



1 Completely randomized design (CRD)

1.1 Introduction

CRD is the basic single factor design. In this design the treatments are assigned completely at random so that each experimental unit has the same chance of receiving any one treatment. But CRD is appropriate only when the experimental material is homogeneous. As there is generally large variation among experimental plots due to many factors CRD is **not preferred in field experiments**. In **laboratory** experiments and **glass house** studies it is easy to achieve homogeneity of experimental materials and therefore CRD is most useful in such experiments.

1.2 Advantages of CRD:

- The CRD is the **simplest** design.
- All **experimental units** are considered the same and **no division** or **grouping** among them exist.
- **Number of replications** for different treatments need not be **equal** and **may vary** from treatment to treatment depending on the knowledge (if any) on the variability of the observations on individual treatments as well as on the accuracy required for the estimate of individual treatment effect (Analysis remains simple when data are missing).
- **All the variability** among the experimental units goes into **experimented error**.
- **CRD is used** when the experimental material is **homogeneous**.

1.3 Disadvantages of the Design (CRD):

- Relatively low accuracy due to lack of restrictions which allows environmental variation to enter experimental error
- Not suited for large numbers of treatments because a relatively large amount of experimental material is needed which increases the variation.

1.4 Layout of CRD

An experiment was conducted on a variety of tomato with 3 treatments of integrated nutrient management t1, t2 and t3. Each with four replications. The objective of the

experiment was to find out the most appropriate integrated nutrient management system for tomato.

Step 1: Determine the **total number of experimental units**.

Number of experimental units = levels (number) of treatments (t) * number of replications.

Number of experimental units = 3*4= 12

t1r1	t1r2	t1r3
t2r1	t2r2	t2r3
t3r1	t3r2	t3r3
t4r1	t4r2	t4r3

Step 2: Assign a **plot number** to each of the **experimental units** starting from **left to right** for all rows.

Step 3: Assign the **treatments** to the **experimental units** randomly by using **LOTEERY PLAN**.

t2r2	t4r1	t3r3
t2r1	t1r1	t3r1
t2r3	t3r2	t4r2
t1r2	t1r3	t4r3

The arrangement of data in CRD is as follows:

replications r	Treatments (T)			Grand Total (GT) = $\sum t_1 + \sum t_2 + \sum t_3$
	t1	t2	t3	
r1	t1r1	t1r2	t1r3	
r2	t2r1	t2r2	t2r3	
r3	t3r1	t3r2	t3r3	
r4	t4r1	t4r2	t4r3	
$\sum t_i$	$\sum t_1$	$\sum t_2$	$\sum t_3$	

OR:

Treatments (t)	Replications (r)				Grand Total (GT) = $\sum t_1 + \sum t_2 + \sum t_3 + \sum t_4$
	r1	r2	r3	$\sum t_i$	
t1	t1r1	t1r2	t1r3	$\sum t_1$	
t2	t2r1	t2r2	t2r3	$\sum t_2$	
t3	t3r1	t3r2	t3r3	$\sum t_3$	

1.5 Linear model of CRD:

$$Y_{ij} = \mu + t_i + e_{ij}$$

μ = overall mean effect

t_i = **true effect** of the i_{th} treatment

e_{ij} = **error** term of the j_{th} unit receiving i_{th} treatment

1.6 Statistical Analysis of the design:

Step 1: putting a hypothesis

The **null hypothesis** will be

H₀: $\mu_1 = \mu_2 = \dots = \mu_k$ or There is **no significant difference** between the treatments

And the **alternative** hypothesis is

H_A: $\mu_1 \neq \mu_2 \neq \dots \neq \mu_k$. There is **significant difference** between the treatments

Step 2: mathematical steps to construct analysis of variance (ANOVA) table for a CRD:

Source of variation (S.O.V.)	Degree of freedom (Df)	Sum of Squares (SS)	Mean of Squares (MS)	Calculated F	Table F
Treatment (t)	Df t = t-1	t SS	t MS	$\frac{t MS}{E MS}$	From F- Table (α , Df E)
Error (E)	Df E = tr - t	E SS	E MS		
Total (T)	Df T = tr - 1	T SS			

1. Degree of freedoms (df):

$$Df \text{ Total} = tr - 1$$

$$Df \text{ Treatment} = t - 1$$

$$Df \text{ Error} = tr - t$$

2. Sum of Squares (SS) and correction factor (C.F):

$$\text{Correction factor (C.F)} = \frac{GT^2}{TR}$$

$$\text{Total Sum of Squares (TSS)} = \sum Tij^2 - C.F$$

$$\text{Treatment Sum of squares (t SS)} = \frac{\sum ti^2}{r} - C.F$$

$$\text{Error sum of squares (E SS)} = \text{Total SS} - \text{Treatment SS}$$

3. Mean squares (MS):

$$\text{Mean Square of Treatment (MS t)} = \frac{\text{Treatment SS}}{\text{Df treatment}}$$

$$\text{Mean square of Error (MS E)} = \frac{\text{Error SS}}{\text{Df Error}}$$

4. Calculated - F:

$$\text{Calculated F} = \frac{MS t}{MS E}$$

5. Tabulated F: we can find it in table-F depending on the level of significant of the experiment (α) and degree of freedom of Error (df E).

6. Constructing ANOVA-table

7. Comparison and decision: Compare the **calculated F** with the **critical** value of **F** corresponding to **treatment degrees of freedom** and **error** degrees of freedom so that acceptance or rejection of the null hypothesis can be determined. If **null hypothesis is rejected** that indicates there is **significant differences** between the different treatments.

EXAMPLE: The following data represents the number of leaves of a given plant under the effect of three types of fertilizer in a pot experiment. Complete ANOVA table and decide if there is significant difference present between the treatments.

	A	B	C
r1	23	42	47
r2	36	26	43
r3	31	47	43
r4	33	34	39
$\sum t_i$	$\sum A = 123$	$\sum B = 149$	$\sum C = 172$

Step 1: putting a hypothesis:

H_A: $\mu_1 \neq \mu_2 \neq \dots \neq \mu_k$. There is **significant difference** between the treatments

Step 2: mathematical steps to construct analysis of variance (ANOVA) table for a CRD:

1. Degree of freedoms (df):

$df_{total} = tr - 1 = (3 * 4) - 1 = 11$

$df_{treatment} = t - 1 = 2$

$df_{error} = tr - t = 3 * 4 - 3 = 9$

2. Sum of Squares (SS) and correction factor (C.F)

$$(C.F) = \frac{GT^2}{tr} = \frac{(444)^2}{3 * 4} = 16428$$

Total Sum of Squares (TSS) = $\sum Y_{ij}^2 - C.F = (23)^2 + (36)^2 + (31)^2 + \dots + (39)^2 - 16428 = 17108 - 16428 = 680$

$$\begin{aligned} \text{Treatment Sum of squares (t SS)} &= \frac{\sum t_i^2}{r} - C.F = \frac{\sum A^2 + \sum B^2 + \sum C^2}{r} - C.F \\ &= \frac{(123)^2 + (149)^2 + (172)^2}{4} - 16428 = 300.5 \end{aligned}$$

Error sum of squares (E SS) = Total SS - Treatment SS = 680 - 300.5 = 379.5

3. Mean squares (MS):

$$\text{Mean Square of Treatment (MS } t) = \frac{\text{Treatment SS}}{\text{Df treatment}} = \frac{300.5}{2} = 150.25$$

$$\text{Mean square of Error (MS } E) = \frac{\text{Error SS}}{\text{Df Error}} = \frac{379.5}{9} = 42.167$$

4. Calculated - F:

$$\text{Calculated } F = \frac{MS t}{MS E} = \frac{150.25}{42.167} = 3.563$$

5. Tabulated F: we can find it in Table-F depending on the level of significant of the experiment (**0.05**) and degree of freedom of treatment (**2**) and Error (**9**); **4.25**.

(S.O.V)	(df)	(SS)	(MS)	Calculated F	Tabulated F
Treatment (t)	2	300.5	150.25	3.563	F_{0.05; 2, 9} = 4.26
Error (E)	9	379.5	42.167		F_{0.01; 2, 9} = 8.02
Total (T)	11	680			

6. Since F-calculated (3.563) < F tabulated (4.26 and 8.02) under both level of significant (0.05 and 0.01) we reject alternative hypothesis; **H_A: μ₁ ≠ μ₂ ≠ ≠ μ_k**. Otherwise we should accept null hypothesis; **H₀: μ₁ = μ₂ = = μ_k**. that the three fertilizers used had the same effect in increasing the number of leaves/ plant.