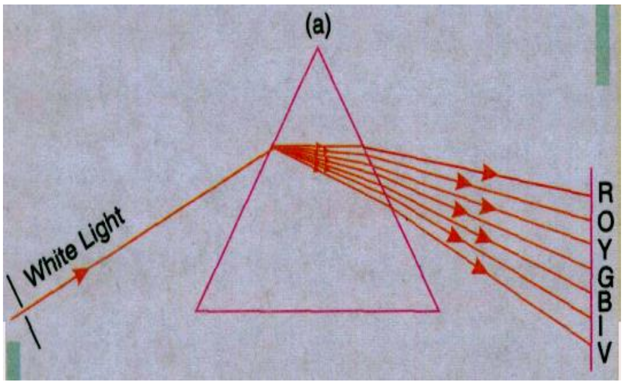
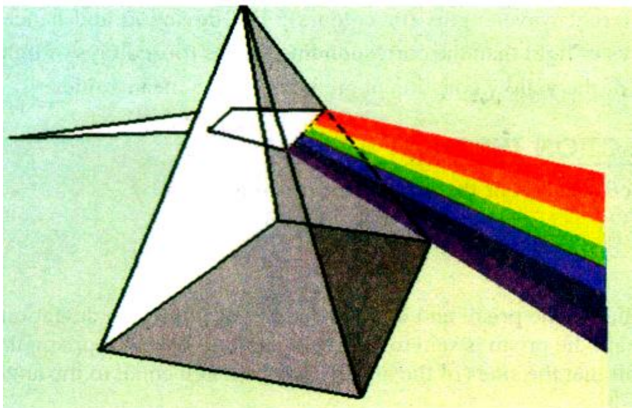
Chapter Five

Dispersion

5.1 Dispersion by A Prism

A beam of white light, when it passes through a prism is split up into its constituent colours. Different colours suffer different amounts of deviation. The spread of colours is called **dispersion of light.**





Light splitting up into its constituent colours.

The principal colours are given by the word **VIBGYOR** (Violet, Indigo, Blue, Green, Yellow, Orange and Red). The deviation produced for the violet rays of light is maximum and for red rays of light it is minimum.

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \text{......................(8.1)}$$

For a thin prism the value of the angles A and δ_m are so small that the sin of the angle.

$$\therefore \quad \sin A \approx A \quad and \ \sin(A+\delta_m) \approx A+\delta_m$$

$$\mu = \frac{A+\delta_m}{A} \Rightarrow \quad \delta_m = A(\mu - 1) \text{........................(8.2)}$$

$$\delta_V = (\mu_V - 1)A \quad \text{..................(8.3)}$$

$$\delta_R = (\mu_R - 1)A \quad \text{..................(8.4)}$$

The total angle through which the spectrum is spread is called as the **angular dispersion**. It is measured by the difference in deviation between the two extreme colours of the spectrum. Therefore, the angular dispersion is given by

$$\theta = \delta_V - \delta_R = (\mu_V - \mu_R)A \quad \text{.................(8.5)}$$

Thus, the angular dispersion depends on the nature of the material of the prism and upon the angle of the prism.

5.2 Dispersive Power

Dispersive power indicates the ability of the material of the prism to disperse the light rays. It is defined as the ratio of the angular dispersion to the deviation of the mean ray.

$$\text{Dispersive power, } \omega = \frac{\delta_V - \delta_R}{\delta} \quad \text{.................(8.6)}$$

Using the set of equations (8.2, 8.3 and 8.4), we can rewrite equ.(8.6) as

$$\omega = \frac{\mu_V - \mu_R}{\mu - 1} \quad \text{....................(8.7)}$$

It is seen that the dispersive power is independent of the angle of the prism and the angle of incidence.

It is also customary to express dispersive power more precisely with reference to C, F and D Fraunhofer lines (dark lines) in the solar spectrum. The F, D and C lines lie in the blue, yellow and red regions of the spectrum and their wavelengths are 4861A and 5893A and 6563A respectively. (lA = 1 Angstrom unit = 10^{-10}m). Thus, the dispersive power may be expressed as:

$$\omega = \frac{\mu_F - \mu_C}{\mu_D - 1}$$

The inverse of dispersive power is called as Abbe's number or V-number and is denoted by *V*.

$$V = \frac{\mu_D - 1}{\mu_F - \mu_C} \quad \text{....................(8.8)}$$

A small Abbe's number means high dispersion. Glasses of low dispersion, having V-number above 55, are called **crowns;** and glasses of high dispersion with V-number below 55 are called **flints.**

5.3 Dispersion without Deviation

Achromatic prisms are not of much practical use. A more useful combination of prisms is the one that produces ***dispersion but no deviation***. It is called a ***direct vision prism***. It consists of two prisms of different dispersive powers and different angles and combined in such a way that the combination does not produce deviation of the mean rays. Let us consider two thin prisms, one made of crown glass and the other of flint glass.

$$\delta = (\mu - 1)A$$

$$\delta' = (\mu' - 1)A'$$

The net deviation produced by the prism combination is to be zero.

That is, $\delta + \delta' = 0$

Or $(\mu - 1)A + (\mu' - 1)A' = 0$

$$\therefore \quad A = -\left(\frac{\mu' - 1}{\mu - 1}\right)A' \quad \text{......................}(8.11)$$

The negative sign shows that the refracting angles of the two prisms are in opposite direction. We calculate the net angular dispersion produced by the combination of prisms as follows:

The total deviation for the violet rays is given by

$$\delta_V = (\mu_V - 1)A + (\mu_V' - 1)A'$$

and the total deviation for the red rays is given by

$$\delta_R = (\mu_R - 1)A + (\mu_R' - 1)A'$$

Hence the total dispersion is equal to

$$\delta_V - \delta_R = (\mu_V - \mu_R)A + (\mu_V' - \mu_R')A'$$

Using equ.(8.11) into the above equation, we obtain

$$\delta_V - \delta_R = (\mu_V - \mu_R)A + (\mu_V' - \mu_R')\left(-\frac{\mu - 1}{\mu' - 1}\right)A$$

or

$$\delta_V - \delta_R = \left[\frac{(\mu_V - \mu_R)}{(\mu - 1)} - \frac{(\mu_V' - \mu_R')}{\mu' - 1}\right](\mu - 1)A$$

It follows from equ.(8.7) that

$$\delta_V - \delta_R = (\omega - \omega')(\mu - 1)A \quad \text{.................(8.12)}$$

Thus the resultant dispersion is equal to the difference between the two dispersions.

Example 5.1: Calculate the dispersive power for crown and flint glass:

	C	*D*	*F*
Crown	1.5145	1.5170	1.5230
Flint	1.6444	1.6520	1.6637

Solution:

$$\omega_1 = \frac{\mu_F - \mu_C}{\mu_D - 1} = \frac{1.5230 - 1.5145}{1.5170 - 1} = 0.01644$$

$$\omega_2 = \frac{\mu_F - \mu_C}{\mu_D - 1} = \frac{1.6637 - 1.6444}{1.6520 - 1} = 0.02961$$

Example 8.2: For a crown and flint glass for C and F lines $\mu_C = 1.515$, and $\mu_F = 1.523$ and $\mu_C = 1.644$, $\mu_F = 1.664$ respectively. Calculate the angle of flint glass prism which may be combined with crown glass prism having refracting angle 20° so that the combination is achromatic for C and F rays.

Solution: The condition for the combination to be achromatic is that

$$(\mu_F - \mu_C)A = (\mu_F' - \mu_C')A'$$

$$(1.523 - 1.515) \times 20° = (1.664 - 1.644)\,\alpha'$$

$$\therefore \qquad \alpha' = \frac{0.008}{0.02} \times 20° = 8°.$$

Example 8.3: A crown glass prism of refracting angle 8° is combined with a flint glass prism to obtain deviation without dispersion. If the refractive indices for red and violet rays for crown glass are 1.514 and 1.524 and for the flint glass are 1.645 and 1.665 respectively, find the angle of flint glass prism and net deviation.

Solution: The condition for deviation without dispersion is

$$(\mu_V - \mu_R)\, A = \left(\mu_V' - \mu_R'\right) A'$$

$$\therefore \qquad A' = \frac{(1.524 - 1.514)\times 8^o}{(1.665 - 1.645)} = \frac{0.08^o}{0.02} = 4^o$$

Mean refractive index for crown glass $\mu = \dfrac{1.514 + 1.524}{2} = 1.519$

Mean refractive index for flint glass $\mu = \dfrac{1.645 + 1.665}{2} = 1.655$

$\therefore$ The net deviation $(\delta - \delta') = (\mu - 1)\, A - (\mu' - 1)\, A'$

$= 0.519 \times 8^o - 0.655 \times 4^o$

$= \mathbf{1.53^o}$.

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The dispersive powers of crown and flint glasses are 0.03 and 0.05 respectively. If the difference in the refractive indices of blue and red colours is 0.015 for crown glass and 0.022 for flint glass, calculate the angles of the two prisms for a deviation of 2° without dispersion. [Ans: A = 10°; A′ = 6.8°]