

SINGLE PHASE INDUCTION MOTOR

Single-phase motors are the most familiar of all electric motors because they are used in home appliances and portable machine tools.

There are many kinds of 1-ph motors on the market, each designed to meet a specific application.

The major problem associated with design of single-phase induction motors is that unlike 3-ph power sources, 1-ph source does not produce a rotating magnetic field, (i.e. we have **NO** starting torque: $T_{start} = \text{ZERO}$).

There are a number of special-purpose motors, these include universal motors, reluctance motors, hysteresis motors, stepper motors, and brushless D.C motor.

3.1 CONSTRUCTION OF 1-PH INDUCTION MOTOR:

Single-phase induction motors, are very similar to 3ph induction motors.

1-ph Induction motors (I.M) are composed of squirrel cage ROTOR (OR may be wound type with slip ring in particular applications), and the stator carries a MAIN WINDING, which create a set of N & S poles. It also carries a smaller auxiliary winding that only operates during the brief period when the motor starts up, Fig(3.1)

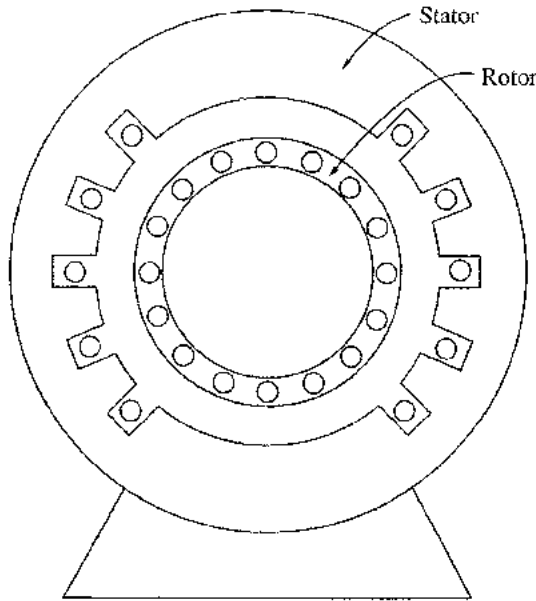


FIGURE 3.1
Construction of a single-phase induction motor. The rotor is the same as in a three-phase induction motor, but the stator has only a single distributed phase.

The auxiliary winding has the same number of poles as the main winding has.

3.2 TORQUE - SPEED CHARACTERISTIC: -

suppose the rotor is locked. IF an A.C Voltage is applied to the stator.

The resulting current (I_s) produces an A.C FLUX (Φ_s).

A schematic diagram of the rotor and main wind of 2-pole 1-ph induction motor is shown Fig(3.2)

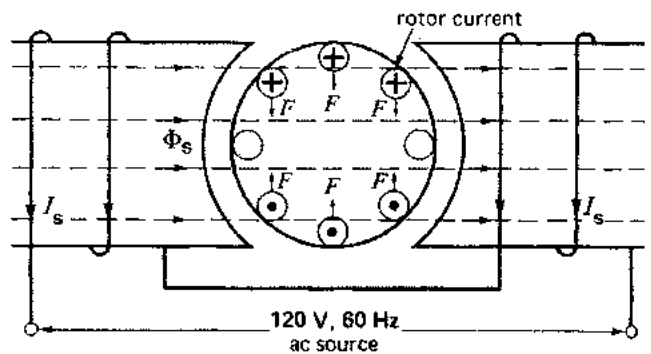
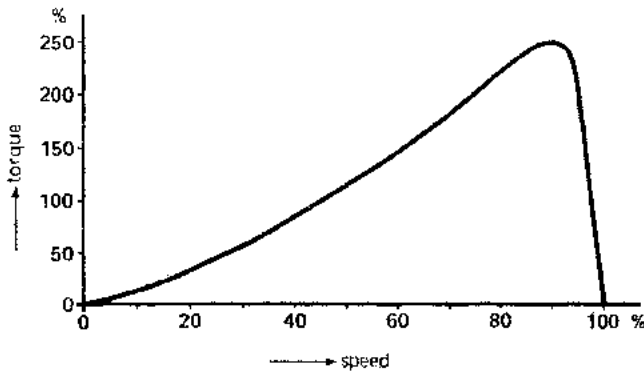


Figure 3-2
Currents in the rotor bars when the rotor is locked. The resulting forces cancel each other and no torque is produced.

Figure 3.3
Typical torque-speed curve of a single-phase motor.

The flux (Φ_s) pulsates back and forth, ³ But, unlike the flux in 3-ph stator, NO revolving field is produced.

The flux induces an A.C Voltage in the stationary rotor which, in turn, creates large A.C rotor currents.

In effect, the rotor behaves like the short-circuited (S.C) secondary of a transformer; consequently, the motor has no tendency to start by itself, see Fig (3.3) -

However, IF we spin the rotor in one direction or the other, it will continue to rotate in the same direction of spin.

The rotor quickly accelerates until it reaches a speed slightly below synchronous speed

The acceleration indicates that the motor develops +ve Torque.

Fig (3.3), shows the typical Torque-speed curve.

When the main winding is excited, although the starting Torque is ZERO, the motor develops a powerful Torque as it approaches synchronous speed.

However, once the rotor begins to turn, an induced Torque will be produced in it. There are two basic theories which explain why a Torque is produced in the rotor once it is turning:

- 1- The double-revolving-field theory
- 2- The cross-field theory.

1- The Double-Revolving-Field Theory: of 4-pole induction motors Basically states that a stationary pulsating magnetic field can be resolved into TWO rotating magnetic fields, each of equal magnitude But rotating in opposite directions.

The induction motor responds to each magnetic field separately, and the net Torque in the machine will be the sum of the Torque due to each of the Two magnetic fields. Fig (3.4) shows how a stationary magnetic field can be resolved

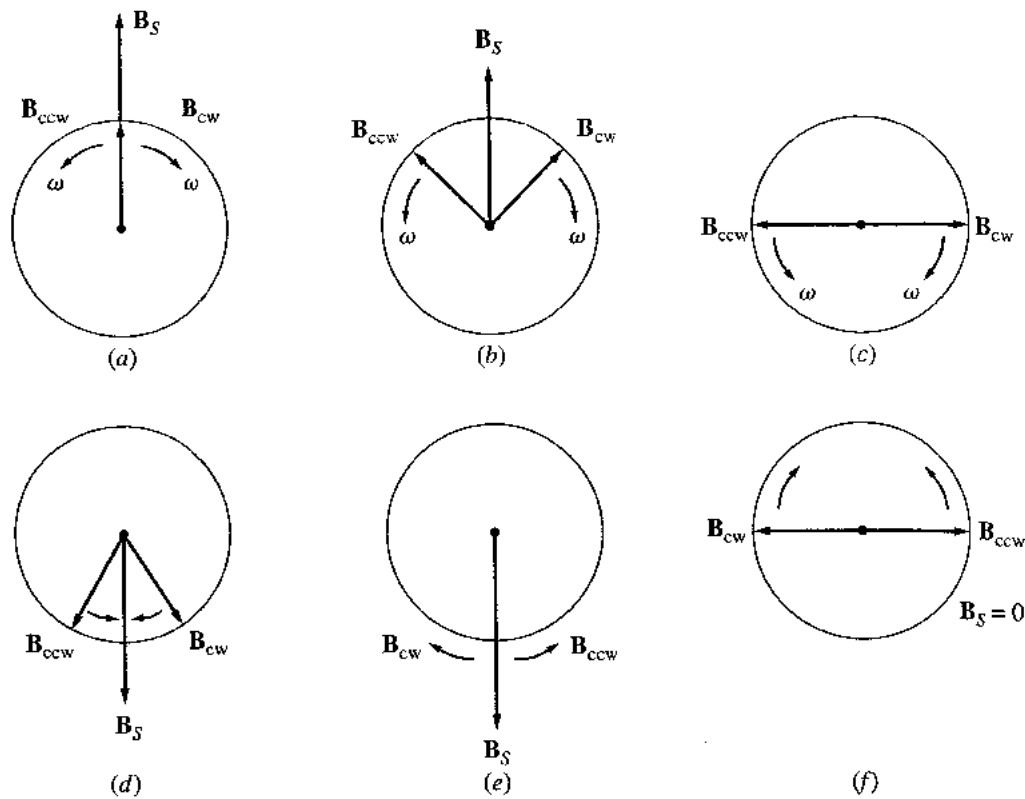


FIGURE 3.4

The resolution of a single pulsating magnetic field into two magnetic fields of equal magnitude by rotation in opposite directions. Notice that at all times the vector sum of the two magnetic fields lies in the vertical plane.

into Two equal and oppositely rotating magnetic fields.

The flux density of the stationary magnetic field is given By:

$$B_s(t) = (B_{\max} \cos \omega t) \hat{j} \quad \dots \dots \dots (3.1) \quad 5$$

A clock wise-rotating field can be expressed as:

$$B_{cw}(t) = \left(\frac{1}{2} B_{\max} \cos \omega t\right) \hat{i} - \left(\frac{1}{2} B_{\max} \sin \omega t\right) \hat{j} \quad \dots \dots \dots (3.2)$$

and a counter-clock wise-rotating magnetic field can be expressed as:

$$B_{ccw}(t) = \left(\frac{1}{2} B_{\max} \cos \omega t\right) \hat{i} + \left(\frac{1}{2} B_{\max} \sin \omega t\right) \hat{j} \quad \dots \dots \dots (3.3)$$

The Torque-speed characteristic of a 3-ph Induction motor in response to its single rotating magnetic field is shown in Fig (3.5a)

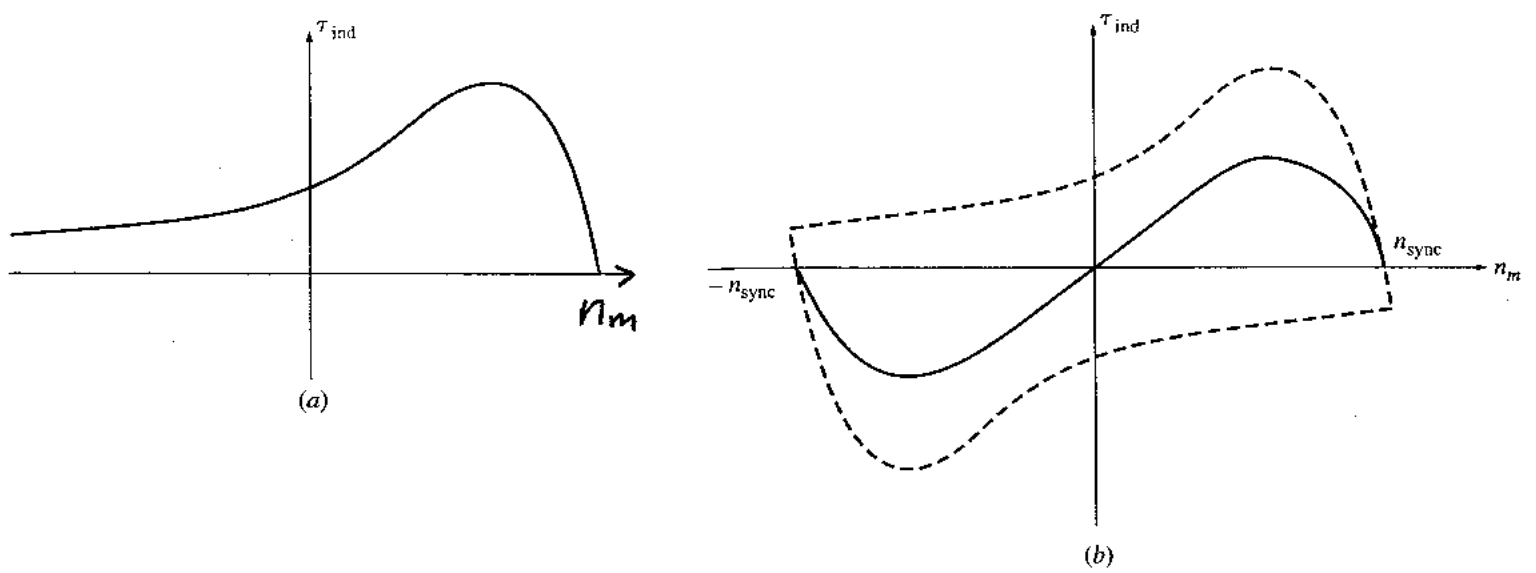


FIGURE 3.5
 (a) The torque-speed characteristic of a three-phase induction motor. (b) The torque-speed characteristic curves of the two equal and oppositely rotating stator magnetic fields.

A single-phase induction motor responds to each of the two magnetic field present within it, so the net induced torque in the motor is the difference between the two-speed curves. This net torque is shown in Fig (3.5b).

Notice that:

There is no net torque at ZERO speed, so this \Rightarrow

⇒ motor has no starting Torque. 6

IF power is applied to the motor while it is forced to turn Back-ward, its rotor currents will be very high,

However, the rotor frequency is also very high, making the rotor's reactance (X_2) is so very high i.e:

The rotor current Lags the rotor voltage By almost 90° .

⇒ producing a magnetic field that is nearly 180° from the stator magnetic field, see Fig(3.6)

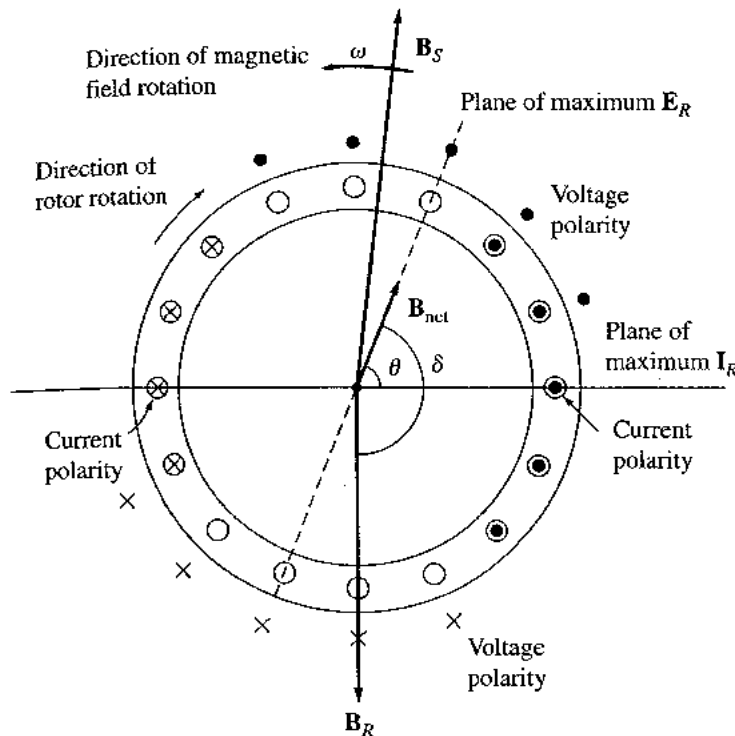


FIGURE 3.6

When the rotor of the motor is forced to turn backward, the angle δ between B_R and B_S approaches 180° .

The induced Torque in the motor is proportional to the sine of the angle between the two fields B_S & B_R

$$\tau_{ind} \propto \sin \delta$$

$$\Rightarrow \delta \text{ is near } 180^\circ$$

$$\Rightarrow \sin(\approx 180^\circ) \approx \text{ZERO}$$

so that τ_{ind} would be very small. 7

The forward and reverse magnetic fields in the motor each contribute a component to the total voltage in stator and, in a sense, are in series each other, because both magnetic fields are present, the forward-rotating magnetic field will limit the stator current flow in the motor, and since the reverse rotor magnetic field is at a very large angle with respect to the reverse stator magnetic field, the torque due to reverse mag. fields is very small near synchronous speed.

2. The cross-field theory of 1-ph I.M.

This theory is concerned with the voltages and currents that the stationary stator magnetic field can induce in the bars of the rotor when the rotor is moving.

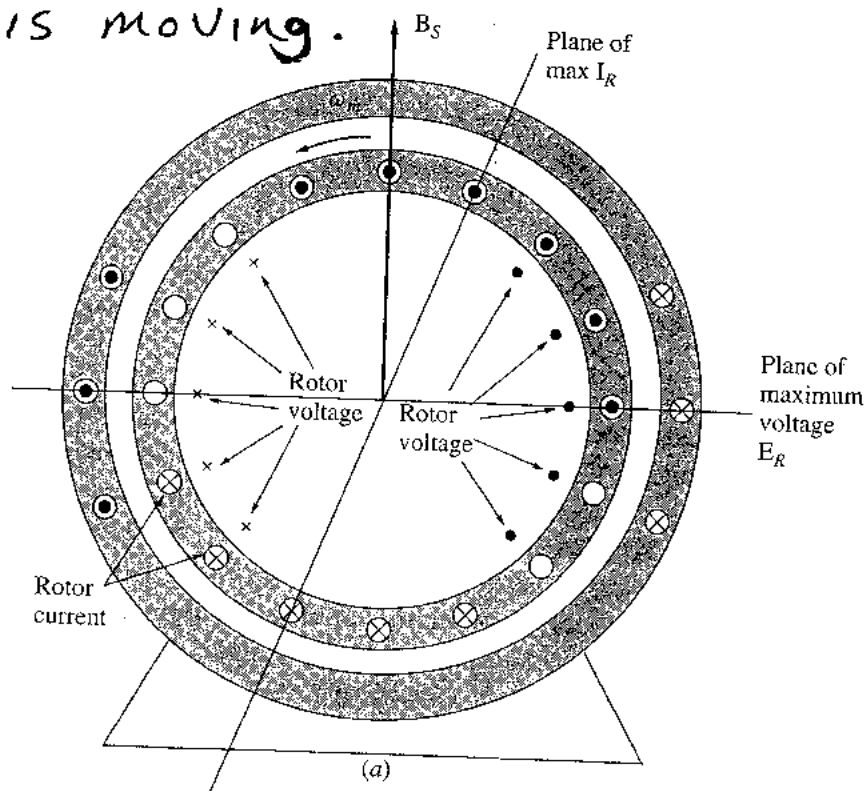


FIGURE 3.7

(a) The development of induced torque in a single-phase induction motor, as explained by the cross-field theory. If the stator field is pulsing, it will induce voltages in the rotor bars, as shown by the marks inside the rotor. However, the rotor current is delayed by nearly 90° behind the rotor voltage, and if the rotor is turning, the rotor current will peak at an angle different from that of the rotor voltage.

consider a single-phase induction motor ⁸ with a rotor which has been brought up to speed By some external method, Fig (3.7a)

voltages are induced in the Bars of this rotor, with peak voltage occurring in the winding passing directly under the stator windings.

These rotor voltages produce a current flow in the rotor. Because of the rotor's high reactance, the current lags the voltage by almost 90° .

Since the rotor is rotating at nearly synchronous speed, that 90° time lag in current produces an almost 90° angular shift between the plane of peak current and the plane of peak rotor voltage.

The resultant magnetic field is shown in Fig (3.7b)

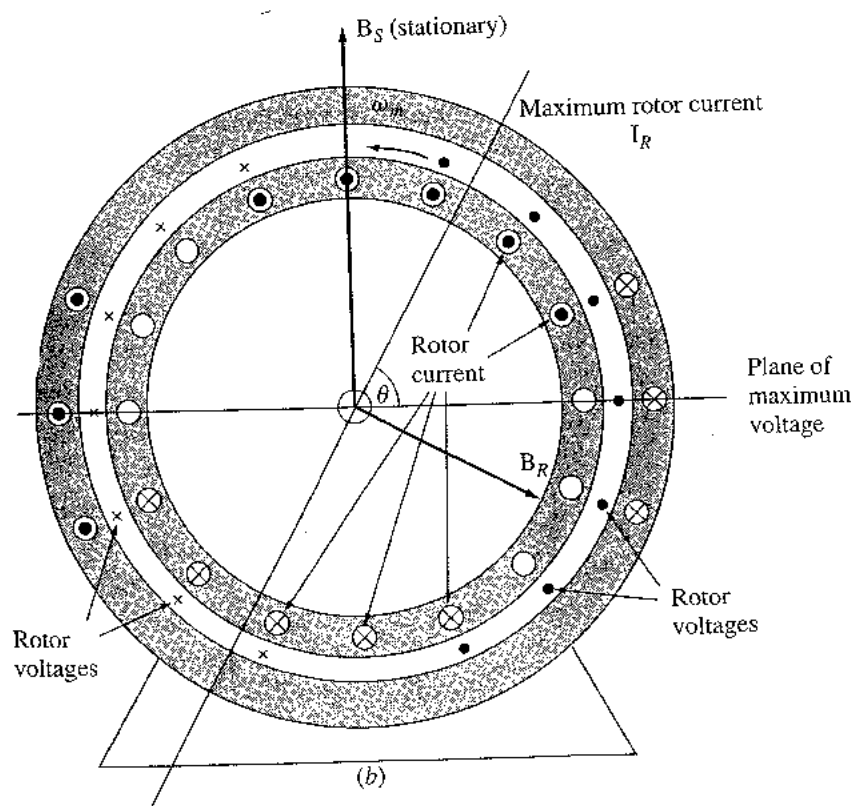


FIGURE 3.7 (concluded)

(b) This delayed rotor current produces a rotor magnetic field at an angle different from the angle of the stator magnetic field.

The rotor magnetic field B_R is somewhat smaller than the stator magnetic field B_S , because of the losses in the rotor, but they differ by nearly 90° in both SPACE & TIME, Fig (3.8a)

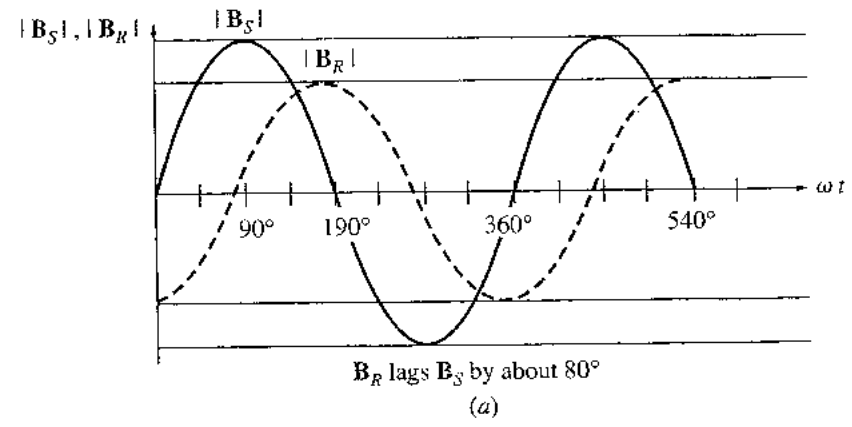


FIGURE 3.8
(a) The magnitudes of the magnetic fields as a function of time.

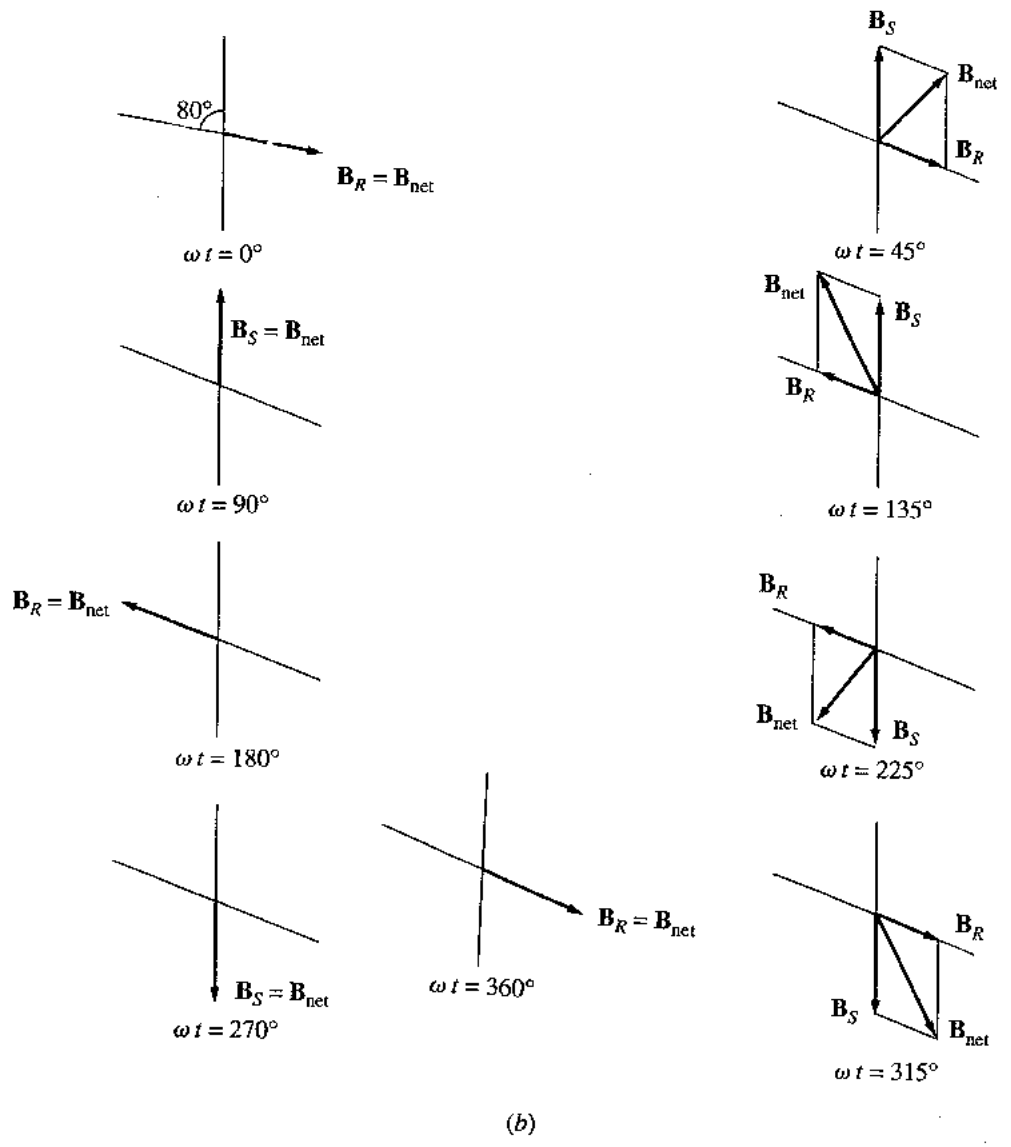


FIGURE 3.8 (concluded)
(b) The vector sum of the rotor and stator magnetic fields at various times, showing a net magnetic field which rotates in a counterclockwise direction.

IF these Two magnetic field are added ¹⁰ at different times, One sees that the total magnetic field in the motor is rotating in a counter clockwise direction, see Fig (3.8b), with a rotating magnetic field present in the motor, the induction motor will develop a net Torque in the direction of motion, and that Torque will keep the rotor turning

IF the motor's rotor had originally been turned in a clock wise direction, the resulting Torque would be clockwise and would again keep the rotor turning.

3.3 SYNCHRONOUS SPEED OF 1-PH I.M.:-

As in the case of 3ph motors, the synchronous speed of all single-phase induction motors is given by the equation:

$$n_s = \frac{120f}{p} \quad \text{--- (3.4)}$$

where

n_s = Synchronous speed [r/min].

f = frequency of the source [Hz].

p = number of poles

The slip (S) of induction motor is the difference between the synchronous speed and the rotor speed.

expressed as a percent [OR Per-unit (P.u)] of synchronous speed.

The slip (S) is given by the Eqn:

$$S = \frac{n_s - n_r}{n_s} \quad \text{--- (3.5)}$$

S = slip

n_s = Synchronous speed. [r/min]

n = rotor speed. [r/min]

The slip is practically at.

NO-Load $S = \text{ZERO}$.

stand still (locked) $S = 1$.

and the full-load slip is typically

3% $\xrightarrow{T_0}$ 5% for fractional h.p motors

Example 3.1:-

calculate the speed of the 4-pole, single-phase motor, if the slip at full-load is 3.4 percent. The line frequency is 60 Hz

Solution:

The motor has 4 poles, consequently

$$n_s = \frac{120f}{P} = \frac{120 \times 60}{4}$$

$$n = 1800 \text{ r/min}$$

The speed n is given by

$$S = \frac{n_s - n}{n_s}$$

$$0.034 = \frac{1800 - n}{1800}$$

$$\therefore n = 1739 \text{ r/min}$$

3.4 STARTING SINGLE PHASE INDUCTION MOTOR

A single-phase induction motor has no intrinsic starting Torque.

There are three techniques commonly used to start these motors, and 1-ph induction motors are classified according to the methods used to produce their starting Torque.

The 3 major starting techniques are:

- 1- split-phase windings.
- 2- capacitor-type windings.
- 3- shaded stator poles

ALL three starting techniques are methods of making one of the two revolving magnetic fields in the motor stronger than the other

3.4.1 SPLIT-PHASE MOTOR: (split-phase wdgs)

A split-phase motor is a single-phase induction motor with two stator windings a main stator winding (M) and an auxiliary starting winding (A), see Fig (3.9)

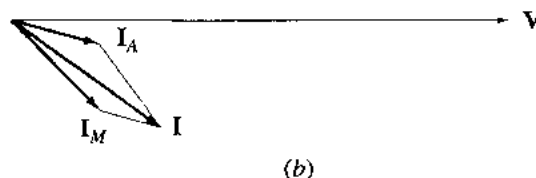
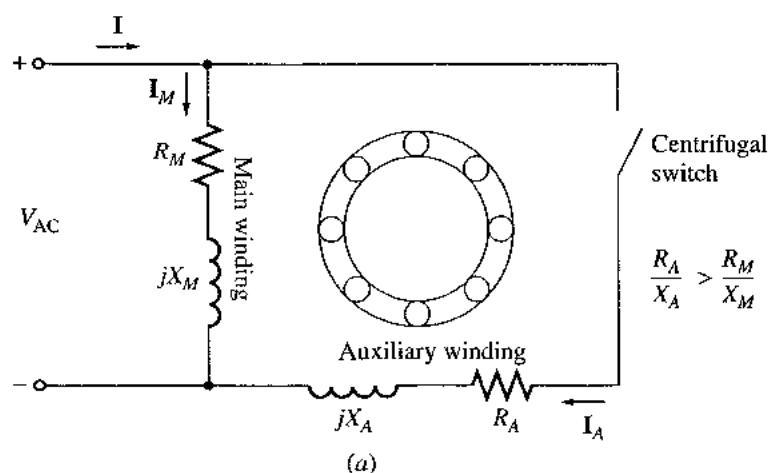


FIGURE 3.9

(a) A split-phase induction motor. (b) The currents in the motor at starting conditions.

These Two windings are set 90° electrical / 4 degrees apart along the stator of the motor. and the auxiliary winding is designed to be switched out of the circuit at some set speed by a centrifugal switch.

The auxiliary winding is designed to have a higher (resistance/reactance) ratio than the main winding.

The higher (R/X) ratio is usually accomplished by using smaller wire for the auxiliary winding.

Smaller wire is permissible in auxiliary winding because it is used only for starting and therefore does not have to take full current continuously.

To understand the function of the auxiliary winding, refer to Fig (3.10), since the current in the auxiliary windings leads the current in the main winding, the magnetic fields (B_A) peaks before the main magnetic field (B_M).

Since (B_A) peaks first and then (B_M) there is a net counter-clockwise rotation in the magnetic field.

In other words, the auxiliary winding makes one of the oppositely rotating stator magnetic fields larger than the other one and provides a net starting torque for the motor

A typical torque-speed characteristic is shown in Fig (3.10c).

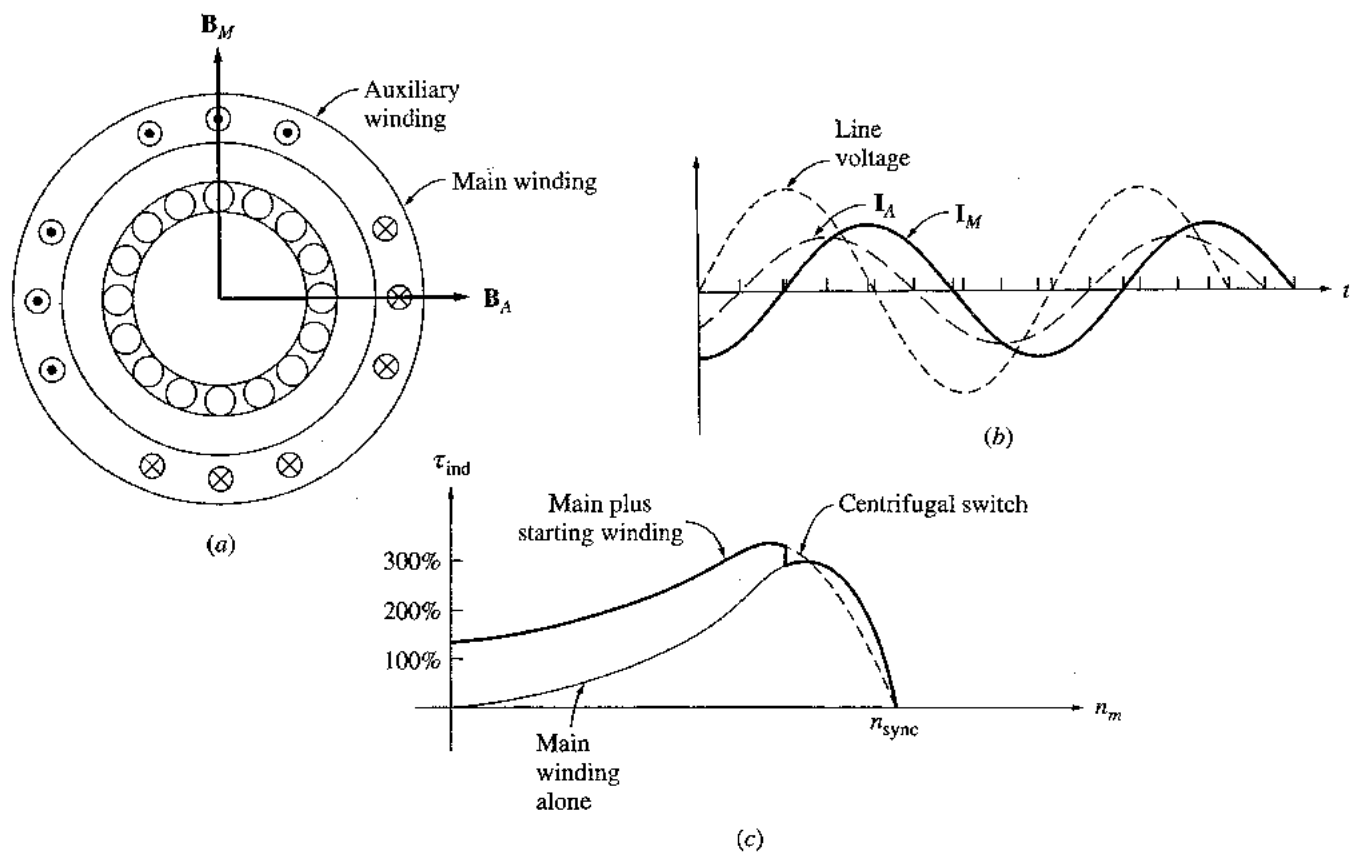


FIGURE 3.10

(a) Relationship of main and auxiliary magnetic fields. (b) I_A peaks before I_M , producing a net counter-clockwise rotation of the magnetic fields. (c) The resulting torque-speed characteristic.

split-phase motors are used for applications which do not require very high starting torque such as:

Fans, Blowers, and centrifugal pumps.

The direction of rotation of the motor can be reversed by changing the connections of the auxiliary winding while leaving the main windings connections unchanged.

Example 3.2 :-

A resistance split-phase motor is rated at 1/4 hp (187W), 1725 r/min, 115V 60 Hz, when the rotor is locked, a test at reduced voltage on the main and auxiliary windings yields the following results:

	main winding	auxiliary winding
applied voltage	$V_{AC} = 23 \text{ V}$	$V_{AC} = 23 \text{ V}$
current	$I_M = 4 \text{ AMP}$	$I_A = 1.5 \text{ AMP}$
active Power	$P_M = 60 \text{ watt}$	$P_A = 30 \text{ watt}$

calculate :-

- The phase angle Between I_A & I_M ?
- The Locked-rotor current drawn from the Line current at 115 V ?

Solution :-

we first calculate the phase angle Φ_M Between I_M & V_{AC} .

a - The apparent power for main wdg is

$$S_M = V_{AC} * I_M = 23 * 4 = 92 \text{ VA}$$

The Power factor P.f

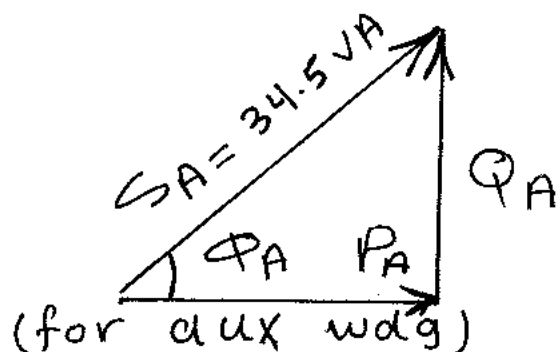
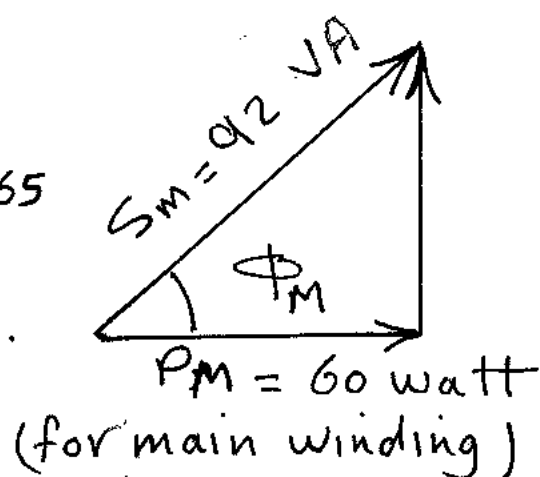
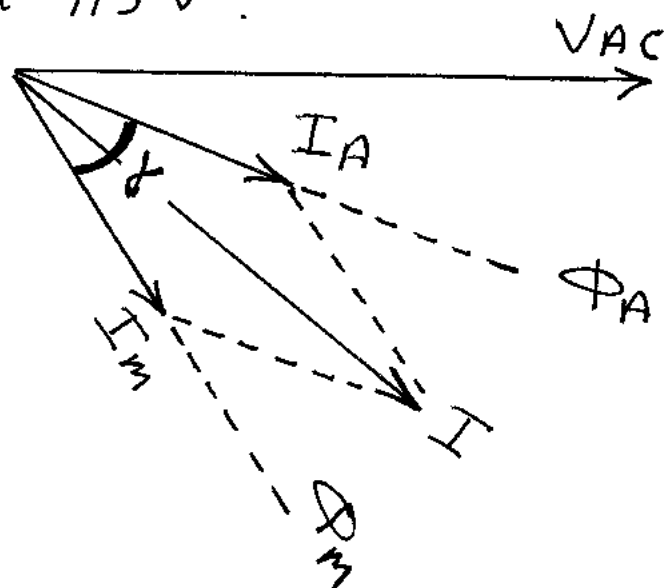
$$\cos \Phi_M = P_M / S_M = 60 / 92 = 0.65$$

Thus $\Phi_M = 49.6$

(i.e I_M Lags 49.6 Behind V_{AC}).

* for the aux. winding

We calculate the phase angle Φ_A Between I_A and V_{AC} .



$$S_A = V_{AC} * I_A = 23 * 1.5 = 34.5 \text{ VA.}$$

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The p.f is

$$\cos \Phi_A = P_A / S_A = 30 / 34.5 = 0.87$$

thus

$$\Phi_A = \cos^{-1} 0.87 = 29.6^\circ$$

I_A Lags 29.6° behind the voltage V_{AC} .

The phase angle Between I_M & I_A is:

$$\alpha = \angle \Phi_M - \angle \Phi_A = 49.6 - 29.6 = 20.0^\circ$$

b. To determine the total Line current, we must calculate total value of P & Q drawn By Both windings and deduce the apparent power S

* The total active power:

$$P = P_M + P_A = 60 + 30 = 90 \text{ watt}$$

The reactive power Q_M & Q_A are:

$$Q_M = \sqrt{S_M^2 - P_M^2} = \sqrt{92^2 - 60^2} = 69.7 \text{ VAR}$$

$$Q_A = \sqrt{S_A^2 - P_A^2} = \sqrt{34.5^2 - 30^2} = 17.0 \text{ VAR}$$

The total reactive power is:

$$Q = Q_M + Q_A = 69.7 + 17.0 = 86.7 \text{ VAR}$$

The total apparent power absorbed is:

$$S = \sqrt{P^2 + Q^2} = \sqrt{90^2 + 86.7^2} = 125 \text{ VA}$$

The Locked-rotor current at $V_{AC} = 23 \text{ V}$ is:

$$I_{\text{Locked}} = S / V_{AC} = 125 / 23 = 5.44 \text{ Amp}$$

and the Locked current at 115 volt is

$$I_{\text{Locked}}(\text{at } 115 \text{ volt}) = 5.44 \times \frac{115 \text{ V}}{23 \text{ V}} = 27.2 \text{ Amp}$$

3.4.2 CAPACITOR-START MOTOR:

For some applications, the starting Torque supplied by a split-phase motor is insufficient to start the Load on a motor's shaft.

In those cases, Capacitor-start motors may be used, Fig (3.11)

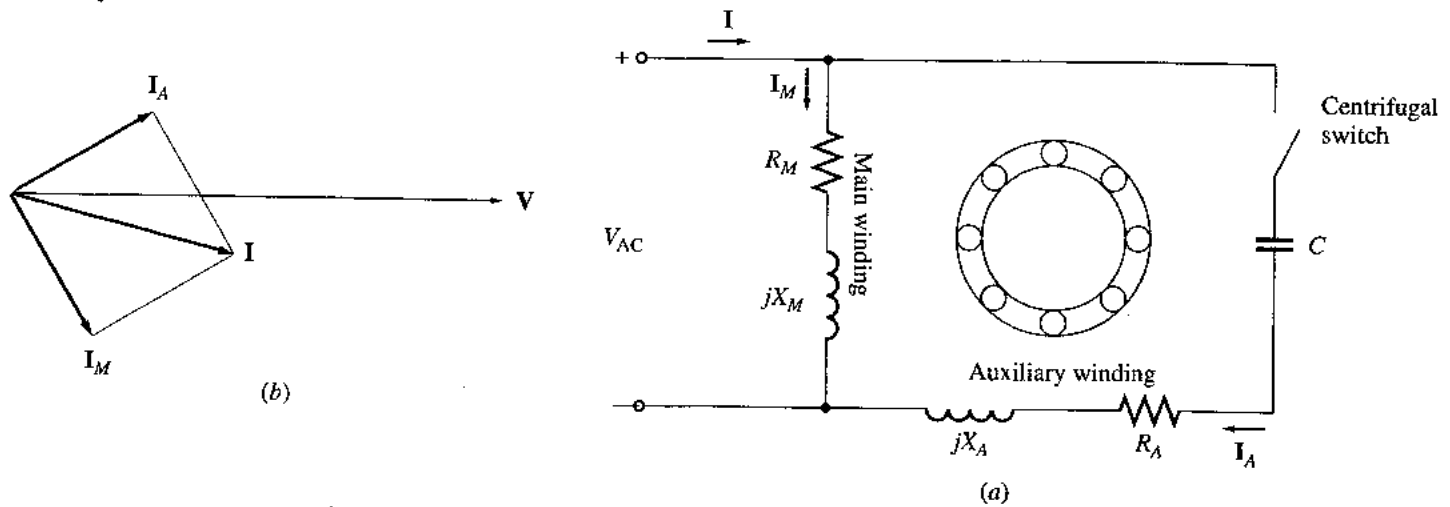


FIGURE 3.11

(a) A capacitor-start induction motor. (b) Current angles at starting in this motor.

In a capacitor-start motor, a capacitor is placed in series with the auxiliary winding of the motor.

By proper selection of capacitor size, the magnetomotive force of the starting current in the auxiliary winding can be adjusted to be EQUAL to the magnetomotive force of the current (I_M) in the main winding, and the phase angle of the current in the auxiliary winding can be made to lead the current in the main winding by 90° .

Since the two windings are physically separated by 90° , a 90° phase difference in current will yield a single uniform \Rightarrow

⇒ rotating stator magnetic field. 19

In this case, the start Torque of the motor can be more than 300% of its rated value, see Fig (3.12).

capacitor-start motors are more expensive than split-phase motors, and they are used in applications where a high starting

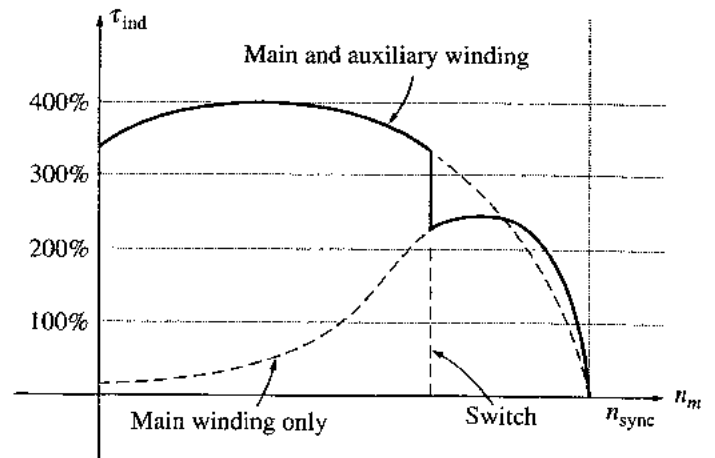


FIGURE 3.12 Torque-speed characteristic of a capacitor-start induction motor.

Torque is absolutely required. applications for such motors are:

Compressors, Pumps, air conditions

Example 3.3 :-

A 2.5 kW, 120V, 60 Hz capacitor-start motor has the following impedances for the main and auxiliary winding (at starting):

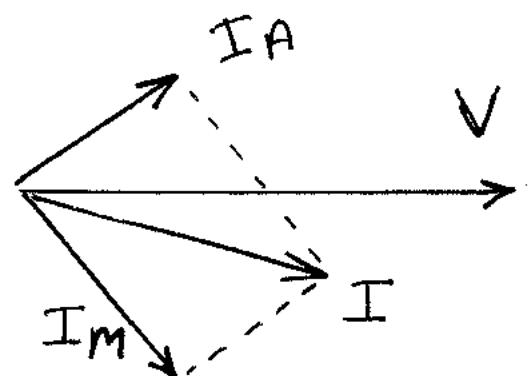
$$Z_{\text{main}} = 4.5 + j3.7 \Omega \quad \text{main winding}$$

$$Z_{\text{aux}} = 9.5 + j3.5 \Omega \quad \text{auxiliary winding.}$$

Find the value of starting capacitance that will place the main and auxiliary winding current in quadrature at starting.

Solution:-

The currents I_M & I_A are shown in Fig(3.11a)



The impedance angle of the main wdg is: 20

$$\Phi_m = \tan^{-1} \left(\frac{3.7}{4.5} \right) = 39.6$$

To produce currents in time quadrature with the main winding, the impedance angle of aux. wdg cct (including the starting capacitor) must be:

$$\phi = 39.6^\circ - 90.0^\circ = -50.4^\circ$$

The combined impedance of the aux. wdg. and starting capacitor is equal to:

$$Z_{\text{Total}} = Z_{\text{aux}} + jX_c = 9.5 + j(3.5 + X_c) \Omega$$

where

$X_c = -\frac{1}{\omega C}$ is the reactance of the capacitor

and $\omega = 2\pi f = 2\pi 60 \approx 377 \text{ rad/sec}$

thus

$$\tan^{-1} \left(\frac{3.5 + X_c}{9.5} \right) = -50.4^\circ$$

$$\therefore \frac{3.5 + X_c}{9.5} = \tan(-50.4^\circ) = -1.21$$

and hence:

$$X_c = -1.21 \times 9.5 - 3.5 = -15.0 \Omega$$

The capacitance C is then:

$$C = \frac{-1}{\omega X_c} = \frac{-1}{377 * (-15.0)} = 177 \mu\text{F}$$

Exercise:- consider the motor of example 3.2. Find the phase angle between the main and aux. wdgs current if the 177 μF capacitor is replaced by a 200 μF capacitor

ANS: 85.2°

3.4.3 : SHADED-POLE MOTORS:

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Ashaded-pole 1-Ph Induction motors is with only a main winding,

Instead of having an auxiliary winding, it has salient poles, and one portion of each pole is surrounded By a short-circuited (s.c) Coil called a SHADING COIL, see Fig (3.13a)

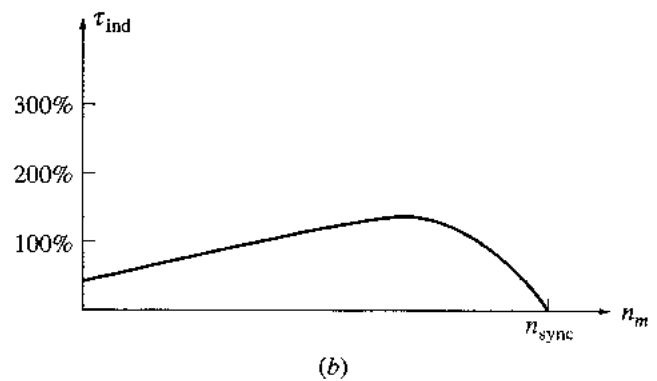
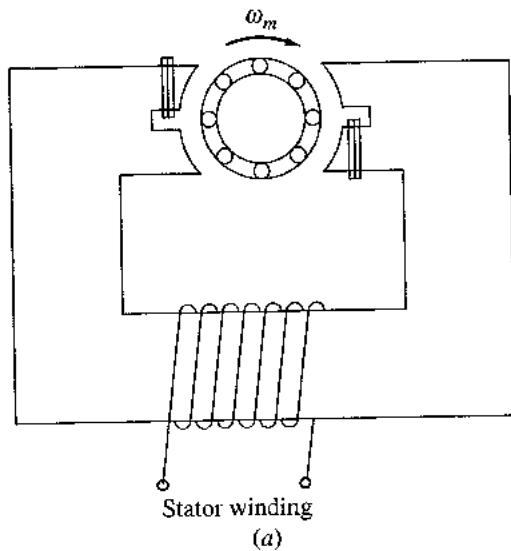


FIGURE 3.13

(a) A basic shaded-pole induction motor. (b) The resulting torque-speed characteristic.

A Time Varying flux is induced in the poles By the main winding.

When the pole flux varies, it induces a voltage and current in the shading coil which opposes the original change in flux.

The opposition retards the flux changes under shaded portions of the coils and therefore produce a slight imbalance between the two oppositely rotating stator magnetic fields.

The net rotation is in the direction from the unshaded to the shaded portion of the pole face.

The Torque-speed characteristic of a shaded-pole motor is shown in Fig (3.13b) 22

shaded poles produce less starting torque than any other type of induction motor starting system.

shaded-pole motors are much less efficient, and have a much higher slip than other types of single-phase induction motors.

Such poles are used only in very small motors (1/20 hp) with very low starting torque requirements.

where it is possible to use them, shaded-pole motors are the cheapest design available

There is no easy way to reverse the direction of rotation of shaded motor.

To achieve reversal, it is necessary to install two shading coils on each pole face and to selectively short one or the other of them

Example 3.4: -

_____ calculate the full-load efficiency and slip of the shaded-pole motor whose properties are listed in the table 18.

Solution:

$$\eta = \frac{P_o}{P_i} \times 100$$

$$= (0.286) \times 100 = 28.6 \%$$

$$S = \frac{n_s - n}{n_s} \Rightarrow S = \frac{3600 - 2900}{3600}$$

$$S = 0.194 \quad \text{OR} \quad 19.4\%$$

TABLE 18

Properties of a Shaded-Pole Motor, Having 2 poles,
Rated 6 W, 115 V, 60 Hz.

No-load	
current	0.26 A
input power	15 W
speed	3550 r/min
Locked rotor	
current	0.35 A
input power	24 W
torque	10 mN·m
Full-load	
current	0.33 A
input power	21 W
speed	2900 r/min
torque	19 mN·m
mechanical power	6 W
breakdown speed	2600 r/min
breakdown torque	21 mN·m

3.5 SPEED CONTROL OF 1-PH I.Ms :-

The speed of single phase Induction motor's may be controlled in the same manner as the speed of poly phase induction motors.

For squirrel-cage rotor motors, the following Techniques are available:

- 1- Vary the stator frequency.
- 2- change the number of poles.
- 3- change the applied Terminal Voltage V_T

In practical for high-slip motors, the usual approach to speed control is to:

Vary the terminal voltage of the motor.

The voltage applied to a motor may be varied in one of 3 ways: Fig (3.14).

1- An autotransformer may be used to adjust the line voltage.

2- An thyristor (SCR) or TRIAC circuit may be used to reduce the v.m.s Voltage (V_{ge}) applied to the motor by A.C phase control

3- A resistor may be inserted in series with the motor's stator circuit.

This is the cheapest method of Voltage control.

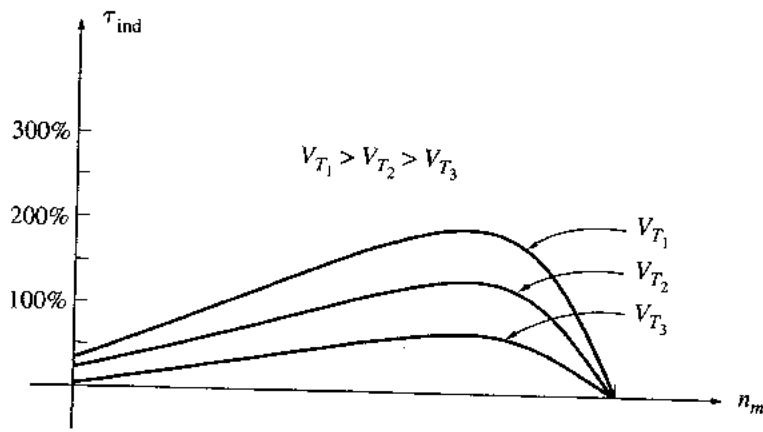


FIGURE 3.14

The torque-speed characteristic of a shaded-pole induction motor as the terminal voltage is changed. Increases in V_T may be accomplished either by actually raising the voltage across the whole winding or by switching to a lower tap on the stator winding.

3.6 REVERSING THE DIRECTION OF ROTATION

In order to reverse the direction of rotation of the 1ph motors we have to interchange the Leads of either the auxiliary winding OR the main winding

In the case of a very small capacitor-run motors, the rotation can be reversed by using a double-throw switch as shown in Fig (3.15)

switch on Position

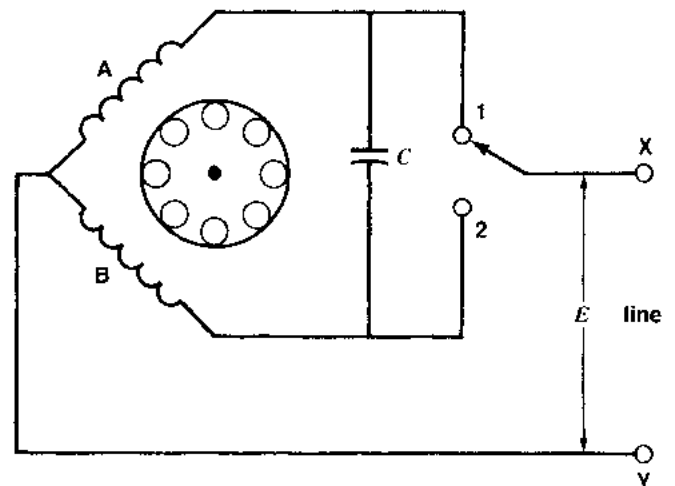
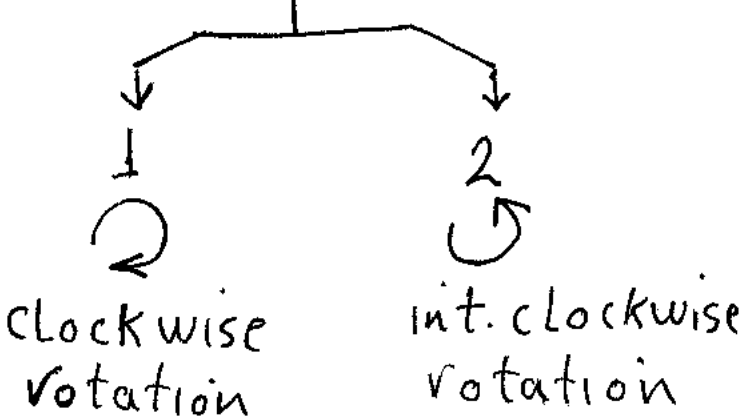


Figure 3.15

Reversible single-phase motor using a 2-pole switch and capacitor.

The role of wdg is reversed, the motor will come to a halt (stop) then run up to speed in the opposite direction

3.7 EFFICIENCY AND POWER FACTOR : - 26

The efficiency (η) and (P.f) Power factor of fractional horse power 1 Ph I.M are usually Low.

Thus, at full-load a 180 watt motor ($\approx 1/4$ h.p) has an efficiency (η) and power factor (P.f) of about 60%.

The Low power factor is mainly due to the large magnetizing current, consequently, even at NO-LOAD these motors have substantial temperature rises.

The relatively low efficiency and power factor of these motors is a consequence of their fractional horse power ratings.

Integral horse power 1 Ph motors can have efficiencies and P.f above 80%.

3.8 EQUIVALENT CIRCUIT OF 1-PH I.M. : -

Consider conditions with the rotor stationary and only the main stator winding excited.

The motor then is equivalent to transformer with its secondary short-circuited.

The equivalent circuit is shown in Fig (3.16a) where: $R_{1\text{main}}$ = The resistance of the main winding.

$X_{1\text{main}}$ = The leakage reactance of the main wdg.

$R_{2\text{main}}$ = The rotor resistance value at standstill

$X_{2\text{main}}$ = The rotor leakage reactance at standstill

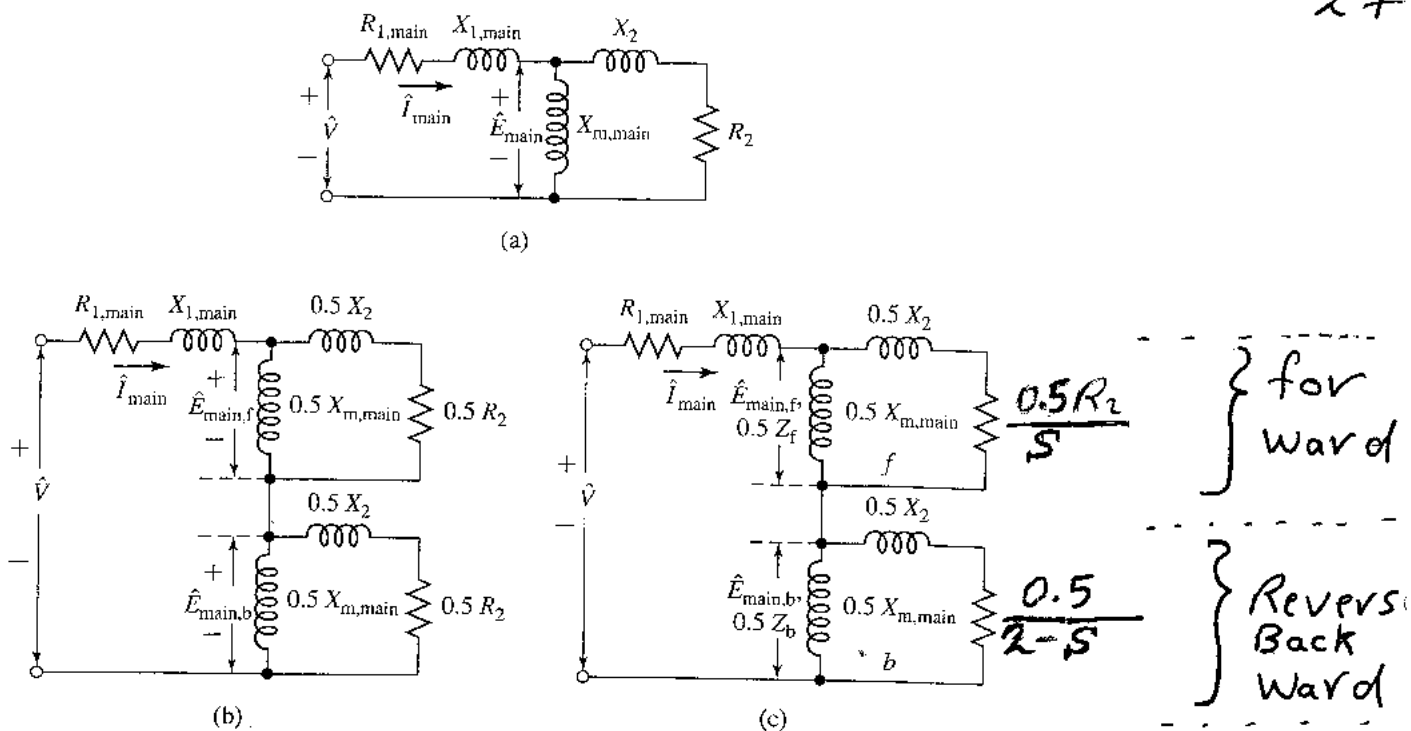


Figure 3.16 Equivalent circuits for a single-phase induction motor: (a) rotor blocker (b) rotor blocked, showing effects of forward and backward fields; (c) running conditions.

$R_{2\text{main}}$ and $X_{2\text{main}}$ referred to the main stator winding by the use of the appropriate turns ratio.

Core Loss, is omitted here (as stand still).

The applied voltage is (\hat{V}) and the main winding current is (\hat{I}).

\hat{E}_{main} = is the counter e.m.f generated in the main wdg by the stationary pulsating air-gap flux.

In accordance with the double-revolving field concept, the stator m.m.f can be resolved into half-amplitude **FORWARD AND BACKWARD** rotating fields.

At stand still the amplitude of the forward and backward resultant air-gap flux waves both equal **HALF** the amplitude of the pulsating field.

In Fig (3.15b), The effects of the air-gap flux is split into TWO EQUAL portions. 28

Now consider the motor has been Brought up to speed By some auxiliary mains and is running on only its main winding in the direction of the forward field are of slip frequency (Sf_e).

f_e = is the stator applied electrical Freq.
0.5 = The factors of resolution of pulsating m.m.F

The rotor is still turning at a slip (S) with respect to the forward field, and (n) is its per-unit (P.U) speed (n) in the direction of the forward field given By:

$$n = 1 - S \quad \text{P.U}$$

speed with Backward field is:

$$= 1 + n$$

or its slip with respect to the Backward field is:

$$1 + n = 2 - S$$

The Backward field then induces rotor current whose frequency is:

$$(2 - S)f_e$$

By use the Equivalent circuit (3.16c), the stator current, Power input (IIP), and Power factor (P.f) can Be computed for any assumed Value of slip when the applied voltage and the motor impedances are known.

Let:

$$Z_F = R_F + jX_F \equiv \left(\frac{R_{2\text{main}}}{s} + jX_{2\text{main}} \right) \text{ in Parallel} \quad (1) \quad jX_{m\text{main}} \quad \dots \dots \dots (3.6)$$

$$Z_b = R_b + jX_b \equiv \left(\frac{R_{2\text{main}}}{2-s} + jX_{2\text{main}} \right) \text{ in Parallel} \quad (1) \quad jX_{m\text{main}} \quad \dots \dots \dots (3.7)$$

The electromagnetic Torque ($T_{\text{main}f}$) of forward field is :

$$T_{\text{main}f} = \frac{1}{\omega_s} P_{\text{gap}f} \quad [\text{N.m}] \quad \dots \dots \dots (3.8)$$

ω_s = Synchronous angular velocity [rad/s]

P_{gap} = air-gap power [watts]

when the magnetizing Impedance is treated as purely inductive ($P_{\text{gap}f}$) is the power absorbed by the impedance ($0.5 Z_F$); that is

$$P_{\text{gap}f} = I^2 (0.5 R_F) \quad \dots \dots \dots (3.9)$$

R_F = is the resistive component of the forward-field impedance.

Similarly:

The internal Torque ($T_{\text{main}B}$) of the Backward field is:

$$T_{\text{main}B} = \frac{1}{\omega_s} P_{\text{gab}B} \quad \dots \dots \dots (3.10)$$

$P_{\text{gab}B}$ = is the power delivered by the stator winding to the Backward field, OR:

$$P_{gap.B} = I^2 (0.5 R_B) \quad \dots \dots \dots (3.11) \quad 30$$

R_B = is the resistive component of the Back ward - field Impedance.

The Torque ($T_{main.B}$) is in opposite direction to that of ($T_{main.F}$).

Therefore The net internal Torque is:

$$T_{mech} = T_{main.F} - T_{main.B} = \frac{1}{\omega_s} (P_{gap.F} - P_{gap.B}) \quad \dots \dots \dots (3.12)$$

The Total rotor $I^2 R$ Loss is the numerical sum of the Losses

Thus

$$\text{Forward - field rotor } (I^2 R) \text{ loss} = S P_{gap.F} \quad \dots \dots \dots (3.13)$$

$$\text{Backward - field rotor } (I^2 R) \text{ Loss} = (2 - S) P_{gap.B} \quad \dots \dots \dots (3.14)$$

$$\text{Total rotor } (I^2 R) \text{ Loss} = S P_{gap.F} + (2 - S) P_{gap.B} \quad \dots \dots \dots (3.15)$$

In general:

Power = angular velocity x Torque

$$P_{mech} = \omega_r \times T_{mech}$$

ω_r - is the angular velocity of the rotor and is given By:

$$\omega_r = (1 - S) \omega_s \quad \dots \dots \dots (3.16)$$

Therefore

$$P_{mech} = (1 - S) \omega_s T_{mech}$$

Using Eqn (3.12) we have:

$$\therefore P_{mech} = (1 - S) \omega_s T_{mech} = (1 - S) (P_{gap.F} - P_{gap.B}) \quad \dots \dots \dots (3.17)$$

The internal Torque (T_{mech}) and internal Power (P_{mech}) are not the output (O/P) values because the rotational losses remain to be accounted for, Fig (3.17) shows the power flow of I.M

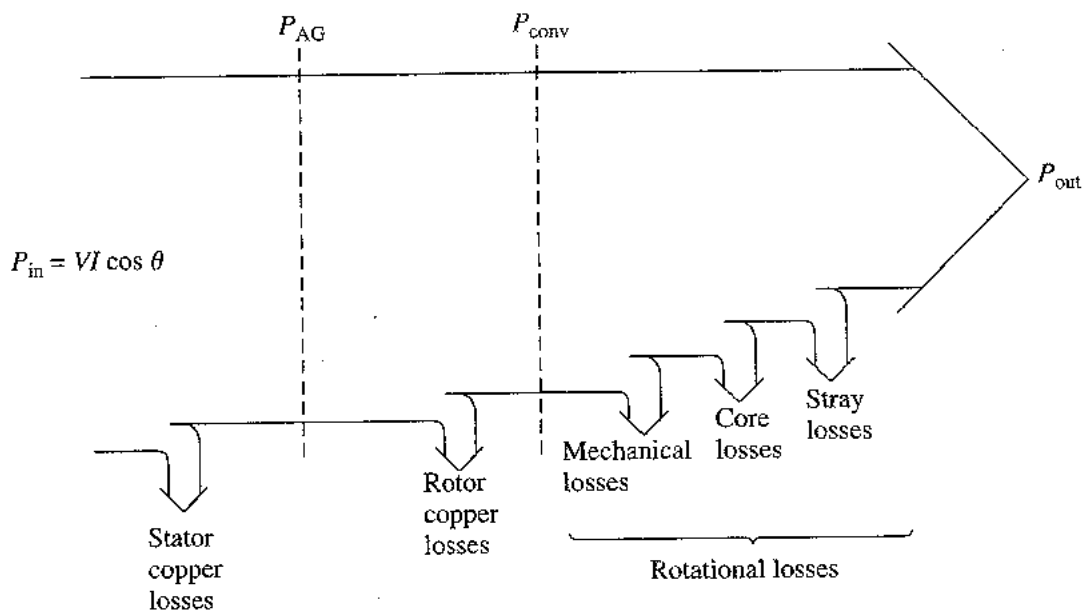


FIGURE 3.17
The power-flow diagram of a single-phase induction motor.

Example 3.5 :-

A $\frac{1}{4}$ h.p, 110 V, 60 Hz, 4 pole capacitor-start motor has the following equivalent circuit parameter values (in Ω) and losses:

$$R_{1main} = 2.02 \quad X_{1main} = 2.79 \quad R_{2main} = 4.12$$

$$X_{2main} = 2.12 \quad X_{mmain} = 66.8$$

$$\text{Core Loss} = 24 \text{ W} \quad \text{Friction and wdg loss} = 13 \text{ W}$$

For a slip of 0.05, determine the stator current, Power factor, Power out put, Speed, Torque, and efficiency when this motor is running as 1 ph motor at rated voltage and freq. with starting

winding open :

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Solution:

The first step is to determine the values of the forward and Backward field impedances at the assigned value of slip

For the forward-field Impedance Z_F

$$R_F = \left(\frac{X_{m.main}^2}{X_{22}} \right) \frac{1}{s \phi_{2main} + \frac{1}{(s \phi_{2main})}}$$

$$X_F = \frac{X_{2main} + X_{m.main}}{X_{22}} + \frac{R_F}{s \phi_{2main}}$$

where:

$$X_{22} = X_{2main} + X_{m.main} \quad \& \quad \phi_{2main} = \frac{X_{22}}{R_{2main}}$$

substitution of numerical values for $s = 0.05$

$$Z_F = R_F + j X_F = 31.9 + j 40.3 \Omega$$

For the Backward-field impedance Z_b :
substituting $(2-s)$ for s i.e. $s \rightarrow (2-s)$

$$R_b = \frac{R_{2main}}{2-s} \left(\frac{X_{m.main}}{X_{22}} \right)^2$$

$$X_b = \frac{X_{2main} X_{m.main}}{X_{22}} + \frac{R_b}{(2-s) \phi_{2main}}$$

substitution of numerical values and $s = 0.05$:

$$Z_b = R_b + j X_b = 1.98 + j 2.12 \Omega$$

Addition of the series elements in Fig(3.16) gives:

$$R_{1main} + j X_{1main} = 2.02 + j 2.79$$

$$0.5(R_F + j X_F) = 15.95 + j 20.15$$

$$0.5(R_b + j X_b) = 0.99 + j 1.06$$

$$\hline \text{Total input } Z = 18.96 + j 24.00 = 30.6 \angle 51.6$$

$$\text{stator current } I = \frac{V}{Z} = \frac{110}{30.6} = 3.59 \text{ Amp.}^{33}$$

$$\text{Power input} = P_{in} = VI \cos \theta = 110 \times 3.59 \times 0.62 = 244 \text{ W}$$

The power absorbed by the forward field Eqn (3.9) is:

$$P_{gap.f} = I^2 (0.5 R_F) = 3.59^2 \times 15.95 = 206 \text{ W}$$

The power absorbed by the Backward field Eqn (3.11)

$$P_{gap.b} = I^2 (0.5 R_B) = 3.59^2 \times 0.99 = 12.8 \text{ W}$$

The internal mech. power Eqn (3.17) is

$$P_{mech} = (1-s)(P_{gap.f} - P_{gap.b}) = 0.95(206 - 13) = 184 \text{ W}$$

Assuming that the core loss can be combined with the friction and windage loss, the rotational loss becomes:

$$24 + 13 = 37 \text{ W}$$

Therefore:

$$P_{shaft} = 184 - 37 = 147 \text{ W} \equiv 0.197 \text{ hp}$$

The speed (n_s) in terms of r/min

$$n_s = \frac{120 f_e}{\text{poles}} = \left(\frac{120}{4}\right) 60 = 1800 \text{ r/min}$$

rotor speed = $(1-s) \times$ synchronous speed

$$n_r = (1-s) n_s = 0.95 \times 1800 = 1710 \text{ r/min}$$

$$\omega_s = \left(\frac{2}{\text{poles}}\right) \omega_e = \left(\frac{2}{4}\right) 120 \pi = 188.5 \text{ rad/s}$$

$$\text{and } \omega_m = (1-s) \omega_s = 0.95 \times 188.5 \quad 34 \\ = 179 \quad \text{rad/sec}$$

The Torque can be found:

$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_m} = \frac{147}{179} = 0.821 \text{ N}\cdot\text{m}$$

and the efficiency is:

$$\eta = \frac{P_{\text{shaft}}}{P_{\text{in}}} = \frac{147}{244} = 0.602 = 60.2\%$$

As a check for Losses:

$$I^2 R_{\text{main}} = (3.59)^2 (2.02) = 26.0$$

$$\text{Forward field rotor } I^2 R = 0.05 \times 206 = 10.3$$

$$\text{Backward " " } I^2 R = 1.95 \times 12.8 = 25.0$$

$$\text{rotational losses} = \underline{37} \\ 98.3 \text{ W}$$

From $P_{\text{in}} - P_{\text{shaft}} = \text{The total Losses}$
 $244 - 147 = 97 \text{ watt}$, which checks within accuracy of computations.

EXERCISE:-

Assume the motor of Example 3.5 to be operating at a slip of 0.065 and at rated voltage and frequency. Determine

- The stator current and power factor
- The power output

Ans:

$$\text{a) } 4.0 \text{ Amp } \text{ p.f.} = 0.7 \text{ lagging}$$

$$\text{b) } 190 \text{ watt}$$

3.9 SELF-STARTING SYNCHRONOUS

RELUCTANCE MOTOR :-

Any one of the Induction motor types described above can be made into a self starting synchronous-reluctance motor.

Any thing which makes the reluctance of the air-gap a function of the angular position of the rotor with respect to the stator coil axis will produce reluctance Torque when the rotor is revolving at synchronous speed.

A simple schematic of a Two-pole reluctance motor is shown in Fig (3.18).

It can be shown that the Torque applied to the rotor of this motor is proportional to:

$$\tau_{ind} \propto \sin 2\delta$$

δ - is the electrical angle between the rotor magnetic field (B_r) and the stator mag. field (B_s).

The reluctance Torque is max when

$$\delta = 45 \quad \Rightarrow \quad \sin 2\delta = \sin 90^\circ = 1$$

For A simple reluctance motor, since the rotor will be locked into the stator magnetic fields as long as the pull out Torque of the motor is not exceeded.

Like a normal synchronous motor it has no starting Torque and will not start by itself.

A self-starting reluctance motor that will operate at synchronous speed until its max. reluctance Torque is exceeded can be build by modifying the rotor of an induction motor, Fig (3.19), Suppose some of the teeth are removed from a squirrel-cage rotor of an induction motor, Leaving the bars and end rings intact.

The motor will start as an induction motor, and because the rotor has salient poles, so the motor at Light Loads will speed up to a small value of slip at steady-state operation near the synchronous speed.

The Torque speed characteristic of this motor, which is sometimes called a Synchronous induction motor is shown in Fig (3.20)

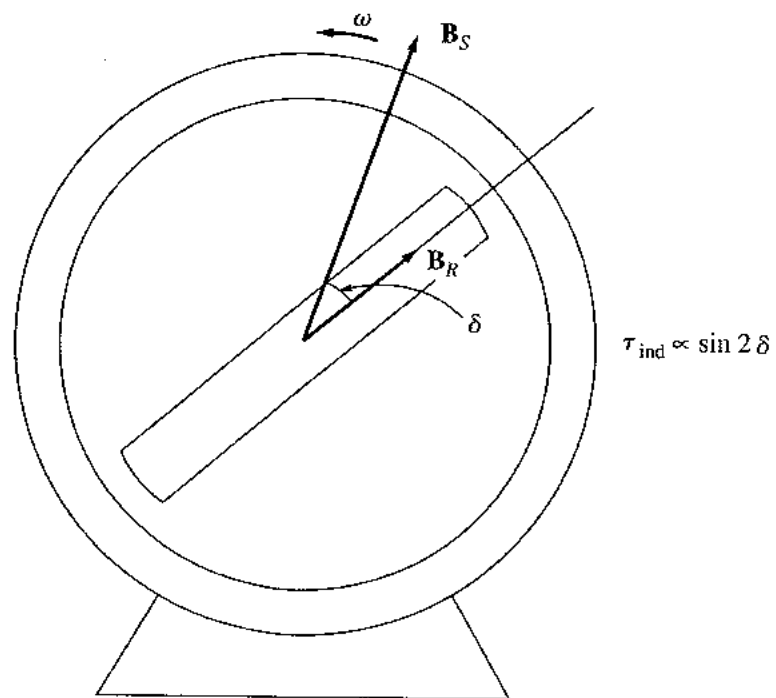


FIGURE 3.18
The basic concept of a reluctance motor.

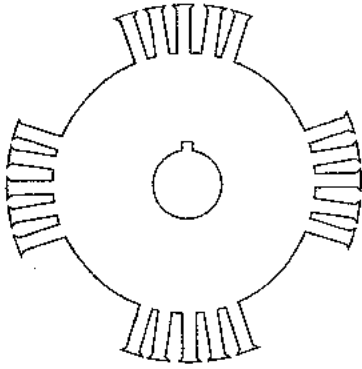


FIGURE 3.19

The rotor design of a "synchronous induction" or self-starting reluctance motor.

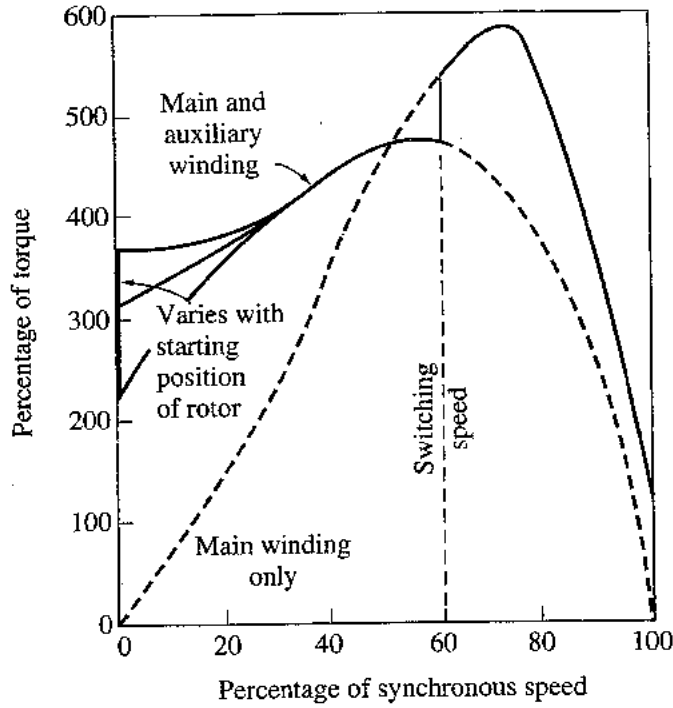


FIGURE 3.20

The torque-speed characteristic of a single-phase self-starting reluctance motor.