Chapter Three

Ray optics

Ray optics is the simplest theory of light. Here light is described in terms of rays, and we ignore the wave and photon character of the light. We also do not bother about the nature of light, but are only interested to understand how it behaves on a large scale. The optics of light rays involves only geometric considerations and the laws are formulated using the language of geometry. Therefore, ray optics is also called **geometrical optics.** It is useful in studying image formation by mirrors, lenses and the working of many optical instruments and devices. However, many aspects of the behavior of light cannot be explained on the basis of ray theory; for example, we cannot explain the phenomenon of interference, diffraction, and polarization using ray concept.

Ray optics is based mainly on the following three simple laws:

1. Law of rectilinear propagation of light:

It is a common sight that an object kept in the path of light coming from a point source produces a sharp shadow. **The law of rectilinear propagation states that *in an optically homogeneous medium light propagates in a straight line*.** A medium is said to be ***optically homogeneous*** if its refractive index is everywhere the same.

2. Law of independence of light rays:

Light rays do not disturb one another when they intersect. If several rays are passing through a medium simultaneously in different directions, then the path of any ray is the same as it would be if all others were absent. For example, when we view an object, light rays passing in other directions does not obstruct the light that comes to us from the object.

3. Law of reversibility of path:

If the path of a light ray is reversed, it will exactly retrace its path, irrespective of the number of reflections and refractions.

Reflection and Refraction

3.1 Reflection at Plane Surfaces (Mirrors)

Any smooth surface acts as a mirror. A mirror may be ***plane* or *curved***. Mirrors were usually made in the past, by coating glass with **silver**. Nowadays, they are made by depositing in vacuum a thin film of aluminum on a polished surface. The reflecting film is protected by deposition of a thin layer of **silicon monoxide or magnesium fluoride** over it. The mirrors used for technological purposes are coated on the front surface so that losses in energy due to transmission through the substrate material are reduced.

3.1.1. Object and Image

An *object* is anything from which light rays radiate. This light could be emitted by the object itself or reflected from it. When light rays proceeding from an object are reflected at a mirror, they *converge to* or *appear to diverge from* a position *different* from that of the original object and give the impression of object being there.

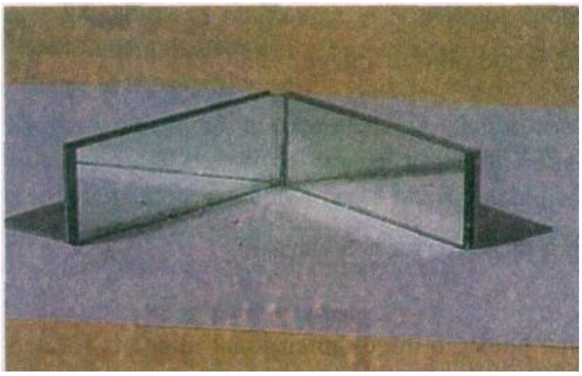
The *apparent object* as seen by the observer is called the *image* of the object.

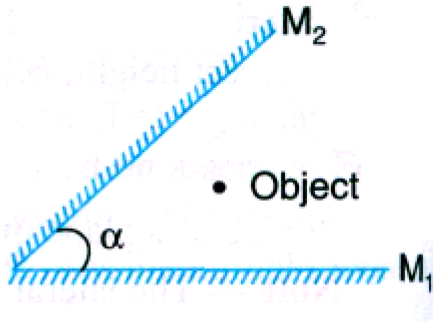
The image formed by an optical component may be **real or virtual**.

	real image	***virtual image***
1	A *real image* contains light energy	A *virtual image dose not contains* light energy
2	The image can be received and seen on a screen.	A *virtual image* cannot be received on a screen.
3	The light rays pass through the real image.	The light rays never actually pass through the virtual image.
4	The real image can be photographed by simply placing a photographic film at the position of the image.	It cannot be recorded on a photographic plate placed at its position.

3.1.2. Multiple Plane Mirrors

If an object is held between two or more plane mirrors, I multiple virtual images are formed because of multiple reflections of light from the mirrors.





The number of images formed depends upon the angle between the mirrors. Let the angle between the mirrors be α , then the number of images, N is given by

$$N = \frac{360^{\circ}}{\alpha} - 1 \quad \text{.................(3.1)}$$

For a pair of parallel mirrors α =0° N is infinity and therefore, infinite number of images will be formed.

3.1.3. Properties of Images in Plane Mirrors

We now summarize the properties of images in plane mirrors:

(a) Image formed by a plane mirror is virtual and erect.

(b) The image is as far behind as the object is in front of it.

(c) The right side of the object is transformed into left side of the image.

(d) Magnification produced by a plane mirror is unity. Hence, the image is as large as the object.

(e) When a plane mirror is rotated through a certain angle, the reflected ray turns through twice the angle.

(f) If an object is held between two or more plane mirrors, multiple virtual images are formed.

3.2 Reflection at Spherical Mirrors

Spherical mirrors are of the simple type among the curved mirrors. A **spherical mirror** is a segment of a spherical surface and usually has circular edges. There are two types of spherical mirrors- ***concave*** **and** ***convex*** mirrors.

Concave Mirror: the reflection takes place from the inner surface of the spherical segment.

Convex Mirror: the light is reflected from the outer of the surface of the spherical segment.

3.2.1. Basic Terms

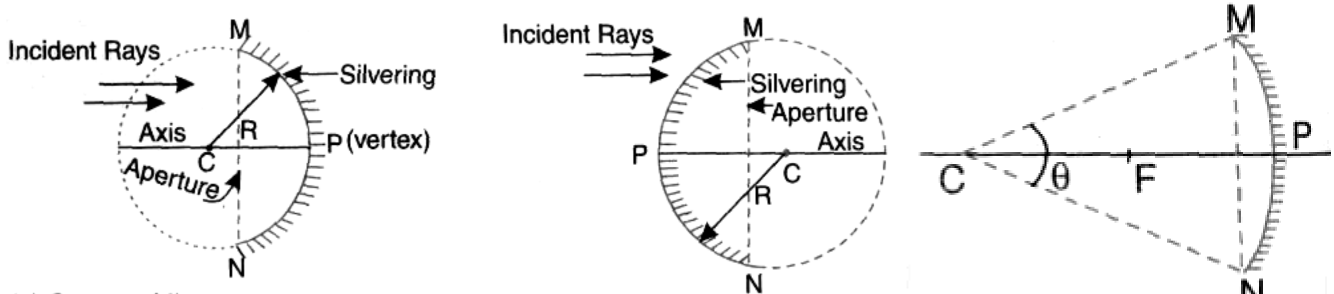
The basic terms associated with spherical mirrors are as follows:

(i) The **centre of curvature, C** is the centre of the sphere of which the mirror is a small segment;

(ii) The **vertex (or pole), P** is the midpoint of the mirror.

(iii) The **radius of curvature**, **R** is the radius of the sphere of which the mirror is a small section.

(iv) The **principle axis** is the line passing through the center of curvature C and the vertex P.



(a) Concave Mirror (b) Convex Mirror (c)

(v) With a concave mirror, all rays parallel to the principal axis pass through a single point F after reflection. F is called the **principal focus.** The parallel rays converge to F after reflection. However, in case of a convex mirror, the reflected rays *appear* to emerge from F behind the mirror. Thus, the reflected rays *appear to diverge* from the principal focus.

(vi) **Focal plane** is a plane passing through the point F and perpendicular to the

principal axis.

(vii) The **focal length *f* of** the mirror is the distance, PF, between the vertex P and the principal focus F. It will be shown later that:

$$f = R/2 \quad \text{.............(3.2)}$$

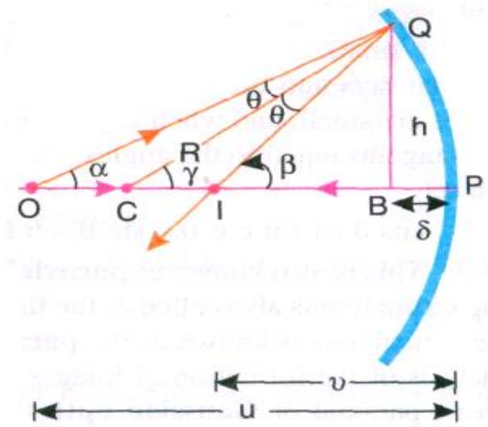
(viii) The reciprocal of the focal length is called the **power** of the mirror. Thus,

$$P = 1/f \quad \text{..........(3.3)}$$

(ix) The diameter MN of the circular outline of the mirror is called the **aperture** of the mirror.

3.4.4. Spherical Mirror Equation

The figure shows a concave mirror with radius of curvature R. The ***centre of curvature*** of the surface is at C and the ***vertex*** of the mirror is at P. The line CP is the ***optic axis***. Point **O** is an object point that lies on the optic axis and we assume that the distance from **O** to **P** is greater than **R**.



Let us now determine the location of the image point **I**. The basic equation governing spherical mirrors relates three lengths, namely **object distance, *u*, image distance *v*, and the radius of curvature of the mirror, R.** An exterior angle of a triangle equals the sum of the two opposite interior angles.

$$\gamma = \alpha + \theta \quad \text{............ (3.4)}$$

Applying this theorem to the triangle OQC, we have

$$\beta = \alpha + 2\theta \quad \text{........... (3.5)}$$

Eliminating angle θ from these equations, we get:

$$\alpha + \beta = 2\gamma \quad \text{............. (3.6)}$$

Let **h** represent the height of point **Q** and let δ represent the short distance BP.

From $\Delta^{le}OQB$, $\tan\alpha = \frac{h}{u-\delta}$

From $\Delta^{le}IQB$, $\tan\beta = \frac{h}{\upsilon-\delta}$ and

From $\Delta^{le}CQB$, $\tan\gamma = \frac{h}{R-\delta}$

By taking the approximation, the angle $\alpha, \beta and \gamma$ very small and we can replace $\tan\alpha$ by α .

When α is small we can also neglect the distance δ in comparison to u, v and R

$$\tan\ \alpha \cong \alpha = \frac{h}{u} \quad ; \quad \tan\ \beta \cong \beta = \frac{h}{\upsilon} \quad \text{and} \ \tan\ \gamma \cong \gamma = \frac{h}{R}$$

Using values of $\alpha, \beta\ and\ \gamma$ into the equation (3.6) we get the spherical mirror equation.

$$\frac{1}{u} + \frac{1}{\upsilon} = \frac{2}{R} \quad \text{………….. (3.7)}$$

Note:

(i) The initial equation (3.7) is a ***numerical relationship*** between the various distances for a particular type of spherical mirror.

(ii) We have to bear in mind that u, v, and R are algebraic quantities.

3.4.4.1. Focal Point and Focal Length

If the object O is taken at a very large distance from the mirror ($u=\infty$), the incoming rays become almost parallel. Then, the image distance *v* is given by

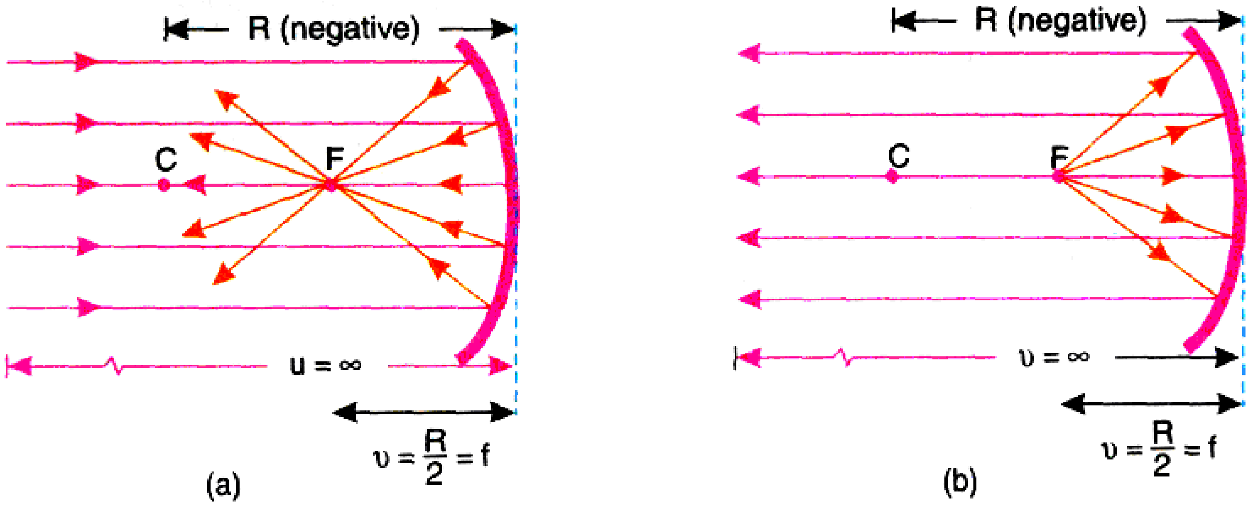
$$\frac{1}{\infty} + \frac{1}{v} = \frac{2}{R} \quad \text{Therefore} \quad f = \frac{R}{2}$$

Focal Point: The point **F** at which the incident parallel rays converge is called **focal point.**

focal length: The distance from the vertex of (lens or mirror) to the focal point is called **focal length** and is denoted by ***f***. the focal length of a concave spherical mirror is half the radius of curvature.

Now let us look at the opposite situation. If the object is placed at the focal point F, the object distance $u = R/2$ the image distance is given by

$$\frac{2}{R} + \frac{1}{\upsilon} = \frac{2}{R} \quad \text{or} \quad \frac{1}{\upsilon} = 0, \quad \text{therefore } \upsilon = \infty.$$



We usually express the relationship between object and image distances in terms of the focal length *f*

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \ldots\ldots\ldots\ldots\ldots \quad (3.8)$$

This is known as the Gauss formula for a spherical mirror.

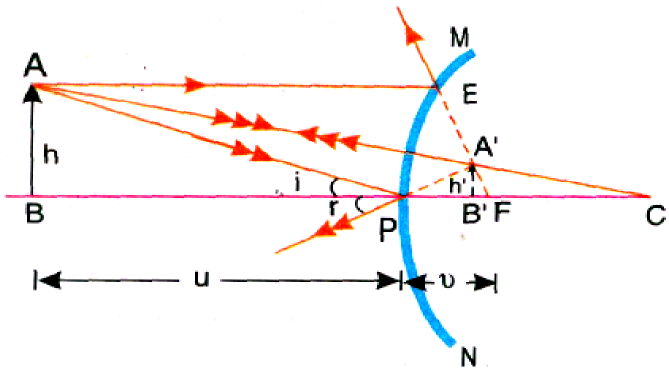
3.4.5. Lateral Magnification

The ratio of the transverse dimensions of the final image formed by an optical system to the corresponding dimension of the object is defined as the **transverse or lateral magnification**. It is noted that the object and image have different sizes and that they have opposite orientations. Because triangles ABP and A' B' P are similar, we have

$$\frac{h}{u} = -\frac{h'}{v} \qquad\qquad \therefore m = \frac{h'}{h} = -\frac{v}{u} \quad \ldots\ldots\ldots\ldots \quad (3.9)$$

3.4.8. Convex Mirror

The figure below shows the formation image with an extend object. The object AB is in front of the mirror and a virtual and erect image $A'B'$ is formed behind the mirror. The triangles ABC and $A'B'C$ are similar, we have:



$$\frac{AB}{A'B'} = \frac{CB}{CB'} \qquad \text{................(3.10)}$$

Also, the triangles ABP and A′ B′ P are similar. Hence we have

$$\frac{AB}{A'B'} = \frac{PB}{PB'} \qquad \text{...........(3.11)}$$

From equation (3.10) and (3.11) we get:

$$\frac{CB}{CB'} = \frac{PB}{PB'} \quad \text{or} \quad \frac{PB + PC}{PC - PB'} = \frac{PB}{PB'}$$

$$\therefore \quad \frac{u + R}{R - \upsilon} = \frac{u}{\upsilon}$$

or $$\upsilon R - uR = -2u\upsilon$$

Dividing the above equation throughout by $u\upsilon R$, we obtain

$$\frac{1}{u} - \frac{1}{\upsilon} = -\frac{2}{R} \qquad \text{.............. (3.12)}$$

Using the sign convention, $u = -ve$, υ = +ve and R = +ve, we get

$$\frac{1}{-u} - \frac{1}{\upsilon} = -\frac{2}{R}$$

or $$\frac{1}{u}+\frac{1}{\upsilon}=\frac{2}{R}$$ ………….. (3.13)

$$\frac{1}{u}+\frac{1}{\upsilon}=\frac{1}{f}$$ …………….. (3.14)

3.4.9. Spherical Mirror Equation Applied to a Plane Mirror

In case of a plane mirror, we may treat that **R** tends towards infinity. Using this value of **R** into the mirror equation (3.13), we get

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{\infty}=0$$
$$\therefore u=-v$$

This result confirms our conclusion about the object-image relationship in case of a plane mirror.

3.4.10. Lateral Magnification

It is seen from Figure (convex mirror) that the object and image have different sizes but both are erect. Because triangles ABP and A' B' P are similar, we have

$$\frac{h}{u}=\frac{h'}{v}$$

$$\therefore m=\frac{h'}{h}=\frac{v}{u}$$ ………….. (3.15)

3.5 Graphical Method

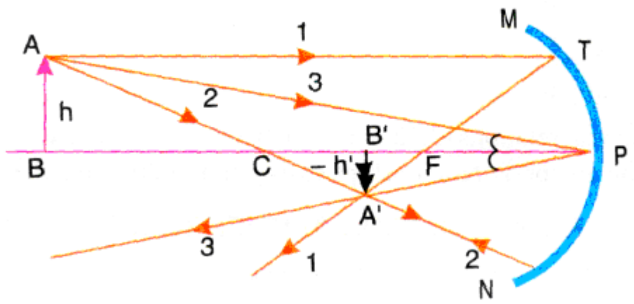
We can determine the properties of the image by a simple ***graphical* method**. This method consists of finding the point of intersection of a few particular rays that diverge from a point of the object and are reflected by the mirror. Then *all* rays from this point that strike the mirror will intersect at the same point. There are four rays, which are useful in locating the corresponding image point. These are called **principal rays or easy rays.**

(a) A ray parallel to the principal axis, after reflection, passes through the focus F in case of concave mirror; or appears as though it came from the focal point F in case of convex mirror **(Ray 1 in Fig.).**

(b) A ray heading towards or away from the centre of curvature C is reflected back along its original path **(Ray 2 in Fig.)**.

(c) A *ray incident at vertex* P at an angle *i* is reflected at an angle *i* **(Ray 3 in Fig.)**.

(d) A ray through the focal point F is reflected parallel to the axis **(Ray 1 in Fig.)**.



3.5.1. Effect of Altering the Object Distance

We will now consider the location of an image graphically when the object lies at various distances from the mirror. In each case we shall find out the position, nature and relative size of the image,

(i) **Object at infinity:**

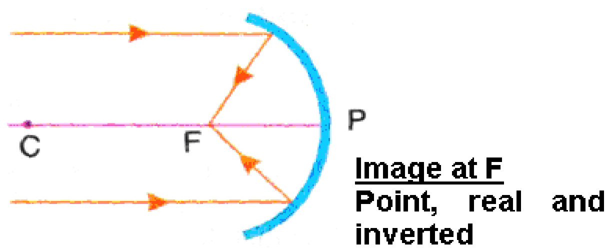
The graphic construction is shown. Since the object is at infinity, the incident rays become parallel. Two such rays are shown in the figure, one along the principal axis and the other parallel to it. The ray parallel to optic axis is incident on the mirror and the reflected ray passes through F.

The two reflected rays intersect at F and the image is formed at point F.

Here, the object distance $u = \infty$ and image distance $\upsilon = f$.

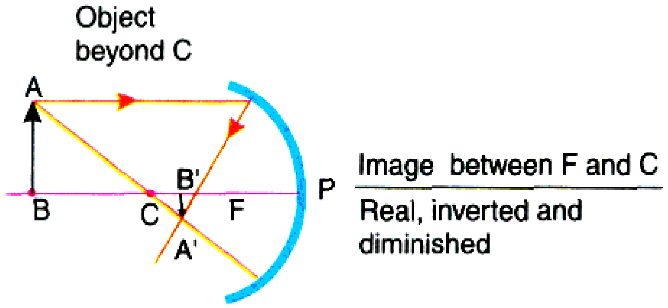
Magnification $m = \frac{\upsilon}{u} = \frac{f}{\infty} = 0$ (i.e. the image is very small in size).

Object at Infinity



(*ii*) Object between infinity and point C:

It is seen from the Fig. that the image has finite dimensions. It is real, inverted and smaller in size than the object. It lies between C and F.

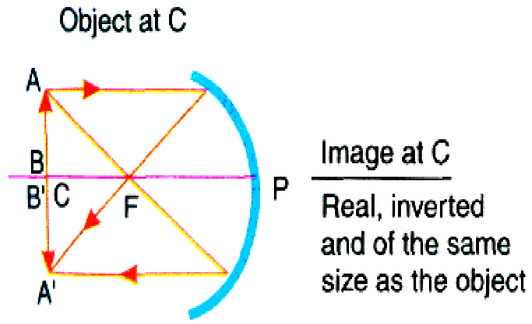


$$\text{Magnification } m = \frac{\upsilon}{u} < 1$$

(*iii*) Object at C:

In this case, the image lies at the centre of curvature. It is real and inverted and of the same size as the object.

$$\text{Magnification } m = \frac{\upsilon}{u} = 1$$

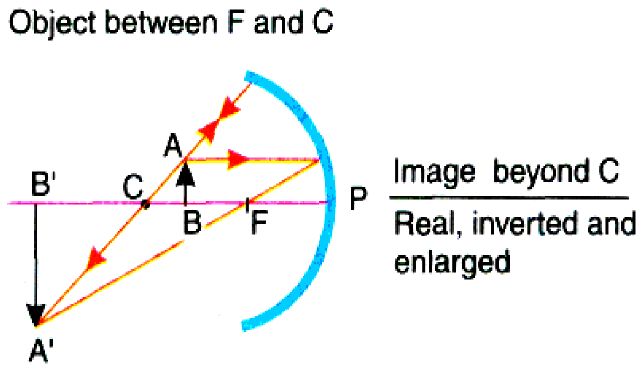


(*iv*) Object between C and F:

The image in this case lies between C and infinity. It is inverted, and real, as shown in Fig.

$$\text{Magnification } m = \frac{\upsilon}{u} > 1$$

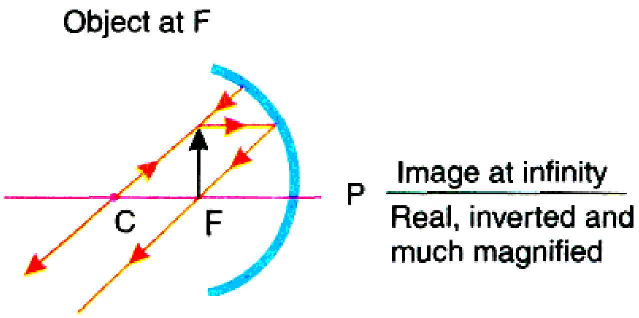
Therefore, the image is larger in size than the object.



(*v*) Object at Focus F:

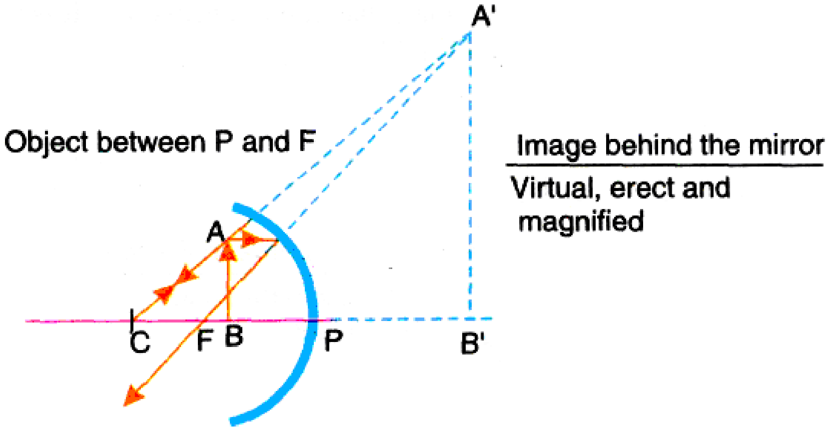
The image forms at infinity, as shown in Fig.

$$\text{Magnification } m = \frac{\upsilon}{u} = \frac{\infty}{f} = \infty$$

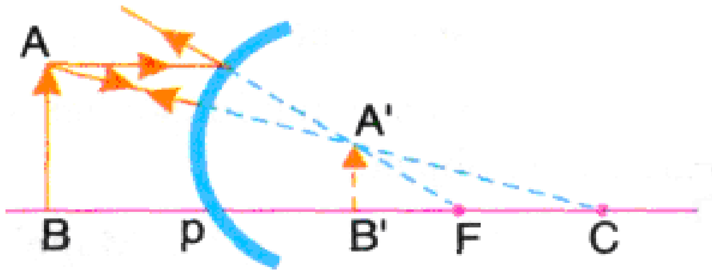


(vi) Object between F and P:

In this case, the image is virtual and erect. It is magnified and lies on the other side of the mirror, as shown in Fig.



In a **convex mirror**, no matter where the object is situated in front of the mirror, **the image is formed behind the mirror. The image is virtual, erect, diminished and is always formed between the vertex P and the focus F**.



3.6 Refraction of Light

3.6.1 Refraction Through a Glass Slab

A light ray that travels through a parallel-sided transparent slab (e.g. plate glass window) is refracted at both faces. The two refractions at the parallel surfaces result in a 'sideways' displacement, but do not change the direction of the ray. To show that there is no change in direction,

we can apply Snell's law at both faces.

At the first face, the Snell's law takes the form

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

Because the faces are parallel, the angle of incidence for the second face equals θ_2 and according to Snell's law

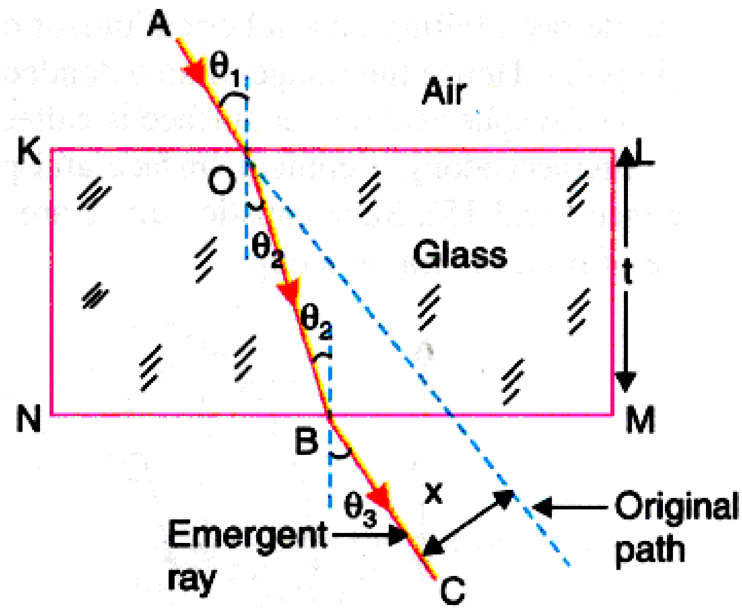
$$\mu_2 \sin\theta_2 = \mu_1 \sin\theta_3$$

By comparing the two equations we see that $\theta_1 = \theta_3$. When a ray of light passes through a parallel-sided slab, the emergent ray is parallel to the incident ray.

The distance *x* between the incident and emergent ray is given by:

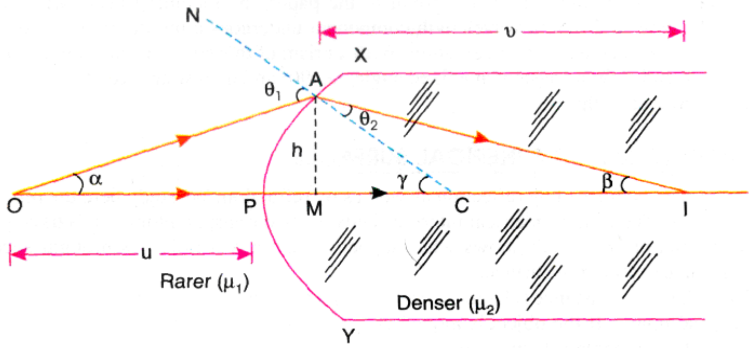
$$x = \frac{t \cdot \sin(\theta_1 - \theta_2)}{\cos \theta_2}$$

where t is the thickness of the glass slab.



3.6.2 Refraction at Spherical Surfaces (convex surface)

From the figure, a convex surface XY with radius R forms an interface between two materials with different indexes of refraction $\mu_1 and \mu_2$. Let **C** be the centre of curvature and **P** the pole of the spherical surface. Consider a point object **O** lying on the principal axis in the rarer medium of refractive index μ_1 at a distance ***u*** from the vertex in front of the convex surface; the distance ***u*** being greater than the radius of curvature **R** of the surface. Ray **OP** travelling along the principal axis strikes the vertex **P** and passes into the second medium without deviation. Ray **OA**, making an angle α with the axis, is incident on the surface at an angle θ_1, with the normal and is refracted along **AI** at an angle θ_2. These rays intersect at **I** at a distance ***v*** to the right of the vertex. The figure is drawn for $\mu_1 \langle \mu_2$.



We use the theorem that an exterior angle of a triangle equals the sum of the two opposite interior angles. Applying this to the triangles OAC and ACI gives

$$\theta_1 = \alpha + \gamma, \quad \gamma = \beta + \theta_2 \quad ....(*)$$

From the law of refraction, $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

Also the tangents of α, β, and γ are

$$\tan\alpha = \frac{h}{u+\delta}, = \tan\beta = \frac{h}{\upsilon-\delta}, \tan\gamma = \frac{h}{R\text{-}\delta}$$

For paraxial rays, θ_1 and θ_2 are both small in comparison to a radian, and we may approximate both the sine and tangent of either of these angles by the angle itself. The law of refraction then gives

$$\mu_1\theta_1 = \mu_2\theta_2 \quad ....(**)$$

Combining this with the first of equation (*), we get

$$\theta_2 = \frac{\mu_1}{\mu_2}(\alpha+\gamma)$$

Substituting this into second of eq. (*), we obtain

$$\mu_1\alpha + \mu_2\beta = (\mu_2 - \mu_1)\gamma$$

Now we use the approximations tan α = α and so on and also neglect the small distance δ.

Thus, $$\alpha = \frac{h}{u}, \quad \beta = \frac{h}{\upsilon}, \quad \text{and} \quad \gamma = \frac{h}{R}.$$

Finally, we substitute these into Eq. (**) and divide out the common factor *h*. We obtain

$$\frac{\mu_1}{u} + \frac{\mu_2}{\upsilon} = \frac{\mu_2 - \mu_1}{R}$$

We now apply the sign convention: *u* is -ve, υ is +ve and R is +ve. It gives us

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad ....(3.16)$$

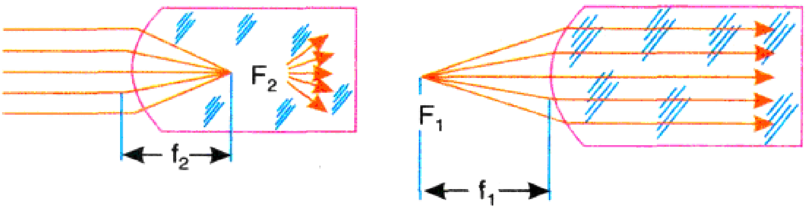
This is the **Gaussian relation** for a single spherical surface.

If the first medium is air and the second medium is of refractive index μ then

$\mu_1 = 1 \; and \; \mu_2 = \mu$ $$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$$

3.6.2.1. Principal Foci

We can use equ. (3.16) to determine the first and second principal foci **F_1**, is the position of the object for which the image lies at infinity, while **F_2** is the position of the image for which the object lies at infinity.



First principal focus:

If the object is placed at **F_1** such that the refracted rays become parallel to the principal axis. The object distance ***u*** is denoted by ***f_1*** and is called the **first principal focal length** of the refracting surface.

$$\frac{\mu_2}{\infty} - \frac{\mu_1}{f_1} = \frac{\mu_2 - \mu_1}{R}$$

$$\therefore \; f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1}$$

The position of the axial point object whose image is formed at infinity is known as the **first principal focus** of the refracting surface.

Second principal focus:

If the object is at infinity, then $u = \infty$ and $1/u = 0$. Let f_2 be the value of υ corresponding to $u = \infty$.

Then $$\frac{\mu_2}{f_2} = \frac{\mu_2 - \mu_1}{R}$$

or $$f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$$

This value of $v = f_2$ is termed as the **second principal focal length** and the position of tie image corresponding to the object at infinity is called the **second principal focus** of the refracting surface.

It is easy to see that **the first focal length f_1, for the spherical refracting surface is** not **equal to the second focal length f_2.**

In case of a convex surface R is positive and f_1 is negative and f_2 is positive. On the other hand, for a concave surface R is negative and hence f_1 is positive and f_2 is negative.

3.6.2.2 Various Relations Involving f_1 and f_2

(*i*) The sum of the two focal lengths equals the radius of curvature.

$$f_1 + f_2 = \left[\frac{-\mu_1}{\mu_2 - \mu_1}\right]R + \left[\frac{\mu_2}{\mu_2 - \mu_1}\right]R = R$$

(*ii*) We see that using relation (3.18) into spherical mirror equation (3.16) and (3.17) we get

$$\frac{\mu_1}{u} - \frac{\mu_2}{\upsilon} = \frac{\mu_1}{f_1}$$

This is known as **Gauss' formula for a spherical surface.**

(*iii*) We may write

$$\frac{f_1}{\mu_1} = \frac{-R}{\mu_2 - \mu_1}, \quad \frac{f_2}{\mu_2} = \frac{R}{\mu_2 - \mu_1}$$

Adding, $$\frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} = 0 \quad \text{or} \quad \frac{f_1}{f_2} = -\frac{\mu_1}{\mu_2}$$

As μ_1 and μ_2 are always positive and unequal, f_1 and f_2 are always of opposite sign and are unequal. The ratio of focal lengths f_1/f_2 is equal to the ratio μ_1/μ_2 of the corresponding refractive indices.

(*iv*) We may write $$\frac{\mu_2}{\upsilon} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

as $$\frac{\mu_2 R}{\upsilon(\mu_2 - \mu_1)} - \frac{\mu_1 R}{u(\mu_2 - \mu_1)} = 1$$

$\therefore$ $$\frac{f_2}{\upsilon} + \frac{f_1}{u} = 1$$

3.6.2.3 Power

The refracting power of a spherical surface is defined as the inverse of the reduced focal length. Any distance divided by the refractive index of the space in which it is measured is called a reduced distance.

the ratios f_1/μ_1 and f_2/μ_2 are the reduced focal lengths. Since $\frac{f_1}{\mu_1} = -\frac{f_2}{\mu_2}$, the refracting power is given by

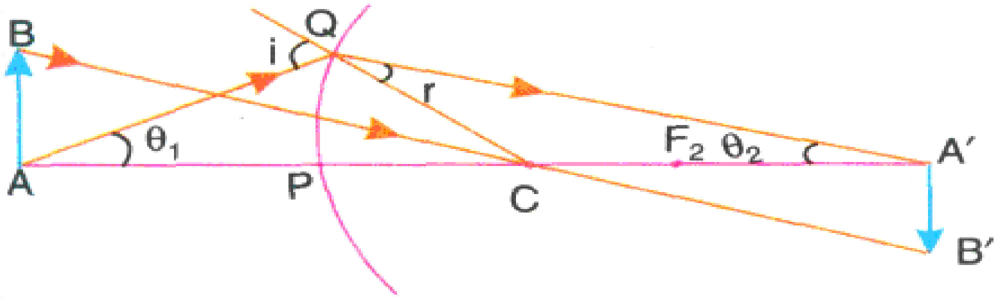
$$P = \frac{\mu_1}{f_1} = -\frac{\mu_2}{f_2}$$

$\therefore$ $$P = \frac{\mu_2 - \mu_1}{R}$$

Power is measured in **diopters**.

3.7 Smith-Helmholtz Equation and Lagrange Law

From the below figure, A'B' is the real inverted image of the object AB. The incident ray AQ after refraction passes through A'. Let the refracted ray QA' be inclined at an angle 0_2 to the axis in the paraxial region.



Angular magnification $\alpha = \frac{\tan\theta_2}{\tan\theta_1}$

or $\alpha = \frac{QP/\upsilon}{AP/u} = \frac{u}{\upsilon}$

Linear magnification $m = \frac{h_2}{h_1} = \frac{\mu_1}{\mu_2}\cdot\frac{\upsilon}{u}$

$\therefore$ $\frac{h_2}{h_1}\cdot\frac{u}{\upsilon} = \frac{\mu_1}{\mu_2}$

or $\frac{h_2}{h_1}\cdot\frac{\tan\theta_2}{\tan\theta_1} = \frac{\mu_1}{\mu_2}$

Thus, linear magnification × angular magnification = μ_1/μ_2.

$\therefore$ $\mu_1 h_1 \tan\theta_1 = \mu_2 h_2 \tan\theta_2$

This is known as *Smith-Helmholtz equation.*

Further, for paraxial rays, $\tan\theta_1 \cong \theta_1$ and $\tan\theta_2 \cong \theta_2$. Therefore, the above relation may be expressed as

$$\mu_1 h_1 \theta_1 = \mu_2 h_2 \theta_2$$

This is known as **Langrange's law**.

3.8 ABBE'S Sine Condition

Referring to Fig. let i and r be the angles of incidence and refraction for the rays AQ and QA′.

Now $$\frac{\sin i}{\sin \theta_1} = \frac{AC}{QC} \text{ and } \frac{\sin r}{\sin \theta_2} = \frac{CA'}{QC}$$

$$\frac{\sin i}{\sin r} = \frac{AC}{CA'} \cdot \frac{\sin \theta_1}{\sin \theta_2}$$

But $$\mu_1 \sin i = \mu_2 \sin r$$

∴ $$\mu_1 \cdot AC \sin \theta_1 = \mu_2 \cdot CA' \sin \theta_2$$

Δ^{les}ABC and A′B′C are similar.

$$\frac{CA'}{AC} = \frac{A'B'}{AB} = \frac{h_2}{h_1}$$

$$\mu_1 h_1 \sin \theta_1 = \mu_2 h_2 \sin \theta_2$$

The above condition is known as *Abbe's sine condition.*

Example 3.2: A point light source lies on the principal axis of a concave spherical mirror with radius of curvature 160 cm. Its vertical image appears to be at a distance of 70 cm from it. Determine the location of the light source.

Solution: $$\frac{1}{u} + \frac{1}{\upsilon} = \frac{2}{R}$$ Here $\upsilon = 70$ cm, $R = -160$ cm

$$\frac{1}{u} = \frac{2}{R} - \frac{1}{\upsilon}$$

∴ $$\frac{1}{u} = \frac{2}{-160\,\text{cm}} - \frac{1}{70\,\text{cm}} = -\frac{15}{560\,\text{cm}}$$

∴ $$u = -\frac{560}{15}\,\text{cm} = -37 \text{ cm}$$

The light is at a distance of 37 cm in front (to the left of the vertex) of the mirror.

Example 3.3: A point source of light is located 20 cm in front of a convex mirror with f = 15 cm. Determine the position and character of the image point.

Solution: $$\frac{1}{u} + \frac{1}{\upsilon} = \frac{1}{f}$$ Here u = – 20 cm, f = 15 cm.

$$\frac{1}{\upsilon} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15\,\text{cm}} - \frac{1}{-20\,\text{cm}} = \frac{35}{300\,\text{cm}} = \frac{7}{60\,\text{cm}}$$

$$\upsilon = 8.6 \text{ cm}$$

As υ is positive, the image is located behind (to the right side of the vertex of) the mirror. Hence, the image is *virtual*.

Example 3.4: A concave spherical surface of radius of curvature 100 cm separates two media of refractive indices 1.50 and 4/3. An object is kept in the first medium at a distance of 30 cm from the surface. Calculate the position of the image.

Solution: $\mu_1 = 1.50$, $\mu_2 = 4/3$, $u = -30$ cm, $R = -100$ cm

$$\frac{\mu_2}{\upsilon} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{4/3}{\upsilon} - \frac{1.50}{-30\,\text{cm}} = \frac{4/3 - 1.50}{-100\,\text{cm}}$$

$$\boldsymbol{\upsilon = -27.58\ \text{cm}}$$

Example 3.6: A convex surface of radius of curvature 40 cm separates two media of refractive indices 4/3 and 1.50. An object is kept in the first medium at a distance of 20 cm from the surface. Calculate the position of the image.

Solution: $\mu_1 = 4/3$, $\mu_2 = 1.50$, $u = -20$ cm, $R = 40$ cm

$$\frac{\mu_2}{\upsilon} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.50}{\upsilon} - \frac{4/3}{-20\,\text{cm}} = \frac{1.50 - 4/3}{40\,\text{cm}}$$

$$\boldsymbol{\upsilon = -24\ \text{cm}}$$

Example 3.7: A convex refracting surface of radius of curvature 15 cm separates two media of refractive indices 4/3 and 1.50. An object is kept in the first medium at a distance of 240 cm from the surface. Calculate the position of the image.

Solution: $\mu_1 = 4/3$, $\mu_2 = 1.50$, $u = -240$ cm, $R = 15$ cm

$$\frac{\mu_2}{\upsilon} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.50}{\upsilon} - \frac{4/3}{-240\,\text{cm}} = \frac{1.50 - 4/3}{15\,\text{cm}}$$

$$\boldsymbol{\upsilon = 270\ \text{cm}}$$

A real image forms in the second medium at a distance of 270 cm from the refracting surface.

Example 3.8: The eye can be regarded as a single spherical refracting surface of radius of curvature of cornea 7.8 mm, separating two media of refractive indices 1.00 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

Solution: $\mu_1 = 1.00$, $\mu_2 = 1.34$, $u = -\infty$, $R = 0.78$ cm

$$\frac{\mu_2}{\upsilon} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.34}{\upsilon} - \frac{1.00}{-\infty} = \frac{1.34 - 1.00}{0.78\,\text{cm}}$$

$$\boldsymbol{\upsilon = 3.075\ \text{cm}}$$

A real image forms in the second medium at a distance of 3.075 cm from the refracting surface.

Example 3.11: A small filament is at the centre of a hollow glass sphere of inner and outer radii 8 cm and 9 cm respectively. The refractive index of glass is 1.50. Calculate the position of the image of the filament when viewed from outside the sphere.

Solution: For refraction at the first surface, $\mu_1 = 1$, $\mu_2 = 1.50$, $u = -8$ cm, $R = -8$ cm

$$\frac{\mu_2}{\upsilon} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{\upsilon'} - \frac{1.00}{-8 \text{ cm}} = \frac{1.50 - 1.00}{-8 \text{ cm}}$$

$$\boldsymbol{\upsilon' = -8 \text{ cm}}$$

It means that due to the first surface the image is formed at the centre of the sphere.

For the second surface, $\mu_1 = 1.50$, $\mu_2 = 1$, $u = -9$ cm, $R = -9$ cm

$$\frac{\mu_2}{\upsilon} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{\upsilon} - \frac{1.50}{-9 \text{ cm}} = \frac{1 - 1.50}{-8 \text{ cm}}$$

$$\boldsymbol{\upsilon = -9 \text{ cm}}$$

Hence, the final image is formed at the centre of the sphere.

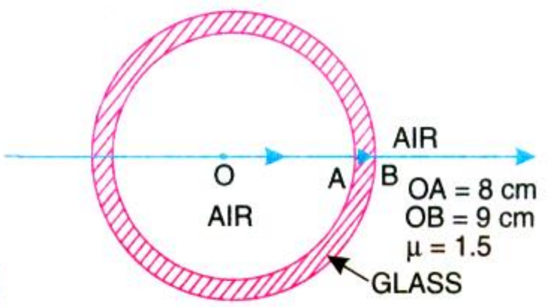


Fig. 3.40

QUESTIONS

1. Discuss refraction at a convex surface when the image is virtual. Show that for air medium
2. Prove that in the case of refraction at a concave surface
$$\frac{\mu}{\upsilon} - \frac{1}{u} = \frac{u-1}{R}$$
3. Discuss refraction at a convex surface when the image formed is real. Derive the necessary formula.
$$\frac{\mu_2}{\upsilon} - \frac{\mu_1}{u} = \frac{u_2 - u_1}{R}$$
4. Explain the terms first principal focus and second principal focus and show that
$$f_1\upsilon + f_2 u = u\upsilon$$
5. Calculate the transverse and the longitudinal magnifications for refraction at a spherical surface.
6. Derive Helmholtz's relation as used for a number of coaxial refracting surfaces.
7. What do you mean by aplanatic points and aplanatic surface? What are the advantages of aplanatic surfaces? **(Nagpur, 2004, 2005; Madhurai Kamaraj, 2003)**
8. Obtain an expression for refraction at a single spherical surface. Hence derive the lens-maker's formula.
9. Derive an expression connecting the object and image distances, when refraction of light takes place at a spherical surface separating two media.
10. State and explain Lagrange's equation for magnification. **(Nagpur, 2005)**

PROBLEMS FOR PRACTICE

1. A light spot is 150 cm away from a convex spherical mirror with radius of curvature 72 cm. Compute the image distance. [Ans: – 29 cm]

2. An object is placed before a concave mirror normal to its principal axis so that its magnification is 1.2. After the object has been moved further from the mirror by 25 cm, the magnification becomes 0.4. Calculate the focal length and radius of curvature of the mirror. [**Ans:** f = 15 cm; R = 30 cm]

3. A convex refracting surface of radius of curvature 50 cm separates two media of refractive indices 1.33 and 1.50. An object is placed at a distance of 100 cm in the first medium. Calculate the position of the image. [**Ans:** A virtual image forms at $\upsilon = -150$ cm.]

4. A convex refracting surface of radius of curvature 20 cm separates two media of refractive indices 1.33 and 1.50. An object is kept at a distance of 240 cm in the first medium. Calculate the position of the image. [**Ans:** a real image forms at $\upsilon = 540$ cm]

5. A convex refracting surface of radius of curvature 20 cm separates two media of refractive indices 1.33 and 1.50. An object is kept at a distance of 160 cm in the first medium. Calculate the position of the image. [**Ans:** $\upsilon = \infty$]

6. A glass dumbbell of length 200 cm and refractive index 1.50 has ends of radius of curvature 10 cm. Calculate the position of the image due to refraction at one end only, when the object is at a distance of 40 cm from one end. [**Ans:** 60 cm]

7. A glass dumbbell of length one metre and refractive index 1.5 has ends of radius of curvature 5 cm. Calculate the position of the image due to refraction at one end only, when the object is at a distance of 50 cm from one end. [**Ans:** 18.75 cm]

8. Obtain the condition for the formation of a real image when paraxial rays of light are refracted at a concave spherical water-air interface. Radius of curvature of the surface is R. [**Ans:** $u > 4R$]

9. A small filament is at the centre of a hollow glass sphere of inner and outer radii 4 cm and 4.5 cm respectively. The refractive index of glass is 1.50. Calculate the position of the image of the filament when viewed from outside the sphere. [**Ans:** At the centre]