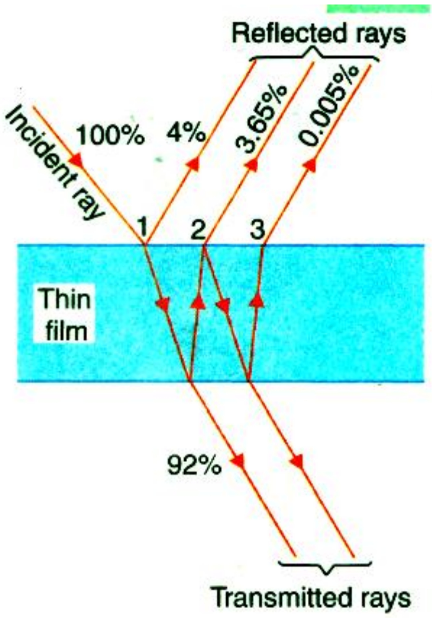
Thin Film

4.1 Introduction

An optical medium is called a **thin film** when its thickness is about the order of 1 wavelength of light in visible region. Thus, a film of thickness in the range 0.5 μm to 10 μm may be considered as a thin film. A thin film may be a thin sheet of transparent material such as glass, mica, an air film enclosed between two transparent plates or a soap bubble. When light is incident on such a film, a small part of it gets reflected from the top surface and a major part is transmitted into the film. Again, a small part of the transmitted component is reflected back into the film by the bottom surface and the rest of it emerges out of the film. In transparent thin films, the two bounding surfaces strongly transmit light and only weakly reflect the incident light. Therefore, only the first reflection at the top surface and the first reflection at the bottom surface will be of appreciable strength. It means that about 4% of the incident light is reflected by the top surface of the glass plate, while 96% of it is transmitted into the plate. Out of the light reaching the bottom surface, again 3.8% is reflected and 92% is transmitted out of the plate. Then, again out of the 3.8% of the light 0.15% is reflected at the inner boundary of the top surface and about 3.65% is transmitted out into the air. After two reflections, the intensity will become insignificantly small. At each reflection, the intensity and hence the amplitude of light wave is divided into a reflected component and a refracted component. The reflected and refracted components travel along different paths and subsequently overlap to produce interference. Therefore, the interference in thin films is called interference by **division of amplitude**. Newton and Robert Hooke first observed the thin film interference. However, Thomas Young gave the correct explanation of the phenomena. A thin film may be uniform or non-uniform in its structure. However, as long as its thickness lies within the specified limits, interference of light occurs.



Application of thin film:

(a) Microelectronic devices

(b) telecommunication devices

(c) wear resistant coatings

(d) decorative coatings

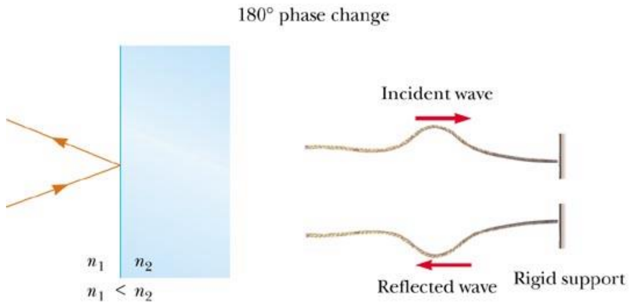
(e) optical coatings (windows, solar cells, etc.)

(f) Sensors

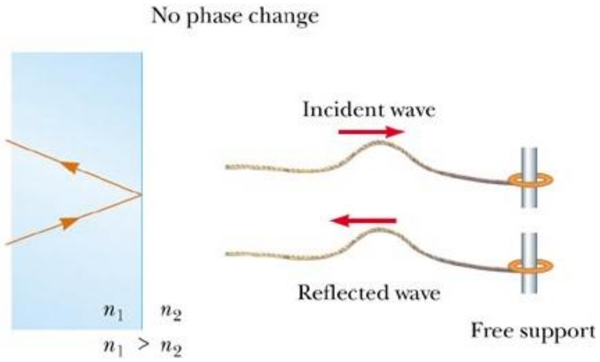
(g) catalysts

Phase Changes Due to Reflection

An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling

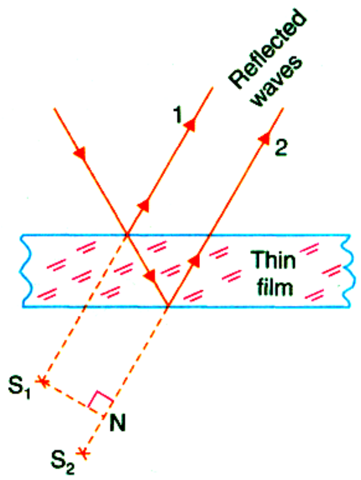


There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction



4.2 Plane Parallel Film

A transparent thin film of uniform thickness bounded by two parallel surfaces is known as a **plane parallel** thin film. When light is incident on a parallel thin film, a small portion of it gets reflected from the top surface and a major portion is transmitted into the film. Again, a small part of the transmitted component is reflected back into the film by the bottom surface and the rest of it is transmitted from the lower surface of the film. Thin films transmit incident light strongly and reflect only weakly.



After two reflections, the intensities of reflected rays drop to a negligible strength. Therefore, we consider the first two reflected rays only. These two rays are derived from the same incident ray but appear to come from two sources located below the film. The sources are virtual coherent sources. The reflected waves 1 and 2 travel along parallel paths and interfere at infinity. This is a case of two-beam interference. The condition for maxima and minima can be deduced once we have calculated the optical path difference between the two rays at the point of their meeting.

4.3 Conditions for Maxima (Brightness) and Minima (Darkness)

Maxima occur when the optical path difference $\Delta = m\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves *interfere constructively*. Thus, when

$$2\mu t \cos r - \frac{\lambda}{2} = m\lambda$$

the reflected rays undergo constructive interference to produce brightness or maxima at the point of their meeting.

$$2\mu t \cos r = m\lambda + \lambda/2$$

or $$2\mu t \cos r = (2m+1)\lambda/2 \qquad \textbf{Condition for Brightness}$$

Minima occur when the optical path difference is $\Delta = (2m+1)\lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave falls on the troughs of the others and *the waves interfere destructively*. Thus, when

$$2\mu t \cos r - \lambda/2 = (2m+1)\lambda/2$$

the reflected rays undergo destructive interference to produce darkness.

$$2\mu t \cos r = (m+1)\lambda$$

The phase relationship of the interfering waves does not change if one full wave is added to or subtracted from any of the interfering waves. Therefore $(m + 1)\lambda$ can as well be replaced by $m\lambda$ for simplicity in expression. Thus,

$$2\mu t \cos r = m\lambda \qquad \textbf{Condition for Darkness}$$

SOME IMPORTANT POINTS

(a) It is seen that the conditions of interference depend on four parameters, namely μ, t, λ and r. In the case of constant thickness (parallel) film, (μ t) is constant. When a parallel beam of light is incident on such a film, r also remains constant. Then the interference conditions depend on the wavelength λ.

(b) When monochromatic light falls on a parallel beam, the whole film will appear *uniformly* dark or bright. If the condition of constructive interference is satisfied, the film will show intense color corresponding to the incident light.

(c) If a parallel beam of white light falls on a parallel film, those wavelengths for which the path difference is m λ, will be absent from the reflected light. The other colours will be reflected. Therefore, the film will appear uniformly coloured with one color being absent.

4.4 Newton's Rings

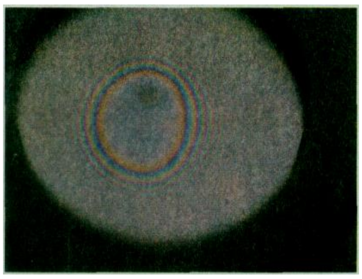
Newton's rings are an example of fringes of equal thickness. Newton's rings are formed when a Plano-convex lens of a large radius of curvature placed on a sheet of plane glass is illuminated from the top with monochromatic light. The combination forms a thin circular air film of variable thickness in all directions around the point of contact of the lens and the glass plate. The locus of all points corresponding to specific thickness of air film falls on a circle. Consequently, interference fringes are observed in the form of a series of concentric rings. Newton originally observed these concentric circular fringes and hence they are called **Newton's rings.**

4.4.1 CONDITION FOR BRIGHT AND DARK RINGS

The optical path difference between the rays is given by $\Delta = 2\mu t \cos r - \lambda/2$. Since $\mu = 1$ for air and $\cos r = 1$ for normal incidence of light,

$$\Delta = 2t - \lambda/2$$

Intensity maxima occur when the optical path difference $\Delta = m\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the other and the waves *interfere constructively*. Thus, if $2t - \lambda/2 = m\lambda$



$$\boxed{2t = (2m+1)\lambda/2}$$

bright fringe is obtained.

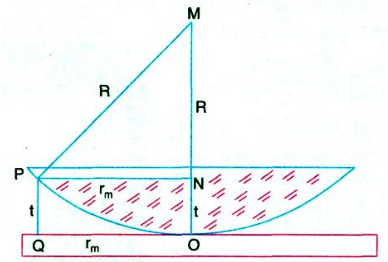
Intensity minima occur when the optical path difference is $\Delta = (2m + 1)\lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave fall on the troughs of the other and *the waves interfere destructively*.

Hence, if $2t - \lambda/2 = (2m + 1)\lambda/2$

$$\boxed{2t = m\lambda}$$

dark fringe is produced.

4.4.3 RADII OF DARK FRINGES



Let R be the radius of curvature of the lens. Let a dark fringe be located at Q. Let the thickness of the air film at Q be PQ = t. Let the radius of the circular fringe at Q be OQ= r_m.

$\therefore \quad R^2 = r_m^2 + (R-t)^2$

or $\quad r_m^2 = 2Rt - t^2$

As $\quad R \gg t, \quad 2Rt \gg t^2$.

$\therefore \quad r_m^2 \cong 2Rt$

The condition for darkness at Q is that

$$2t = m\lambda$$

$\therefore \quad r_m^2 \cong m\lambda R$

$$r_m = \sqrt{m\lambda R}$$

The radii of dark fringes can be found by inserting values 1,2,3, ...for m. Thus,

$$r_1 = \sqrt{1\lambda R} \quad \text{or} \quad r_1 \propto \sqrt{1}$$

$$r_2 = \sqrt{2\lambda R} \quad \text{or} \quad r_2 \propto \sqrt{2}$$

$$r_3 = \sqrt{3\lambda R} \quad \text{or} \quad r_3 \propto \sqrt{3} \quad \text{and so on}$$

It means that *the radii of the dark rings are proportional to under root of the natural numbers.* The above relation also implies that

$$r_m \propto \sqrt{\lambda}$$

Thus, *the radius of the m^{th} dark ring is proportional to under root of wavelength.*

Ring Diameter: Diameter of m^{th} dark ring $D_m = 2r_m$

$$D_m = 2\sqrt{2Rt}$$

$$D_m = 2\sqrt{m\lambda R}$$

4.4.4 SPACING BETWEEN FRINGES

It is seen that the diameter of dark rings is given by

$$D_m = 2\sqrt{m\lambda R}$$

where m = 1,2,3, ...

The diameters of dark rings are proportional to the square root of the natural numbers. Therefore, the diameter of the ring does not increase in the same proportion as the order of the ring, for example, if m increases as 1,2,3,4, ...the diameters are

$$D_1 = 2\sqrt{\lambda R}$$

$$D_2 = 2\,(1.4)\,\sqrt{\lambda R}$$

$$D_3 = 2\,(1.7)\,\sqrt{\lambda R}$$

$$D_4 = 2\,(2)\,\sqrt{\lambda R} \quad \text{and so on.}$$

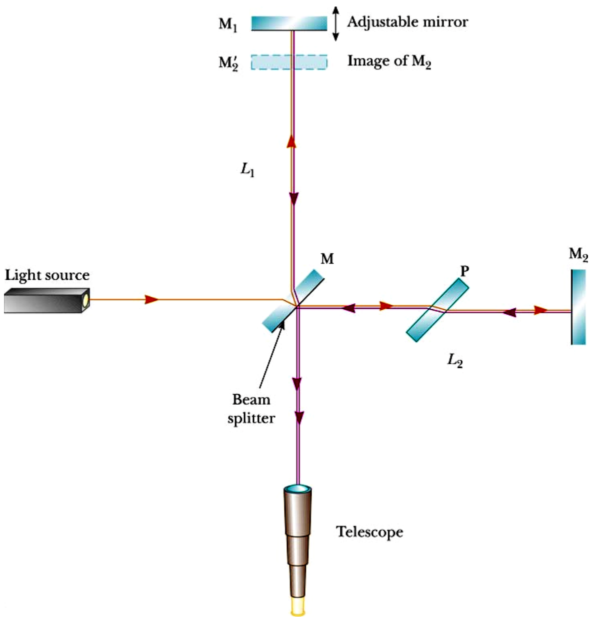
Therefore, the rings get closer and closer, as m increases. This is why the rings are not evenly spaced.

4.5 MICHELSON'S INTERFEROMETER

An interferometer is an instrument in which the phenomenon of interference is used to make precise measurements of wavelengths or distances.

4.5.1 PRINCIPLE

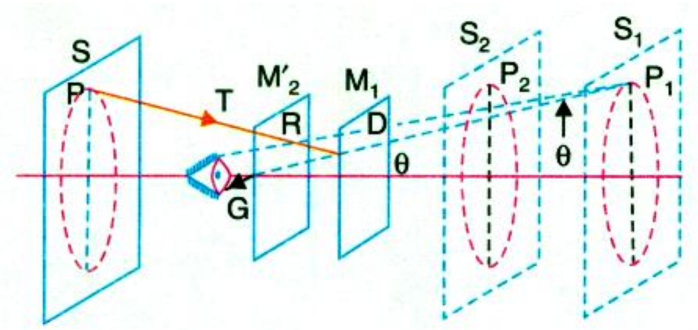
In Michelson interferometer, a beam of light from an extended source is divided into two parts of equal intensities by partial reflection and refraction. These beams travel in two mutually perpendicular directions and come together after reflection from plane mirrors. The beams overlap on each other and produce interference fringes.



4.5.2 CIRCULAR FRINGES

Circular fringes are produced with monochromatic light when the mirrors M_1 and M_2 are exactly perpendicular to each other. The origin of the circular fringes can be understood as follows.

If we look into the instrument from T, we see mirror M_1 directly, and in addition we will see the virtual image M'_2 of mirror M_2 formed by reflection in the glass plate G_1. It means that one of the interfering beams come from M_1 and the other beam appears to come from the virtual image M'_2. The situation is similar to an air film enclosed between mirrors M_1 and M'_2 with the difference that in case of a real film between two surfaces, multiple reflections take place, whereas in this case only two reflections take place.



If the two arms of the interferometer are equal in length, image M_2 coincides with mirror M_1. If M_2 and M_1 do not coincide, the distance between them is finite, $M'_2\ M_1 = d$. Now if a light ray comes from a point S and is reflected by both M'_2 and M_1 the observer will see two virtual images S_1 due to reflection at M_2 and S_2 due to reflection at M_1. The virtual images are separated by a distance 2d. If the observer looks into the system at an angle θ, the path difference between the two beams will be 2d cosθ. The light that comes from M_2 and goes to T undergoes rare-to-dense reflection and therefore a π-phase change occurs. In view of this, the total path difference between the two beams is given by:

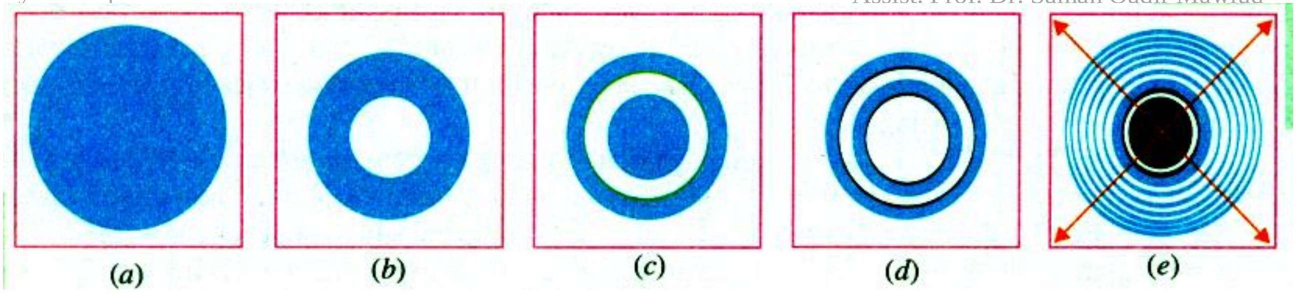
$$\Delta = 2d\cos\theta + \lambda/2$$

The condition for obtaining brightness

$$2d\cos\theta + \lambda/2 = m\lambda$$

Where m = 0, 1, 2, ...

For a given mirror separation d, a given wavelength λ, and order m, angle θ is constant. This means that the fringes are of circular shape. They are called fringes of equal inclination. In case the mirror M_1 coincides with the virtual image M'_2, d=0. The path difference between the interfering beams will be λ/2. Consequently, we obtain a minimum at the coincidence position and the centre of the field will be dark.



If one of the mirrors is now moved through a distance λ /4, the path difference changes by λ /2 and therefore a maximum is obtained. By moving the mirror through another λ /4, a minimum is obtained; by moving it by another λ /4 again a maximum is obtained and so on. Therefore, a new ring appears in the centre of the field each time the mirror is moved through λ /2. As d increases new rings appear in the centre faster than rings already present disappear in the periphery; and the field becomes more crowded with thinner rings. Conversely, as d is made smaller, the rings contract and disappear in the centre.

4.5.6 Applications of Michelson Interferometer

Michelson interferometer can be used to determine:

(i) The wavelength of a given monochromatic source of light

(ii) The difference between the two neighboring wavelengths or resolution of the spectral lines.

(iii) Refractive index and thickness of various thin transparent materials.

Example 15.5: Newton's rings are observed in reflected light of $\lambda = 5.9 \times 10^{-5}$ cm. The diameter of the 10^{th} dark ring is 0.5 cm. Find the radius of curvature of the lens and the thickness of the air film.

Solution: Given that $\lambda = 5.9 \times 10^{-5}$ cm, m = 10.

The radius of m^{th} dark ring is given by

$$\therefore \qquad R = \frac{r^2}{m\lambda} = \frac{(0.5\,\text{cm})^2}{10\times 5.9\times 10^{-5}\,\text{cm}} = 106\text{ cm} = \mathbf{1.06m}$$

The thickness of air film is given by

$$\therefore \qquad t = \frac{m\lambda}{2} = \frac{10\times 5.9\times 10^{-5}\,\text{cm}}{2} = \mathbf{2.95\mu m.}$$

Example 15.6: In a Newton's rings experiment, the diameter of 10th dark ring due to wavelength 6000 Å in air is 0.5 cm. Find the radius of curvature of the lens.

Solution: Radius of curvature, $R = \frac{(D/2)^2}{m\lambda} = \frac{(0.5\times10^{-2}/2)^2\,\text{m}^2}{10\times6000\times10^{-10}\,\text{m}} = \mathbf{1.04m}$

Example 15.9: In a Michelson interferometer 200 fringes cross the field of view when the movable mirror is moved through 0.0589 mm. Calculate the wavelength of light used.

Solution: $\lambda = \frac{2d}{m} = \frac{2\times0.0589\times10^{-3}\,\text{m}}{200} = \mathbf{5890\,\text{Å}}$

QUESTIONS

1. Give the construction and working of Michelson Interferometer with neat diagram.
2. Show that the diameters of Newton's dark rings are proportional to the square roots of natural numbers.
3. Explain the formation of circular fringes in the Michelson Interferometer.
4. Give the theory of Newton's rings.
5. How is the wavelength of sodium light determined by Newton's rings method? Derive the formula used. Why are the rings circular?
6. Soap bubble or a thin film of oil spread over the surface of the water appears colored in sunlight. Why?
7. Describe the Michelson interferometer and explain the formation of fringes in it.
8. What are Newton's rings?