Interference of Light

3.1 Introduction

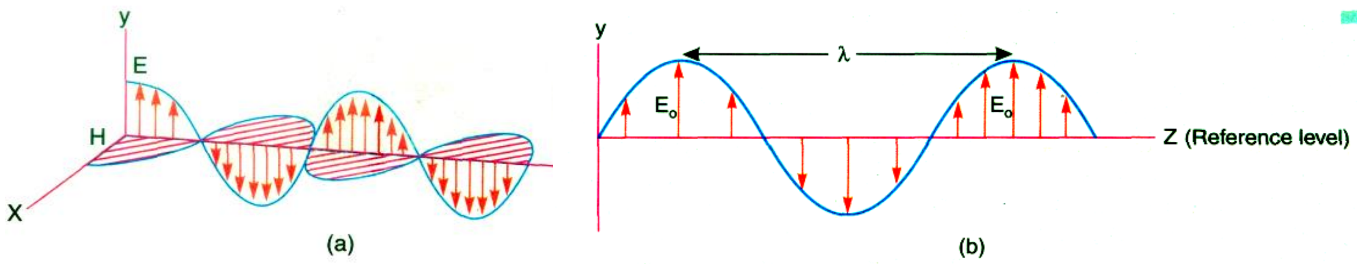
In 1680 Christian Huygens suggested that light propagates in the form of waves. Huygens did not know anything about the nature of the light wave; whether it is a transverse wave or a longitudinal wave; he had no knowledge about the speed of light or its wavelength. The wave theory of light received the first experimental evidence in 1801 from the interference experiments conducted by Thomas Young.

3.2 Light Waves

A light wave is a harmonic electromagnetic wave consisting of periodically varying electric and magnetic fields oscillating at right angles to each other and also to the direction of propagation of the wave. The electric field in the wave is defined by the electric field strength vector E and the magnetic field by the vector of magnetic induction B. Vectors E and B are of equal importance to the wave. The vector E is often referred to as the **light vector or optical vector.** The electric field is known as the **optical Held,** radiation field, wave field or light field. The magnetic field is implied to be oscillating in a plane normal to the plane of the electric field oscillations.

The light wave depicted is mathematically represented by the expression

$$E = E_0 \sin (k z - \omega t)$$



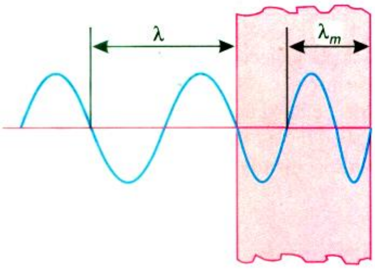
1. We note the following features of such a mathematical wave.

(i) The wave has a *single* definite frequency $\nu = \omega / 2\pi\ r$. In optics, waves having a single frequency and wavelength are called **monochromatic** waves.

(ii) It is a *harmonic* wave and is of **infinite extension** consisting of a continuous **train of waves.** At any instant, the wave extends from $z = -\infty$ *to* $z = +\infty$.

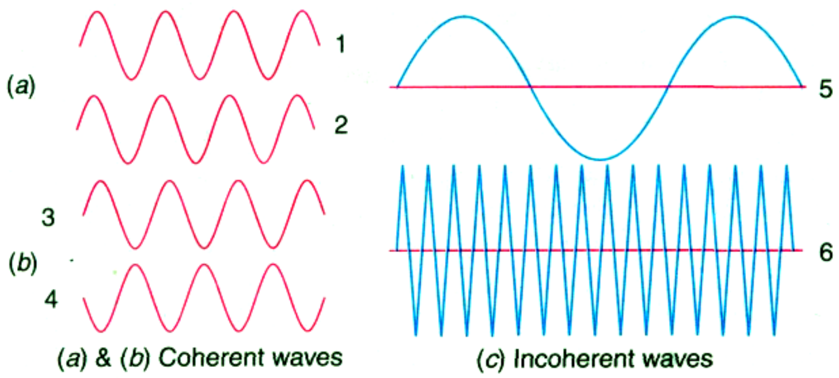
(iii) The amplitude of the wave E_o stays constant as the wave propagates through air. Hence it is a plane wave and its wavefront is normal to the z-axis.

(iv) The electric vector E of the wave oscillates always parallel to a fixed direction in space, i.e. y-direction. In other words, the E-vibrations are confined to the *yz* - plane. Therefore, the wave is **plane-polarized (or linearly polarized).**



2. A light wave travels *slower* in an optical medium than in air or a vacuum. **The wavelength of the light wave *decreases in the medium*, while its frequency remains constant.**

3. Phase difference and coherence: Suppose two waves are passing through a point in space. If the frequencies of the two waves are different, the phase difference between the vibrations changes with time. The two waves are said to be **incoherent.**



(a) Waves 1 and 2 are in-phase, (b) Waves 3 and 4 are in opposite phase and (c) Waves 5 and 6 are incoherent.

4. Optical path and Phase change: Optical path length indicates the number of light waves that fit into that path. Thus, $\Delta = N\lambda$, where Δ is the optical path length and *N* is an integer or a mixed fraction. Optical path is related to the geometric path and the relation may be found as follows. The distance traversed by light in a medium of refractive index μ in time *t* is given by

$$L = \upsilon t$$

where υ is the velocity of light in the medium. The distance travelled by light in a vacuum in the same time *t*, is $\Delta = c\,t = c\cdot\frac{L}{\upsilon} = \mu L$

The distance L is called the *geometric path length* (GPL). Δ is the equivalent distance in a vacuum and is called *optical path length* (OPL). Thus,

$$O.P.L. = \mu \times G.P.L.$$

or

$$\Delta = \mu L$$

(a) Effect of Optical path: Optical path determines **the phase** of a light wave arriving at a point. We know that if a wave covers in air a distance of one wavelength, its phase changes by 2π radians. Therefore, we compute that if a wave travels a distance L in air, its phase change is given by

$$\delta = \frac{2\pi L}{\lambda}$$

When the wave travels the distance L in a medium, then

$$\delta = \frac{2\pi\Delta}{\lambda} = \frac{2\pi\mu L}{\lambda}$$

where Δ is the optical path or optical path difference.

(b) Effect of reflection: The process of reflection also affects the phase of a light wave. When light is incident on a surface, part of the light gets reflected while a major portion may be transmitted or absorbed. The quantity characterizing the reflectivity of a surface is called the **reflection coefficient** ρ depends on the nature of surface. For normal incidence it was shown that:

$$\rho = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}$$

It may be seen that ρ is positive when $\mu_2 \langle \mu_1$. It implies that the oscillations in the incident and reflected waves occur in the same phase. On the other hand, when $\mu_2 \rangle \mu_1$, ρ is *negative* signifying that the *oscillations in the incident and reflected waves are in the opposite phase.*

3.3 Superposition of Waves

Principle of superposition: *when two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.*

3.4 Interference

If two or more light waves of the same frequency overlap at a point, the resultant effect depends on the *phases* of the waves as well as their *amplitudes.* The resultant wave at *any point* at any instant of time is governed by the **principle of superposition.** The combined effect at each point of the region of superposition is obtained by adding algebraically the amplitudes of the individual waves.

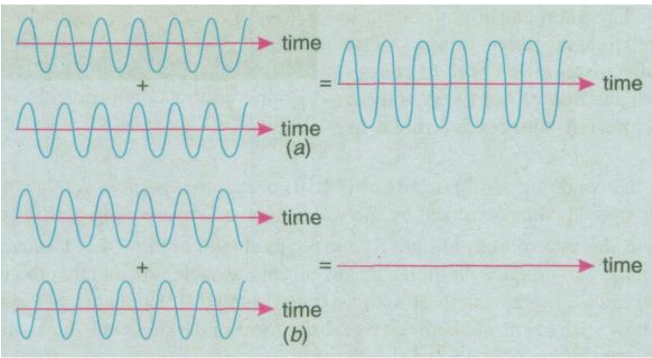
When two waves are in same phase:

$$A_R = A + A = 2A$$
$$I_R \alpha A_R^2 = 2^2 A^2 = 2^2 I$$
$$I_R \rangle I + I = 2I$$

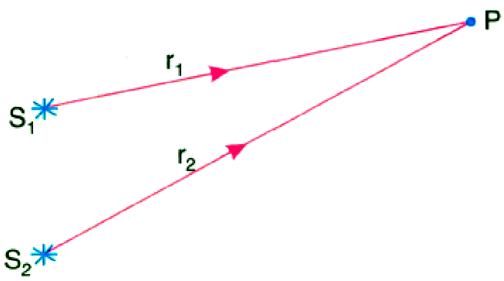
When two waves are in opposite phase:

$$A_R = A - A = 0$$
$$I_R \alpha 0^2 = 0$$
$$I_R \langle 2I$$

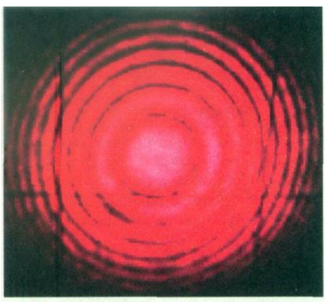
Therefore, the interference produced at these points is known as **constructive interference. A stationary bright band** of light is observed at points of constructive interference.



Therefore, the interference produced at these points is known as **destructive interference. A stationary dark band** of light is observed at points of destructive interference. Thus, we see that a redistribution of energy took place in the region.



Thus, when two or more coherent waves of light are superposed, the resultant effect is **brightness** in certain regions and **darkness** at other regions. The regions of brightness and darkness alternate and may take the form of straight bands, or circular rings or any other complex shape. The alternate bright and dark bands are called **interference fringes. The phenomenon of redistribution of light energy due to the superposition of light waves from two or more coherent sources is known as interference.**



In the central bright spot, there is constructive interference and then a destructive interference ring and then constructive, and so on.

If the optical path difference $\Delta = (\mu_2 r_2 - \mu_1 r_1)$ is equal to zero or an integral multiple of wavelength λ, then the waves arrive in phase at P and superpose with crest-to-crest correspondence. That is, if

$$\Delta = m\lambda$$

where m is an integer and takes values, m = 0,1,2,3,4,5,……,then the waves are in phase and their overlapping at P produces constructive interference or brightness.

On the other hand, if the optical path difference $\Delta = (\mu_2 r_2 - \mu_1 r_1)$ is equal to an odd integral multiple of half-wavelength, $\lambda/2$, then the waves arrive out of phase at P and superpose with crest-to-trough correspondence. That is, if

$$\Delta = (2m+1)\frac{\lambda}{2}$$

where m is an integer and takes values, m = 0,1,2,3,4,5,…….,then the waves are inverted with respect to each other and their overlapping at P produces destructive interference or darkness.
The regions of brightness and darkness are also known as regions of **maxima** and **minima**.

3.5 Theory of Interference using Analytic Method

Let us assume that the electric field components of the two waves arriving at point P vary with time as

$$E_A = E_1 \sin \omega t$$

and

$$E_B = E_2 \sin (\omega t + \delta)$$

where δ is the phase difference between them. According to Young's principle of superposition, the resultant electric field at the point P due to the simultaneous action of the two waves is given by

$$\begin{aligned} E_R &= E_A + E_B \\ &= E_1 \sin \omega t + E_2 \sin (\omega t + \delta) \\ &= E_1 \sin \omega t + E_2 (\sin \omega t \cos\delta + \cos \omega t \sin \delta) \\ &= (E_1 + E_2 \cos \delta) \sin \omega t + E_2 \sin \delta \cos \omega t \end{aligned}$$

Let $\quad E_1 + E_2 \cos \delta = E \cos \phi$

and $\quad E_2 \sin \delta = E \sin \phi$

where E is the amplitude of the resultant wave and ϕ is the new initial phase angle. In order to solve for E and ϕ, we square the equ. and add them.

$$(E_1 + E_2 \cos \delta)^2 + E_2^2 \sin^2 \delta = E^2 \left(\cos^2 \phi + \sin^2 \phi\right)$$

or

$$E^2 = E_1^2 + E_2^2 \cos^2 \delta + 2E_1E_2 \cos \delta + E_2^2 \sin^2 \delta$$

or

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos \delta \quad \cdots\cdots\cdots(*)$$

Thus, it is seen that the square of the amplitude of the resultant wave is not a simple sum of the squares of the amplitudes of the superposing waves, there is an additional term which is known as the *interference term*.

3.6 Intensity Distribution

The intensity of a light wave is given by the square of its amplitude.

$$I = \frac{1}{2}\varepsilon_o c E^2 \propto E^2$$

Using this relation into (∗), we get

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

We see that the resultant intensity at P on the screen is not just the sum of the intensities due to the separate waves. The term $2\sqrt{I_1 I_2}\cos\delta$ is known as the **interference term**. Whenever the phase difference between the waves is zero, i.e. $\delta = 0$, we have maximum amount of light. Thus,

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

When $I_1 = I_2 = I_o$ $\qquad I_{max} = 4I_o$

It means that the resultant intensity I will be *more than the sum* of the intensities due to the two sources.

When the phase difference is $\delta = 180°$, $\cos 180° = -1$ and we have a minimum amount of light.

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

which, when $I_1 = I_2$ becomes

$$I_{min} = 0$$

It means that the resultant intensity I will be *less than the sum* of the intensities due to the two sources.

At points that lie between the maxima and minima, when $I_1 = I_2 = I_0$, we get

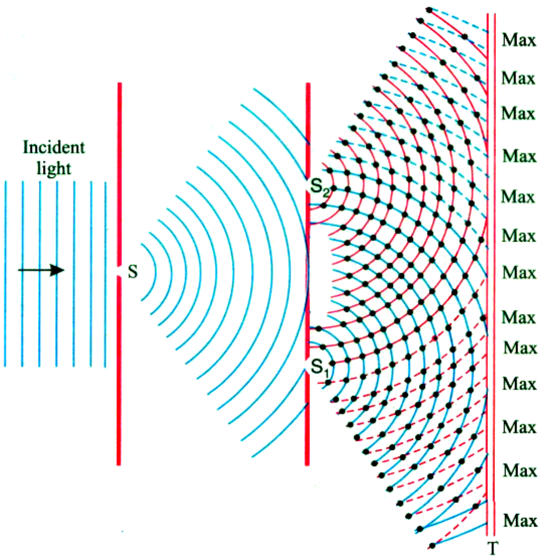
$$I = I_o + I_o + 2I_o\cos\delta$$
$$= 2I_o(1 + \cos\delta)$$

Then using the identity $1 + \cos\delta = 2\cos^2\left(\frac{1}{2}\delta\right)$, we get

$$I = 4I_o\cos^2\left(\frac{1}{2}\delta\right)$$

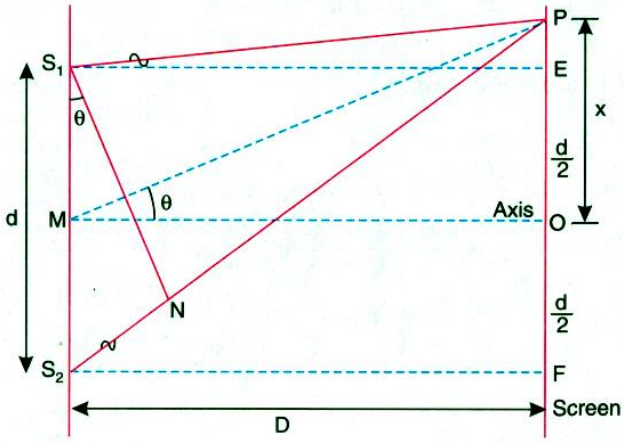
the intensity varies along the screen in accordance with the ***law of cosine square.***

3.7 YOUNG'S DOUBLE SLIT EXPERIMENT



As early as in 1665 Grimaldi attempted to produce interference between two beams of light. He directed sunlight into a dark room through two pinholes in a screen, with an expectation that bright and dark bands would be observed in the area where the beams overlap on each other. He observed uniform illumination instead. In 1801, about one hundred thirty-six years later, Thomas Young gave the first demonstration of the interference of light waves. Young admitted the sunlight through a single pinhole and then directed the emerging light onto two pinholes. Finally the light was received on a screen. The spherical waves emerging from the pinholes interfered with each other and a few coloured fringes were observed on the screen. The amount of light that emerged from the pinhole was very small and the fringes were faint and difficult to observe. The pinholes were later replaced with narrow slits that let through much more light. The sunlight was replaced by monochromatic light. Young's experiment is known as **double-slit experiment.**

12.7.1 Optical Path Difference Between the Waves At Point P:



Let the point P be at a distance x from O

PE = $x - d/2$ and PF = $x + d/2$.

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$(S_2P)^2 - (S_1P)^2 = 2xd$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

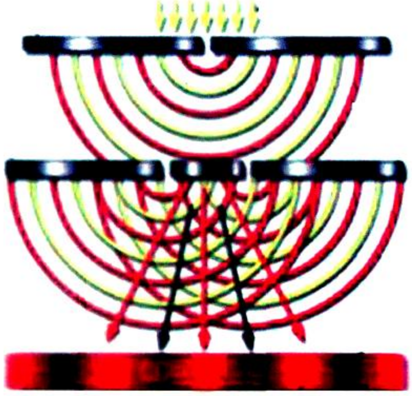
We can approximate that $S_2P \cong S_1P \cong D$.

$\therefore$ Path difference = $S_2P - S_1P = \dfrac{xd}{D}$

We now find out the conditions for observing bright and dark fringes on the screen.

3.7.2 BRIGHT FRINGES

In general, constructive interference occurs if S_1P and S_2P differ by a whole number of wavelengths.



The condition for finding a bright fringe at P is that

$$S_2P - S_1P = m\lambda$$

it means that

$$\frac{xd}{D} = m\lambda$$

where m is called the **order of the fringe**.

3.7.3 Dark Fringes

The first dark fringe occurs when $(S_2P - S_1P)$ is equal to $\lambda/2$. The waves are now in opposite phase at P. The second dark fringe occurs when $(S_2P - S_1P)$ equals $3\lambda/2$. The m^{th} dark fringe occurs when

$$(S_2P - S_1P) = (2m + 1)\,\lambda / 2$$

The condition for finding a dark fringe is $\frac{xd}{D} = (2m+1)\frac{\lambda}{2}$

3.7.4 Separation Between Neighboring Bright Fringes

The m^{th} order fringe occurs when $x_m = \frac{m\lambda D}{d}$

and the $(m+1)^{th}$ order fringe occurs when $x_{m+1} = \frac{(m+1)\lambda D}{d}$

The fringe separation, β is given by $\beta = x_{m+1} - x_m = \frac{\lambda D}{d}$

From the last equation we find the following:

(i) The fringe width β is independent of the *order* of the fringe. It is directly proportional to the wavelength of light.

(ii) The width of the fringe is *directly proportional* to the distance of the screen from the two slits.

(iii) The width of the fringe is *inversely proportional* to the distance between the two slits.

3.8 Conditions for Interference

We may now summarize the conditions that are to be fulfilled in order to observe a distinct well-defined interference pattern.

(A) Conditions for sustained interference:

(i) The waves from the two sources must be of the same frequency.

(ii) The two light waves must be coherent.

(iii) The path difference between the overlapping waves must be less than the coherence length of the waves.

(iv) If the two sets of waves are plane polarized, their planes of polarization must be the same.

(B) Condition for the formation of distinct fringe pattern:

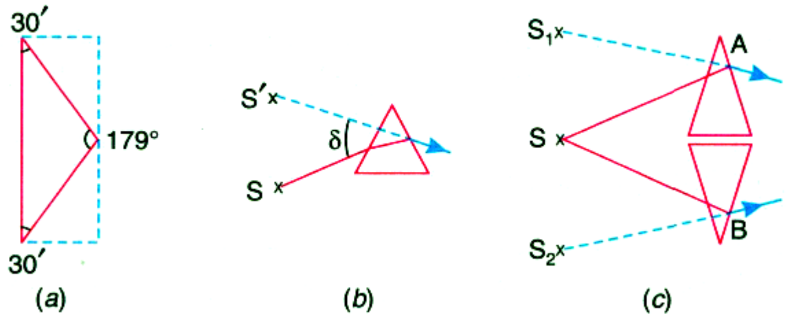
(i) The two coherent sources must lie close to each other in order to discern the fringe pattern. If the sources are far apart, the fringe width will be very small and fringes are not seen separately.

(ii) The distance of the screen from the two sources must be large.

(iii) The vector sum of the overlapping electric field vectors should be zero in the dark regions for obtaining distinct bright and dark fringes.

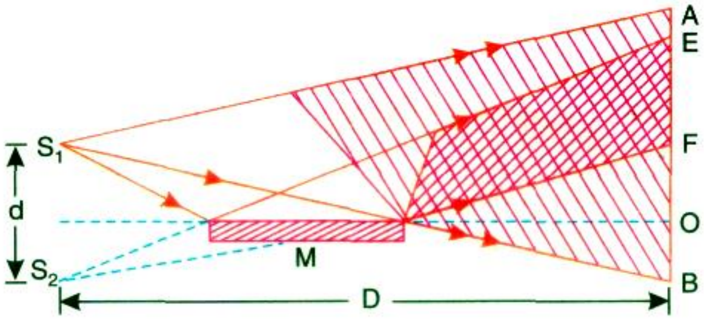
3.9 Fresnel Biprism

Fresnel used biprism to show the interference phenomenon. The biprism consists of two prisms of very small refracting angles joined base to base. In practice, a thin glass plate is taken and one of its faces is ground and polished till a prism is formed with an obtuse angle of about 179°.



3.10 Lloyd's Single Mirror

In 1834, Lloyd devised an interesting method of producing interference, using a single mirror and using almost grazing incidence. Lloyd's mirror consists of a plane mirror about 30 cm in length and 6 to 8 cm in breadth. It is polished on the front surface and blackened at the back to avoid multiple reflections.



By moving the screen nearer to the mirror such that it comes into contact with the mirror, the point O can be just brought into the region of interference.

With white light the central fringe at O is expected to be white but in practice it is *dark*. The occurrence of dark fringe can be understood taking into the consideration of the phase change of *n* that light suffers when reflected from the mirror. The phase change leads to a path difference of λ/2 and hence destructive interference occurs there.

Comparison between the fringes produced by biprism and Lloyd's mirror:

1. In biprism the complete set of fringes is obtained. In Lloyd's mirror a few fringes on one side of the central fringe are observed, the central fringe being itself invisible.
2. In biprism the central fringe is bright whereas in case of Lloyd's mirror, it is dark.
3. The central fringe is less sharp in biprism than that in Lloyd's mirror.

3.11 FRESNEL'S DOUBLE MIRROR

Fresnel's double mirror is an arrangement for obtaining two coherent sources by using the phenomenon of reflection. It consists of two plane mirrors inclined to each other at a very small angle. The mirrors are silvered on their front surfaces and are arranged at nearly 180° such that their surfaces are nearly coplanar.

Comparison between the fringes produced by biprism and double mirror:

The fringes in both cases are similar in appearance. However, the double mirror fringes are narrower than the biprism fringes.

3.12 VISIBILITY OF FRINGES

The contrast of the interference fringes can be quantitatively described by the parameter called *visibility*. Visibility is defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

The value of visibility varies between 0 and 1. When the fringes are of maximum intensity in the bright areas and totally dark in the dark areas, the visibility is equal to 1. As the phase difference increases, the coherence between the light waves decreases and the visibility is reduced. Finally, when the coherence between the two light waves disappears, I_{max} and I_{min} become equal and the visibility goes to zero. Then fringes are not observed and instead we observe uniform illumination.

The visibility of the fringes produced by two beams can be expressed as

$$V = \frac{(E_1 + E_2)^2 - (E_1 - E_2)^2}{(E_1 + E_2)^2 + (E_1 - E_2)^2} = \frac{2E_1E_2}{E_1^2 + E_2^2} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{2\sqrt{I_1 / I_2}}{1 + I_1 / I_2}$$

It is seen from the above equation that the closer the intensities of the two waves, the higher is visibility of the fringes. When $I_1 = I_2$, $V = 1$. It may be noted that V is always equal to 1 when monochromatic light is used.

WORKED OUT PROBLEMS

Example 14.1: Green light of wavelength 5100 Å from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, find the slit separation.

Solution: The fringe width $\beta = \frac{\lambda D}{d}$

It is given that D = 200 cm, $\lambda = 5100 \times 10^{-8}$cm and $10\beta = 2$ cm.

$\therefore \quad \beta = 0.2$ cm.

The slit separation $d = \frac{\lambda D}{\beta} = \frac{5100 \times 10^{-8} cm \times 200 cm}{0.2 cm}$ = **0.05 cm.**

Example 14.2: Two coherent sources are 0.18 mm apart and the fringes are observed on a screen 80 cm away. It is found that with a certain monochromatic source of light, the fourth bright fringe is situated at a distance of 10.8 mm from the central fringe. Calculate the wavelength of light.

Solution: The distance of the nth fringe from the central fringe is $x = \frac{n\lambda D}{d}$.

It is given that D = 80 cm, d = 0.18 mm = 0.018 cm, x = 10.8 mm = 1.08 cm and n = 4.

$$\therefore \quad \lambda = \frac{xd}{nD} = \frac{1.08\,\text{cm} \times 0.018\,\text{cm}}{4 \times 80\,\text{cm}} = 6075 \times 10^{-8}\text{cm} = \mathbf{6075\ Å}.$$

Example 14.3: A light source emits light of two wavelengths 4300Å and 5100Å. The source is used in a double slit experiment. The distance between the sources and the screen is 1.5 m and the distance between the slits is 0.025mm. Calculate the separation between the third order bright fringes due to these two wavelengths.

Solution: It is given that $\lambda_1 = 4300Å = 4300 \times 10^{-8}$cm, $\lambda_2 = 5100Å = 5100 \times 10^{-8}$cm, $n = 3$, D = 1.5 m = 150 cm and d = 0.025mm = 0.0025cm.

Now $\quad x_1 = \frac{n\lambda_1 D}{d}$ and $x_2 = \frac{n\lambda_2 D}{d}$

$$\therefore \quad x_2 - x_1 = \frac{nD}{d}(\lambda_2 - \lambda_1) = \frac{3 \times 150\,\text{cm}}{0.0025\,\text{cm}}(5100 - 4300) \times 10^{-8}\,\text{cm} = \mathbf{1.44\ cm}$$

QUESTIONS

1. What are the conditions necessary for observing interference fringes?
2. Why is the condition of coherence necessary to observe interference fringes?
3. Is it possible to observe interference fringes with light emanating from two independent sources? If not, why?
4. How can coherent sources be obtained in practice?
5. Is it necessary that the interfering waves should have the same frequency? If so, why?
6. Is it necessary that the interfering waves should have equal amplitudes? Explain.

7. What are coherent sources? How are they realized in practice? Describe a method for determining the refractive index of a gas using the interference phenomenon.
8. Describe Fresnel's biprism. Explain how the wavelength of light can be determined with its help.
9. What are coherent sources? Explain the importance of such sources in the interference phenomenon. Two coherent sources form interference fringes. Obtain an expression for the distance between two consecutive bright fringes.
10. What is meant by the interference of light? State the fundamental conditions for the production of interference fringes.
11. Discuss the conditions for interference. Describe Young's experiment and derive an expression for (i) intensity at a point on the screen and (ii) fringe width.
12. Derive an expression for the resultant intensity when two coherent beams of light are superposed. What is the visibility of fringes:
    (a) For two slits of equal intensities?
    (b) If the intensity of one slit is 4 times the other?
    (c) What will be the intensity when the two sources are incoherent?
13. State the principle of superposition of light waves.
14. Why two independent sources cannot produce an observable interference pattern?
15. How are the fringes of equal inclination obtained?
16. Explain the optical path of light in the denser medium.
17. What are the conditions for a sustained interference pattern'?
18. State and explain conditions for the interference of light.
19. Calculate the displacement of fringes when a thin transparent film is introduced in the path of one of the interfering beams in biprism.