 \times 10^{-14} \times \cos^2 12 \times (0.7)^2 = 1.08 \times 10^{-14}$

We calculate the output SNR from equation (3.56) but with $J = 0$ since no interference is applied:

$$(SNR)_0 = \frac{P_r'}{\frac{N_0}{2} \times B_b} = \frac{1.08 \times 10^{-14}}{\frac{10^{-18}}{2} \times 9600} = 2.25 = 3.5\,\text{dB}$$

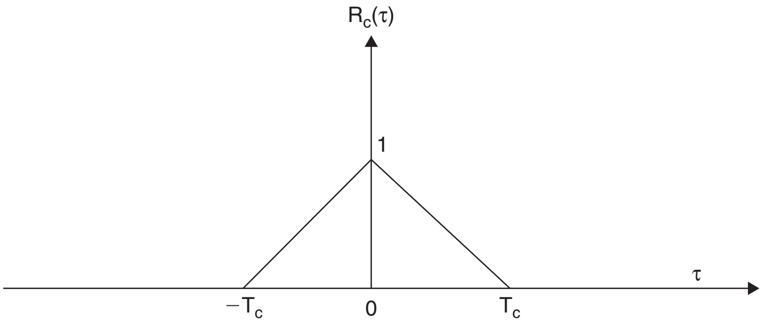


Figure 3.10 *Signal-to-noise ratio at receiver output (SNR)$_0$ in dB.*

ii. The maximum possible $(\mathrm{SNR})_0$ is evaluated when system employed coherent reference carrier and the code synchronization error is zero.

$$(\mathrm{SNR})_0|_{\max} = \frac{P_r}{\frac{N_0}{2} \times B_b} = \frac{2.3 \times 10^{-14}}{\frac{10^{-18}}{2} \times 9600} = 4.79 = 6.8\,\mathrm{dB}$$

iii. The new value for $(\mathrm{SNR})_0$ is $6.8 - 1.5 = 5.3\,\mathrm{dB} = 3.39$

$$(\mathrm{SNR})_0 = 3.39 = \frac{P'_r}{\frac{N_0}{2} \times B_b} = \frac{P'_r}{\frac{10^{-18}}{2} \times 9600}$$

Thus $P'_r = 16.27 \times 10^{-15}\,\mathrm{W} = 2.3 \times 10^{-14} \times R_c^2(\tau)$

Thus $R_c^2(\tau) = 0.707$ giving $R_c(\tau) = 0.84 = 1 -$ synchronization error percent.

Therefore synchronization error = 16%.

3.7 Performance of spread-spectrum systems (Pursley, 1977)

The performance of a spread-spectrum system is measured in terms of the bit error rate (BER), a quantity related to the theoretically computed probability of detection error.

The main parameter that influences the performance of the system when the link shared by a group of users is the level of interference generated by the multiple user-access, due to the fact that signals from individual users in spread-spectrum transmission co-exist within the same frequency band. The transmission from the users can take one of two possible modes: the first mode is when transmissions from individual users are coordinated in time and frequency and this mode results in a synchronous spread-spectrum system. Furthermore, if the synchronous signals are generated using orthogonal sequences (i.e. the cross-correlation between any pair is zero), then such signals do not interfere with

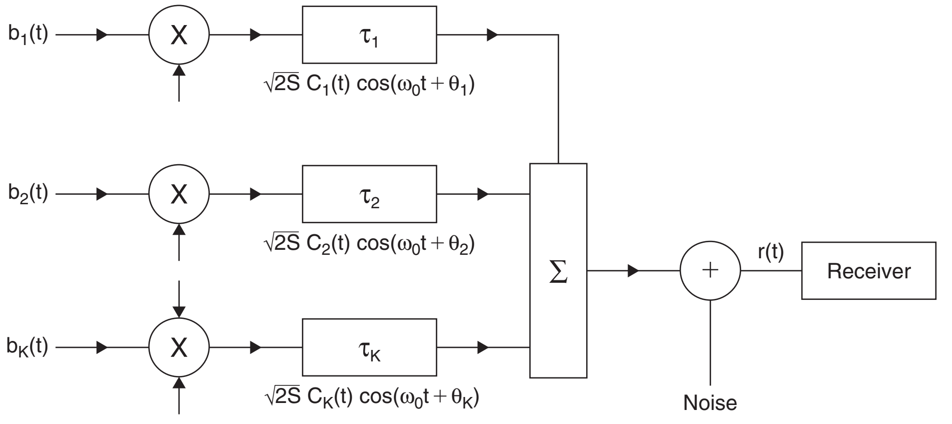


Figure 3.11 *Spread-spectrum system model (Pursley 1977).*

each other. Consequently, the channel thermal noise exclusively determines the performance of the system. The second mode is when the transmissions are uncoordinated and results in an asynchronous system and the multiple access interference has to be considered when evaluating system performance.

The performance of an asynchronous spread-spectrum system is analysed by Pursley (1977) using the basic model shown in Figure 3.11. It has been shown (Mowbray and Grant, 1992) that the distribution of the multiple access interference generated by a large group of users simultaneously accessing the network, tends to be Gaussian.

Let us consider the Pursley model with K users such that data $d_k(t)$ generated by k^{th} user is transmitted using code sequence $C_K(t)$, at a time delay τ_K and carrier phase offset θ_K relative to the intended user. Pursley has shown that for a large community of users ($N >> 1$), the worst-case probability of error P_{max} is given by:

$$P_{max} \leq 1 - \Phi\left(\left[1 - (K-1)\left(\frac{2C_c}{N}\right)\right] \cdot \sqrt{\frac{2E_b}{N_0}}\right) \tag{3.61}$$

Where $\frac{N_0}{2}$ is channel two-sided thermal noise spectral density, E_b is the energy per data bit, and $\Phi(\cdot)$ is the zero mean, unit variance Gaussian distribution, C_c is the maximum magnitude of the aperiodic cross-correlation given by:

$$C_c = \max\{|C_{k,i}(\tau)|\} \quad 1 - N \leq \tau \leq N - 1 \tag{3.62}$$

$|C_{k,i}(\tau)|$ is the magnitude of the cross-correlation between code sequences that belong to users k and i. Furthermore, Pursley has shown that the amount of multiple access interference, Q_a, from $(K-1)$ other active users is:

$$Q_a = \frac{1}{6 \cdot N^3} \cdot \sum_{\substack{k=1 \\ k \neq i}}^{K} r_{k,i} \approx \frac{(K-1)}{3N} \tag{3.63}$$

Where $r_{k,i}$ is given by the cross-correlation $C_{k,i}(\tau)$ and N is length of the spreading sequence. The signal-to-noise ratio for the ith channel, SNR_i, is given by:

$$SNR_i = \left[\frac{N_0}{2E_b} + Q_a\right]^{-1} \approx \left[\frac{N_0}{2E_b} + \frac{K-1}{3N}\right]^{-1} \tag{3.64}$$

For a single user, $K=1$, the multiple access interference is nil $(Q_a = 0)$ and the

$$SNR = \frac{E_b}{\frac{N_0}{2}}.$$

The probability of error P_e is given by:

$$P_e = Q(\sqrt{SNR}) \quad \text{single user} \tag{3.65}$$

$$= Q(\sqrt{SNR_i}) \quad \text{K users} \tag{3.66}$$

Example 3.10

Three users share a spread-spectrum transmission link. The system performance is to be evaluated while the users are sharing the link. The code sequences for the users are chosen from the family of Walsh-Hadamard sequences of period $N = 64$. The code sequences for the active users are:

{x} is sequence number 7
{y} is sequence number 11
{g} is sequence number 32

{x} = {0110 1001 0110 1001 0110 1001 0110 1001 0110 1001 0110 1001 0110 1001 0110 1001}

{y} = {0110 0110 1001 1001 0110 0110 1001 1001 0110 0110 1001 1001 0110 0110 1001 1001}

{g} = {0000 0000 0000 0000 0000 0000 0000 0000 1111 1111 1111 1111 1111 1111 1111 1111}

The signal power to channel thermal noise is 3 dB.

i. Find the probability of bit error assuming transmission is conducted on an asynchronous link.
ii. What would be the worst-case error probability of the asynchronous transmission?
iii. Find the probability of error when users are transmitting on a synchronous link.

Solution

i. Signal power to channel thermal noise ratio = 3 dB = 2 in ratio

Thus $\frac{2E_b}{N_0} = 2$

$N = 64$

$K = 3$

$$SNR_i = \left(\frac{1}{2} + \frac{2}{3 \times 64}\right)^{-1} = 1.959$$
$$P_e = Q(\sqrt{1.959}) = Q(1.4) = 0.081$$

ii. The highest magnitude of the cross-correlation between any pair of the active users can be shown to be 4.

$$C_c = 4$$
$$N = 64$$
$$K = 3$$

Thus
$$P_{max} = 1 - \Phi\left(\left[1 - \frac{2 \times 2 \times 4}{64}\right] \times \sqrt{2}\right)$$
$$= 1 - \Phi(0.75 \times \sqrt{2}) = 1 - \Phi(1.0607)$$
$$= Q(1.0607)$$

Thus
$$P_{max} = Q(1.0607) = 0.145$$

iii. When transmission is synchronous, the cross-correlation is zero since code sequences are orthogonal at zero time shift. Therefore:

$$P_e = Q(\sqrt{2}) = Q(1.414) = 0.074$$

3.8 Summary

A brief historical background to the development of the spread-spectrum technique and the deployment of a commercial system that implements the spread-spectrum technique was given in Section 3.1. The benefits acquired in the use of such a system were discussed in Section 3.2. The principles of the spread-spectrum techniques were explained in terms of Shannon bandwidth efficiency diagram in Section 3.3. We have also shown the trade-off between the system bandwidth and the channel signal-to-noise power ratio so that a reduction in SNR can be compensated by an increase in the system bandwidth.

After a brief description of the most common types of spread-spectrum systems in Section 3.4, our attention was focused on the direct sequence spread-spectrum system model.

One of the most exciting phenomenon's at the heart of this technique, namely, the processing gain that is introduced through the process of spread–despread scheme was treated in depth in Section 3.5.

Since a spread-spectrum system employs bandwidth exorbitantly, the frequency spectrum is generally shared by a number of users leading to a spread-spectrum system based on CDMA operation.

The other important phenomenon, namely the correlations between different code sequences used by users accessing the system, was treated in more detail in Section 3.6 and information from this section will be cited in forthcoming sections in Chapters 5 and 6.

One of the spread-spectrum factors that are exposed to ongoing research is the choice of family of the spreading code sequences for multiple users. Conventionally this research is carried out by exhaustive computer searches. One of the parameters used for benchmark comparisons of the outcome of the search is the Merit factor which was discussed in Section 3.6.

An important facet of the spread-spectrum technique is its ability to resist interference/jamming so that the desired signal can be recovered with minimum distortion provided that there is enough processing gain to eradicate the effects of the interfering signal. This anti-jamming scheme was defined in terms of the jamming margin in Section 3.6. Finally, a scheme for evaluating the performance of the spread-spectrum system in terms of bit error rate based on Purley's model was introduced in Section 3.7.

Laboratory session III: Introduction to spread-spectrum techniques

The aim of this laboratory is to present a laboratory-intensive understanding of the spread-spectrum techniques and of the main concepts involved in such systems. These concepts are fundamental to direct-sequence multiple access systems such as IS-95 and the wideband code division multiple access used in 3G cellular networks. For background information, please refer to the material in this chapter.

Part A: Simulation of system parameters

Write and RUN MATLAB code to model the following system:

i. Generate random data of (± 1).
ii. Generate user code sequence from 64×64 Walsh matrix.
iii. Perform the spreading process on the data using one out of the 64 Walsh code sequences.
iv. Use FFT, overlay the spectrum of the baseband data and the spread-spectrum signal in the frequency domain.
v. Measure the processing gain from FFT plot and compare this value with computed value of $G_p = N = 64$.

vi. Add Gaussian noise to the system at specified $\frac{E_b}{N_0}$ setting (=3 dB).
vii. Measure the BER for $\frac{E_b}{N_0}$ between 1 to 8 dB in steps of 0.5 dB.
viii. Plot BER vs. $\frac{E_b}{N_0}$ in dB.

Part B: Evaluation of system BER performance

In this part of the laboratory, the signature waveforms are chosen from 31 × 31 Gold code matrix where each user uses 31-chip Gold sequence. The focus is on the distribution of the multiple access interference and its impact on the system BER performance.

1. Gold Sequence matrix

i. Generate 31 ×31 Gold code matrix using MATLAB code.
ii. Extend the M-file to compute the Autocorrelation function for shift delay (τ) between 0 to 31 chip in a step of one chip duration.
iii. Plot the magnitude of the Autocorrelation function versus τ.
iv. Modify the M-file in (ii) to compute the cross-correlation function between code sequence 1 and code sequence 4 for shift delay (τ) between 0 to 31 chip in a step of one chip duration.
v. Plot the magnitude of the cross-correlation function versus τ.

2. Distribution of multiple access interference

i. Write a MATLAB code to generate transmitter signal for k users sharing the spread-spectrum channel.
ii. Set k = 31 so you have one desired user (say user 1) and interference from 30 users.
iii. Compute the interfering signal for each transmission cycle defined by: Interference = estimated data – transmitted data.
iv. Run (iii) for 4000 data symbols.
v. Plot the distribution (Histogram) of the multiple access inference.

3. Simulation of BER of multiple access spread-spectrum system

i. Modify MATLAB in 2.i to vary k from 1 up to 31 in a step on 1.
ii. Measure BER of the system for each value of k. Assume input noise power set to zero.
iii. Plot BER versus number of users.
iv. Plot theoretical BER, calculated from equations in Section 3.7, on the same graph of (iii).
v. Comment of your results.

Problems

3.1 Binary data is transmitted at the rate of 16.6 kb/s over a Gaussian channel that has a signal-to-noise power ratio (SNR) equal to 2.7 dB. Channel coding is employed to ensure error-free data detection with a matched filter receiver. If the channel SNR is reduced by 1.5 dB, find the new bandwidth required to keep the transmission rate constant.

3.2 A direct sequence spread-spectrum system is transmitting binary data at a bit rate of 9.6 kb/s and code sequence chip rate of 1.22 Mc/s. At the input of the matched filter receiver, the received power is 3.7 μW and the noise spectral density N_0, is 0.65 μW. Find the signal-to-noise ratio at the input of the receiver.

3.3 A frequency hopping spread-spectrum system is transmitting binary data at the rate of 4.8 kb/s. The system operates with 4 hops per data bit and final frequency is multiplied by 8. The processing gain is 36 dB. Find the:
(a) number of hopping channel frequencies used. Assume this number to be a power of 2.
(b) bandwidth of the frequency-hopped signal.

3.4 A hybrid system, consisting of a direct sequence and a frequency hopping spread-spectrum system, is transmitting binary data at the rate of 9.6 kb/s and chip rate 1.228 Mc/s. The number of hopping frequencies used is 1024 and the system hops once every time a single symbol is being transmitted. Find the:
(a) system processing gain.
(b) bandwidth of the system.

3.5 A group of 23 users are sharing a direct sequence spread-spectrum link. Each user transmits at a binary bit rate of 9.6 kb/s using a binary PSK carrier. If the users transmit at equal power and the additive Gaussian channel noise is to be ignored, what chip rate of the code sequence is required to ensure a bit error probability not greater than 10^{-3}.

3.6 A group of 13 equal-power users are sharing a direct sequence spread-spectrum channel and transmitting binary data at the rate of 12.6 kb/s on a binary PSK carrier. The system is operating at a chip rate of 1.2 Mc/s.
(a) Compute the processing gain.
(b) Find the ratio of energy per bit to power spectral density of the access interference $\left(\frac{E_b}{J_0}\right)$.
(c) If the number of users is to double, keeping the output signal-to-noise ratio (SNR) fixed, how much should the processing gain be increased?

3.7 A group of 27 users is sharing a direct sequence spread-spectrum channel. The signal to Gaussian noise power ratio is 2.5 dB. Each user is using a code sequence of period 128 and chip rate of 1.22 Mc/s. Find the:
(a) maximum magnitude of cross-correlation allowed if the worst-case probability of error is not to exceed 10^{-3}.
(b) average probability of error.
(c) increase in processing gain required to keep the probability of error equal to the value determined by the channel noise only.

3.8 Binary data is transmitted at the bit rate of 9.6 kb/s over a spread-spectrum channel that employs a code sequence of chip rate 1.228 Mc/s. The received signal at the input of a matched filter receiver is 3.1 μW. The noise power spectral density, N_0, is 2.9 μWHz, and the signal-to-noise ratio at the receiver output, $(SNR)_0$, is 27 dB. Find the:

(a) ratio of energy per bit to jamming power spectral density, $\frac{E_b}{J_0}$, assuming that the jamming power spectral density is uniform over the system bandwidth.

(b) jamming margin of the system.

3.9 Consider the following two Walsh-Hadamard sequences {A} and {B}, each of period $N = 8$ where:

$$\{A) = \{0101\ 1010\}$$
$$\{B\} = \{0011\ 1100\}$$

Find the:

(a) periodic autocorrelation and periodic cross-correlation of both sequences.

(b) aperiodic autocorrelation and the aperiodic cross-correlation of both sequences.

(c) even and odd cross-correlation.

3.10 Consider the following two Walsh-Hadamard sequences, each with period $N = 8$ where sequences {a} and {b} are given by:

$$\{a\} = \{0011\ 0011\}$$
$$\{b\} = \{0110\ 0110\}$$

Find the:

(a) merit factor for each sequence.

(b) DFT of the cross-correlation and thus show that it can be expressed in terms of the DFT of both sequences.

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