

## EXERCISES

**4.4.1.** Let  $\bar{X}$  denote the mean of a random sample of size 100 from a distribution that is  $\chi^2(50)$ . Compute an approximate value of  $P(49 < \bar{X} < 51)$ .

**4.4.2.** Let  $\bar{X}$  denote the mean of a random sample of size 128 from a gamma distribution with  $\alpha = 2$  and  $\beta = 4$ . Approximate  $P(7 < \bar{X} < 9)$ .

**4.4.3.** Let  $Y$  be  $b(72, \frac{1}{3})$ . Approximate  $P(22 \leq Y \leq 28)$ .

**4.4.4.** Compute an approximate probability that the mean of a random sample of size 15 from a distribution having pdf  $f(x) = 3x^2$ ,  $0 < x < 1$ , zero elsewhere, is between  $\frac{3}{5}$  and  $\frac{4}{5}$ .

**4.4.5.** Let  $Y$  denote the sum of the observations of a random sample of size 12 from a distribution having pmf  $p(x) = \frac{1}{6}$ ,  $x = 1, 2, 3, 4, 5, 6$ , zero elsewhere. Compute an approximate value of  $P(36 \leq Y \leq 48)$ .

*Hint:* Since the event of interest is  $Y = 36, 37, \dots, 48$ , rewrite the probability as  $P(35.5 < Y < 48.5)$ .

**4.4.6.** Let  $Y$  be  $b(400, \frac{1}{5})$ . Compute an approximate value of  $P(0.25 < Y/n)$ .

**4.4.7.** If  $Y$  is  $b(100, \frac{1}{2})$ , approximate the value of  $P(Y = 50)$ .

**4.4.8.** Let  $Y$  be  $b(n, 0.55)$ . Find the smallest value of  $n$  which is such that (approximately)  $P(Y/n > \frac{1}{2}) \geq 0.95$ .

**4.4.9.** Let  $f(x) = 1/x^2$ ,  $1 < x < \infty$ , zero elsewhere, be the pdf of a random variable  $X$ . Consider a random sample of size 72 from the distribution having this pdf. Compute approximately the probability that more than 50 of the observations of the random sample are less than 3.

**4.4.10.** Forty-eight measurements are recorded to several decimal places. Each of these 48 numbers is rounded off to the nearest integer. The sum of the original 48 numbers is approximated by the sum of these integers. If we assume that the errors made by rounding off are iid and have a uniform distribution over the interval  $(-\frac{1}{2}, \frac{1}{2})$ , compute approximately the probability that the sum of the integers is within two units of the true sum.