Salahaddin University – Erbil

College of Engineering

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Third year Students

D.C Motor

DIRECT CURRENT MOTOR

The earliest Power systems were DC system, but by the 1890s A.C. Power systems clearly wining out over DC. System. D.C Motors continued to be significant fraction of the machinery purchased each year through 1960s, after that D.C. Motors not so common, when DC. Power system themselves were fairly rare!.

DC. Motor are often compared by their speed regulations. Speed regulation (SR) of a motor is defined as:

$$SR = ([(\boldsymbol{\omega}_{nL} - \boldsymbol{\omega}_{FL})/\boldsymbol{\omega}_{FL}]) \times 100\% \quad \text{.......... (3.1)}$$

$$= ([(\mathbf{n}_{nL} - \mathbf{n}_{FL})/\mathbf{n}_{FL}]) \times 100\% \quad \text{.......... (3.2)}$$

It is a rough measure of the shape of a motor's Torque – speed characteristic.

* A positive (+Ve) speed regulation means that a motor speed drops with increasing load.

* A negative (– Ve) speed regulation means that a motor speed increasing with increasing load.

The Equivalent Circuit of a DC. Motors

The Equivalent Circuit of a DC. Motor is shown in Fig (3.1).

The armature circuit is represented by:

$\mathbf{E_a}$ **–** Armature Voltage.

$\mathbf{R_a}$ – Armature Resistance.

This representation is the Thevenin equivalent of the entire rotor structure (including rotor coils, inter poles, and compensating windings).

$\mathbf{V_{BR}}$ = The Bruch Voltage drop. $\mathbf{V_{BR}}$ is represented by a small Battery opposing the current flow direction in the machine.

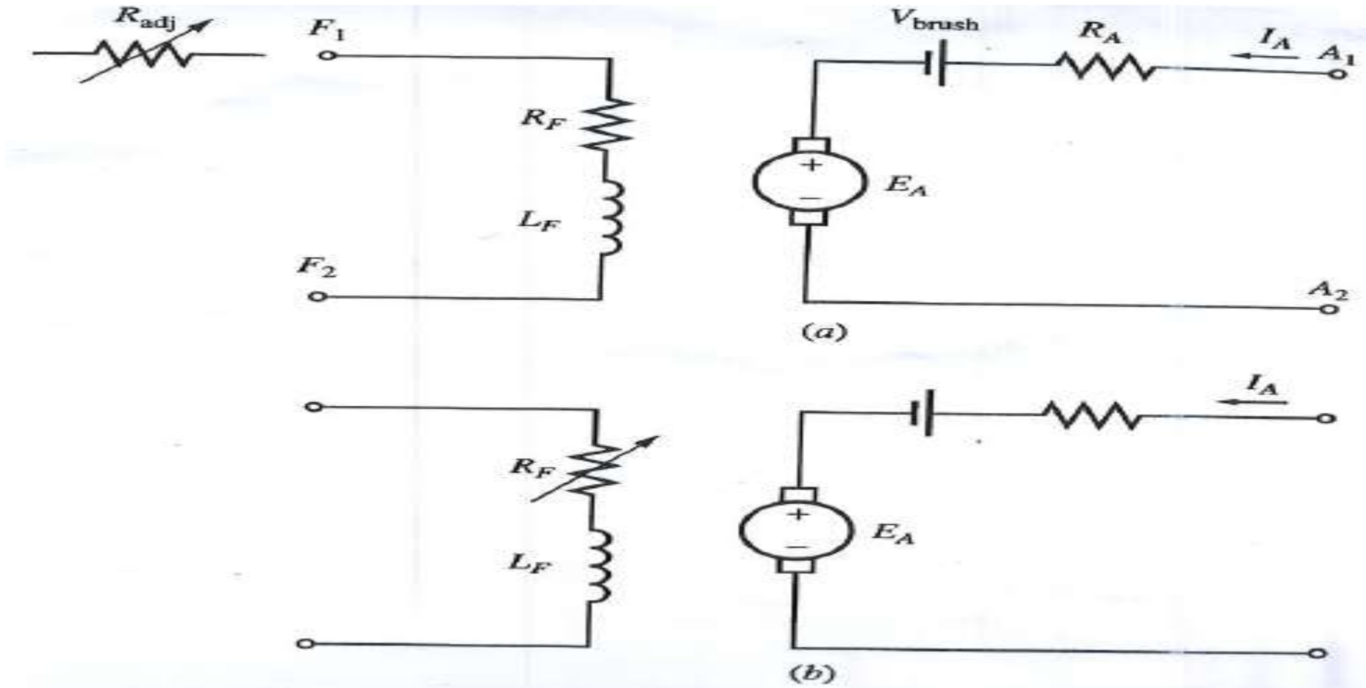


Fig (3.1). (a) Equivalent Circuit of a DC. Motors. (b)A simplified Equivalent Circuit eliminating the Brush voltage drop and combining R_{adj} with the field resistance.

The field circuit, which produce magnetic flux in the generator, are represented by:

$\mathbf{L_F}$ = Field inductor. : $\mathbf{R_F}$= Field Resistor. : $\mathbf{R_{adj}}$ = Separate external Variable Resistor.

$\mathbf{R_{adj}}$ used to control the amount of current in the field circuit.

There are a few variations and simplifications of this basic Equivalent Circuit.

The Bruch Voltage drop is often only a very tiny fraction of the generated voltage in a machine. The inter resistance of the field Coils is sometimes lumped together with the variable resistor:

R_{adj} + R_F = R_F

And the total is called R_F, see **Fig (3.1b).** The internal generated voltage (Vge) in this machine is given by:

$$\mathbf{E_a = K\, \phi\, \omega} \qquad \text{.........} \ (3.3)$$

And the induced torque developed by the machine is given by:

$$\tau_{ind} = K \times \phi \times I_a \qquad \text{.........(3.4)}$$

These two equation's, with the Kirchhoff's Voltage machine's magnetization curve, are all the tools necessary to analyze the behavior and performance of a DC. Motor.

Mechanical Power and Torque

The Power and Torque of a DC. Motor are two of its most important properties.

We now derive two simple equations that enable us to calculate them. According to equation (1.4)

$$E_a = (Z \times n \times \phi) / 60 \qquad \text{........... (1.4)}$$

The electrical Power P_a supplied to the armature:

P_a (supplied to armature) = Supply Voltage (or Terminal voltage) ×Armature current.

$$P_a = V_T \times I_a \qquad \text{....... (3.5)}$$

However, V_T is equal to sum of E_a +the $I_a R_a$ drop on the armature:

$$V_T = E_a + I_a R_a \qquad \text{...... (3.6)}$$

It follows that:

$$P_a = V_T \times I_a = (E_a + I_a R_a) \times I_a = (E_a \times I_a) + (I_a^2 \times R_a) \qquad \text{......(3.7)}$$

The ($I_a^2 \times R_a$) term represents heat dissipated in the armature, but the very important term ($E_a \times I_a$) is the electrical power that is converted into mechanical power.

The mechanical power of the motor is therefore exactly equal to the product of e.m.f. induced by the armature current. i.e.

$$P_a = E_a \times I_a \qquad \text{....... (3.8)}$$

P_a - The mechanical power developed by the motor [Watt].

E_a - Induced Voltage in the armature [Volt].

I_a – Total current of the armature [Amp].

We know that the mechanical power (**P**) related to the Torque by the equation:

$$P = (n \times T)/9.55 \qquad \text{........ (3.9)}$$

n-The speed of rotation R.P.M.

9.55- Constant (exact value= 60/2 **π**).

Combining's equations (3.8) and (3.9), we obtain:

$$(n\times T)/9.55= E_a \times I_a$$

$$= [(Z \times n\times \phi)/ 60] \times I_a$$

And so: $T = (Z \times \phi\times I_a)/ 6.28$

The torque developed by a Lap - wound motor is therefore given by the expression:

$$T = (Z \times \phi\times I_a)/ 6.28 \quad \text{[N.m]} \qquad \text{........ (3.10)}$$

T- Torque **[N.m]**

Z- Number of conductors in the armature.

ϕ- Effective flux per pole [wb.] Is given by: $\phi = (60\times E_a)/ Z \times n$

6.28-constant, to take cure of unit, exact value = 2 **π.**

Equation (3.10) shows that we can raise the Torque of a motor either by raising armature current I_a or by raising the flux **ϕ** created by the poles.

Example 3.1:

The following details are given on a 225 Kw (300 h.p), 250V, 1200 RPM. DC Motor:

Armature coils; 243 Turns per coil; 1

Type of winding; Lap Armature slots; 81

Field poles; 6

Calculate:

a) The rated armature current.

b) The number of conductors per slot.

c) The flux per pole.

Solution:

a) The rated armature current is :(Assume E_a= 250V).

$I_a = P/ E_a$ = 225000/250 = 900 Amp.

b) Each coil is made up of 2 conductors, so altogether there are:

243×2= 486 conductors on the armature.

Conductors per slot = 486/81 = 6.

c) The motor torque is:

$$T= (9.55\times P)/ n \rightarrow\rightarrow T = (9.55\times225000)/1200 = 1791 \ [N.m].$$

The flux per pole is: $\phi = (6.28\times T)/Z \times I_a = (6.28\times1791)/ (486\times900) = 25.7$ wb.

Armature Speed Control

According to the equation (1.4) replacing the internal induced Voltage (E_a) by the source [or terminal] Voltage V_T we obtain:

$$V_T = (Z \times n\times \phi)/ 60$$

That is:

$$n= (60\times V_T)/ Z \times \phi \quad \text{(approximately)} \qquad .... (3.11)$$

Where: n-The speed of rotation RPM.

V_T-Armature Voltage or supply Voltage or terminal Voltage [Volt].

Z-Total number of armature conductors.

This important equation shows that the Motor speed is directly proportional to the armature supply voltage and inversely proportional to the flux per pole (**ϕ).**

If the flux per pole (**ϕ**) is kept constant (permanent magnet field or field with fixed excitation), the speed depends only upon the Armature Voltage $\mathbf{V_T}$.

By raising or lowering $\mathbf{V_T}$, the motor speed will rise and fall in proportion.

In practice, we can vary the supply Armature Voltage $\mathbf{V_T}$ by connecting the motor armature to a Separately Excited variable voltage DC. Generator, Fig (3.2). The field excitation of the motor is kept constant.

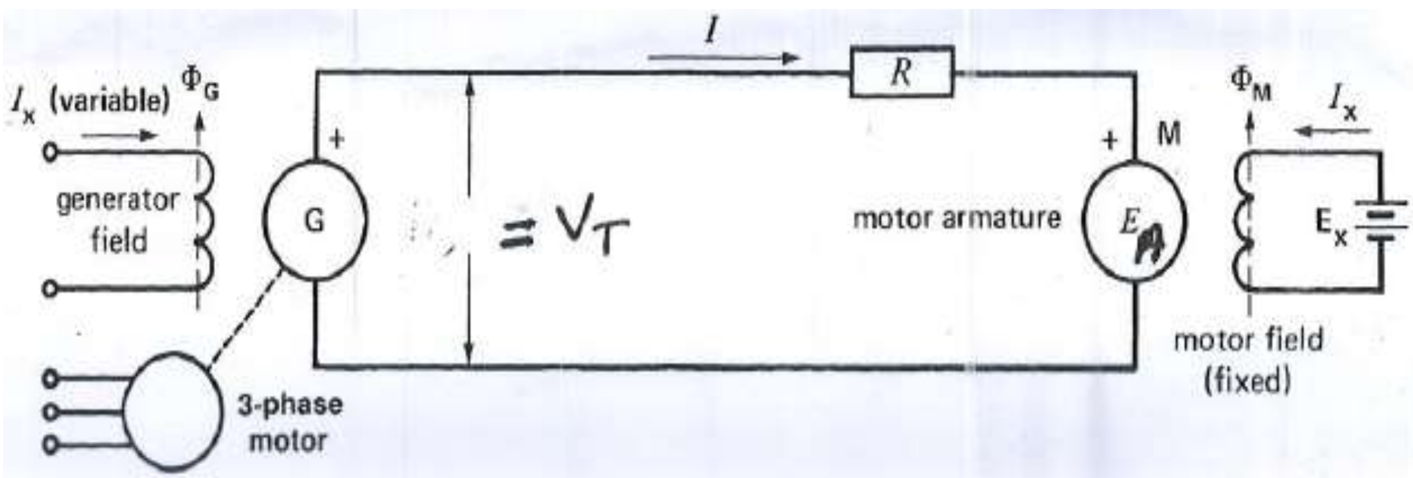


Fig (3.2) Ward – Leonard speed control system

But the generator excitation $\mathbf{I_X}$ can be varied from ZERO to MAXIMUM and even reversed.

The generator output (O/P) voltage $\mathbf{V_T}$ can therefore be varied from ZERO to MAXIMUM, with either positive or negative polarity. Consequently; the motor speed can be be varied from ZERO to MAXIMUM in either direction.

Note that; the generator is driven by a 3 – phase AC. Motor. This method of speed control known as: THE WARD – LEONARD SYSTEM.

The ward – Leonard System is found in steel mills, high – rise elevators (Lifts), mines, and paper mills.

In modern installations the generator is often replaced by high – power electronic converter (with SCR or TRIAG).

[SCR or TRIAG] changes AC. Power →To→ DC. Power

Example 3.2:

A 2000Kw, variable – speed motor is driven by a 2500 Kw generator, using a ward – Leonard System. The total resistance of the motor and generator armature circuit is 1mΩ. The motor turns at a nominal speed of 300 RPM, when $\mathbf{E_a}$ =500 V.

Calculate:

a) The motor torque and speed when: $\mathbf{V_T}$= 400V and $\mathbf{E_a}$ =380 V.

b) The motor torque and speed when: $\mathbf{V_T}$= 350V and $\mathbf{E_a}$ =380 V.

Solution:

a) The armature current is: $I_a = (V_T - E_a)/R_a = (400 - 380)/0.01 =$ 2000 Amp.
The power to the motor armature is: $\mathbf{P_a = E_a \times I_a = 380 \times 2000 = 760}$ **Kw.**

The motor speed is:

$n = (380V/500V) \times 300 = 228$ R.P.M

The motor Torque is: $T = 9.55 \times P/n = (9.55 \times 760000)/228 = 31.8$ KN.m.

b) Because $\mathbf{E_a}$ =380 V, the motor speed is still 228 R.P.M

The armature current is: $I_a = (V_T - E_a)/R_a = (350 - 380)/0.01 = -3000$ Amp.

The current is (– ve) so it flows in reverse, the motor torque also reverse.

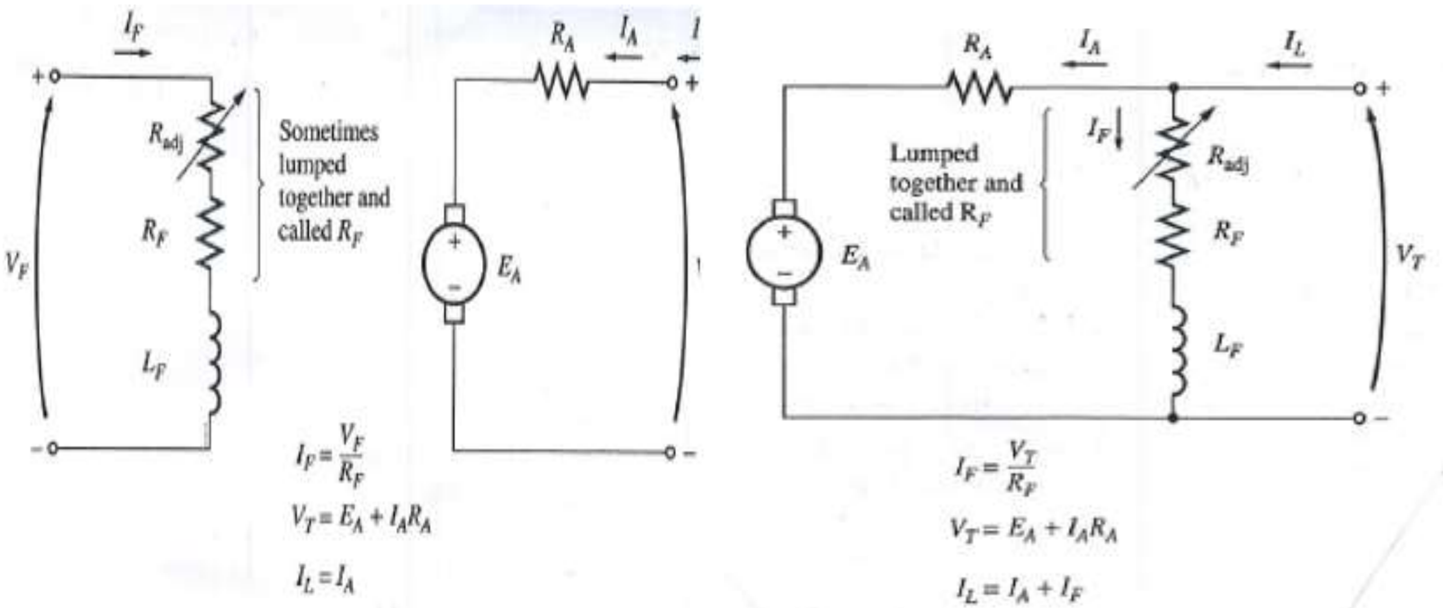
$\mathbf{P_a = E_a \times I_a = 380 \times 3000 = 1140}$ **Kw.**

Braking Torque developed by the motor:

$T = 9.55 \times P/n = (9.55 \times 1140000)/228 = 47.8$ KN.m.

Separately Excited and Shunt DC. Motors

The equivalent circuit of Separately Excited DC. Motor is shown in Fig (3.3a).



(a) (b)

Fig (3.3) **a)** equivalent circuit of Separately Excited DC Motor. **b)** Shunt DC Motor equivalent circuit.

A Separately Excited d.c Motor is a motor whose field circuit is supplied from a separate constant – voltage power supply.

The shunt DC. Motor is a motor whose field circuit gets its power directly across the armature terminals of the motor, Fig (3.3b).when the supply voltage to a motor is assumed constant, and there is no practical difference in behavior between these two Machines.

The Kirchhoff's Voltage Law (KVL) equation for armature circuit of these motor is:

$$V_T = E_a + I_a R_a$$

The Terminal Characteristic of Shunt DC. Motor

A terminal characteristic of a machine is a plot of the machine's output (o/p) quantities versus each other.

For a motor, the output (o/p) quantities are shaft Torque, and speed, so the terminal characteristic of a motor is a plot of its output Torque versus speed.

If the shaft load of a shunt motor is increased. Then the load Torque (τ_{Load}) will exceed the induced Torque (τ_{ind}) in the machine, the motor will start to slow down.

When the motor slows down, it internal generated voltage drops:

$E_a = K\,\phi\,\omega$; so the armature current in the motor increases:

$V_T = E_a + I_a R_a \rightarrow\rightarrow \quad I_a = (V_T - E_a)/R_a$; as the armature current rises, the induced torque will equal the load torque at a lower mechanical of rotation (**ω**).

The (o/p) characteristic of a shunt DC. Motor can be derived from the induced voltage and torque equations of the motor plus Kirchhoff's Voltage Law (KVL).

The Kirchhoff's Voltage Law (KVL) shunt motor is:

$$V_T = E_a + I_a R_a$$

The induced voltage: $E_a = K\,\phi\,\omega$

So: $V_T = K\,\phi\,\omega + I_a R_a$ .... (3.12)

$$\tau_{ind} = K \times \phi \times I_a$$

Current I_a can be expressed as:

$I_a = \tau_{ind}/(K \times \phi)$ .... (3.13)

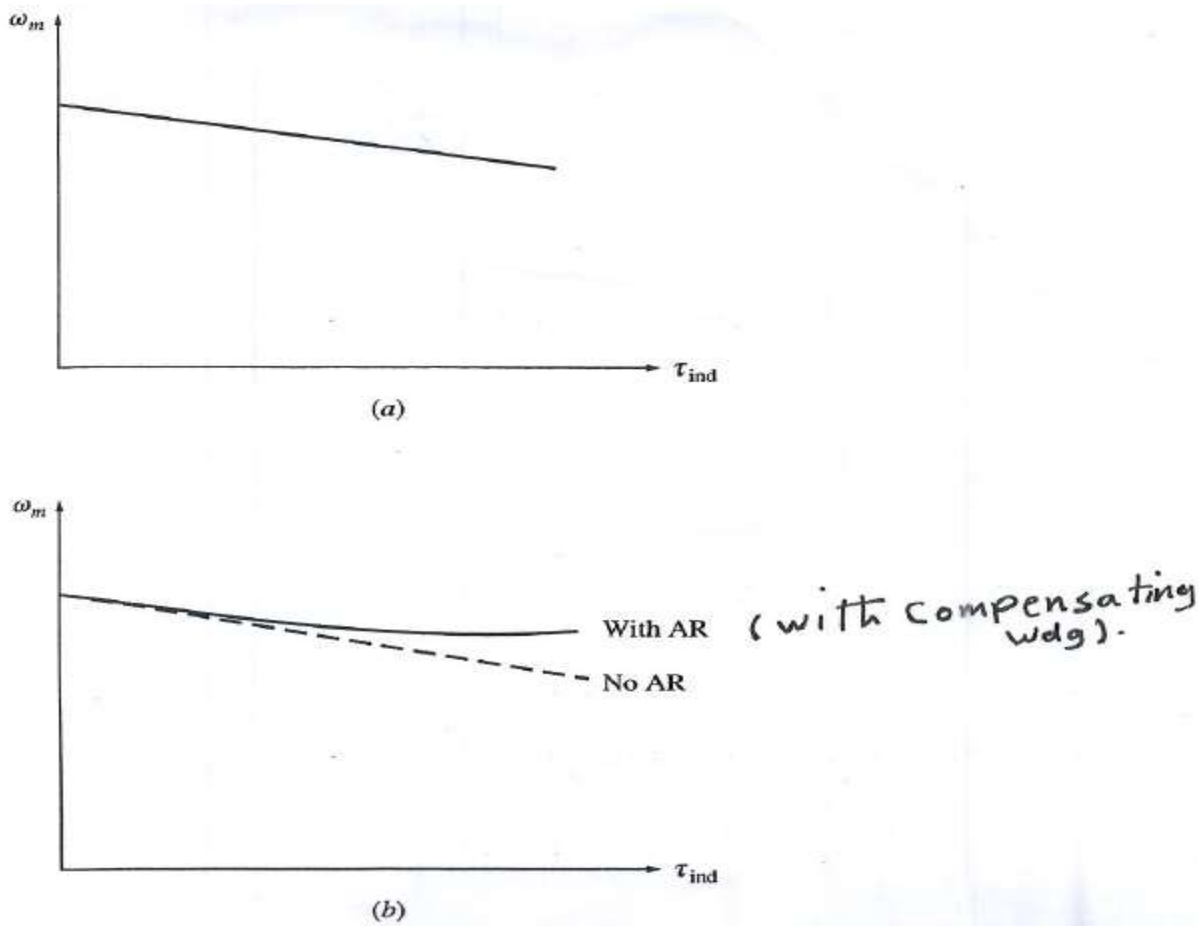
Combining equation's (3.12) and (3.13) produces:

$$V_T = K\,\phi\,\omega + [\tau_{ind}/(K \times \phi)]\,R_a$$

Finally, solving for the motor's speed yields:

$\omega = (V_T/K\,\phi) - [R_a/(K\,\phi)^2] \times \tau_{ind}$ .... (3.14)

This equation (3.14) is just a straight line with a negative slop. The resulting Torque – Speed characteristic of a shunt DC. Is shown in Fig (3.4).



Fig(3.4).a)Torque -speed characteristic of a shunt or separately excited d.c. motor with compensating windings to eliminate armature reaction(b) Torque - speed characteristic with armature reaction present.

It is important to realize that, in order for the motor speed to vary linearly with Torque, the other terms in this expression must be constant as the load changes.

The terminal voltage supplied by DC. Power source is assumed to be constant, but if it is not constant, then voltage variations will affect the shape of Torque – speed curve. Another effect internal to the motor that can also affect the shape of the Torque – speed is ARMATURE REACTION.

If a motor has armature reaction, then as its load increase, the flux – weaking effects reduce its flux.

As equation (3.14) shows, the effect of a flux reduction is to increase the motor's speed at any given load over the speed it would run at without armature reaction.

The Torque -speed characteristic of a shunt DC motor with armature reaction is shown in Fig (3.4).b). If a motor has compensating windings, of course there will be no flux – weaking problems in the machine, and the machine flux will be constant.

Also if a Shunt DC. Motor has compensating windings so that its flux is CONSTANT REGARDLESS OF LOAD, and the motor's speed, armature current are known at any one value of load, then it is possible to calculate its speed at any other value of load, as long as the armature current at that load is known or can be determined.

Example 3.3:

A 50 hp, 250V, 1200 RPM,DC Shunt motor with compensating windings has an armature resistance (including the Bruch, compensating windings, and inter poles) of 0.06Ω. Field circuit has a total resistance (**R_{adj}+R_F**) of 50 Ω, which produces a no load speed of 1200 RPM. There are 1200 turns per pole on shunt field winding, Fig (3.5).

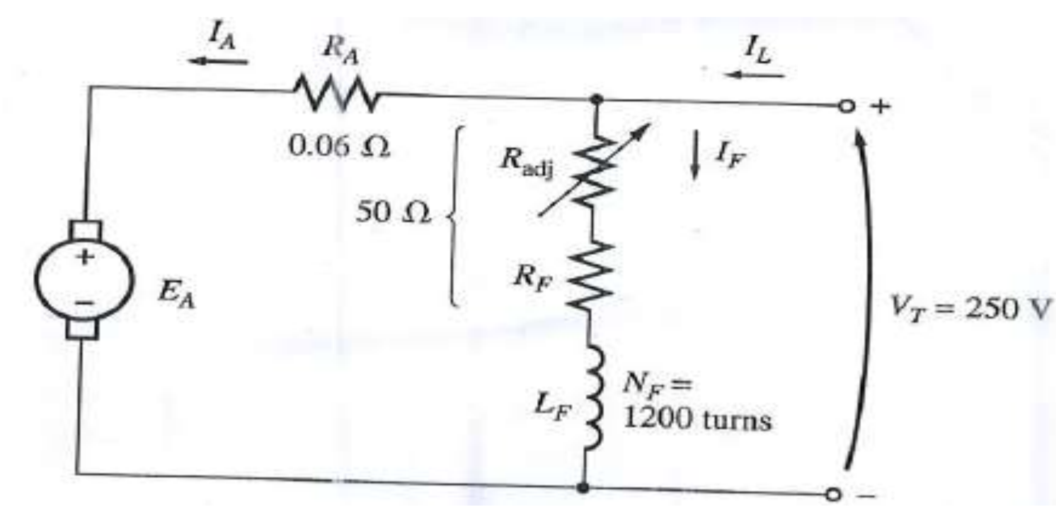


Fig (3.5). The shunt motor in Example 3.3

a) Find the speed when input current is 100A
b) Find the speed when input current is 200A.
c) Find the speed when input current is 300A.
d) Plot the torque – speed characteristic of this motor.

Solution:

The internal generated voltage of a DC machine with it speed expressed in revolutions per minute is given by:

$$E_a = K' \times \phi \times n$$

Since the field current in the machine is constant (because V_T and the field resistance are both constant) and since there are No armature reaction effects.

The flux in this motor is constant. The relationship between the speeds and internal voltages of the motor at two different load is thus:

$$E_{a2} / E_{a1} = (K' \times \phi \times n_2) / (K' \times \phi \times n_1)$$

Since K' is constant for any given machine, and the ϕ cancels from the above equation therefore:

$$n_2 = (E_{a2} / E_{a1}) \times n_1$$

At No – load, the armature current= ZERO.

$E_{a1} = V_T = 250$ V , while the speed $n_1 = 1200$ RPM.

If we can calculate the internal generated voltage (E_a) at any other load, we can then find the speed.

a) If $I_L = 100$ A, then the armature current I_a:

$$I_a = I_L - I_F = I_L - (V_T/R_F) = 100\text{A} - (250\text{ V}/50\Omega) = 95\text{ Amp.}$$

Therefore

$$E_a = V_T - I_a R_a = 250-(95\times0.06) = 244.3 \quad \text{Volt.}$$

The resulting speed is:

$$n_2 = (E_{a2} / E_{a1}) \times n_1 = (244.3\text{V}/ 250\text{V}) \times 1200\text{ RPM.} = 1173\text{ RPM.}$$

b) If $\mathbf{I_L}$ = 200 A, then the armature current $\mathbf{I_a}$:

$$I_a = I_L - I_F = I_L - (V_T/R_F) = 200A - (250\ V/50\Omega) = 195\ \text{Amp.}$$

Therefore, $\mathbf{E_a}$ at this load will be:

$$E_a = V_T - I_a R_a = 250-(195\times 0.06) = 238.3\quad \text{Volt.}$$

The resulting speed is:

$$n_2 = (E_{a2}/ E_{a1}) \times n_1 = (238.3V/ 250V) \times 1200\ \text{R.P.M.} = 1144\ \text{R.P.M.}$$

c) If $\mathbf{I_L}$ = 300 A, then the armature current $\mathbf{I_a}$:

$$I_a = I_L - I_F = I_L - (V_T/R_F) = 300A - (250\ V/50\Omega) = 295\ \text{Amp}$$

Therefore, $\mathbf{E_a}$ at this load will be:

$$E_a = V_T - I_a R_a = 250-(295\times 0.06) = 232.3\quad \text{Volt.}$$

The resulting speed is:

$$n_2 = (E_{a2}/ E_{a1}) \times n_1 = (232.3V/ 250V) \times 1200\ \text{R.P.M.} = 1115\ \text{R.P.M.}$$

d) To plot the output Torque, it is necessary to find the Torque corresponding to each value of speed. At No – load →→ τ_{ind} = ZERO.

The induced torque for any other load can be found, the power converted in DC Motor:

$$P_{conv} = E_a \times I_a = \tau_{ind} \times \omega$$

$$\rightarrow \quad \tau_{ind} = (E_a \times I_a)/\omega$$

When $\mathbf{I_L}$ = 100 A:

$$\tau_{ind} = (244.3V \times 95A)/\{(1173\ \text{RPM.}) \times [1\text{min.}/60\text{sec.}] \times (2\pi\ \text{rad/r})\} = 190\ \text{N.m}$$

When $\mathbf{I_L}$ = 200 A:

τ_{ind} =(238.3V×195A)/{(1144 R.P.M.)× [1min. /60sec.] × (2 π rad/r)} = 388 N.m

When I_L = 300 A:

τ_{ind} =(232.3V×295A)/{ (1115 R.P.M.)× [1min. /60sec.] × (2 π rad/r)} = 587 N.m

The resulting Torque – speed characteristic is plotted in Fig (3.6).

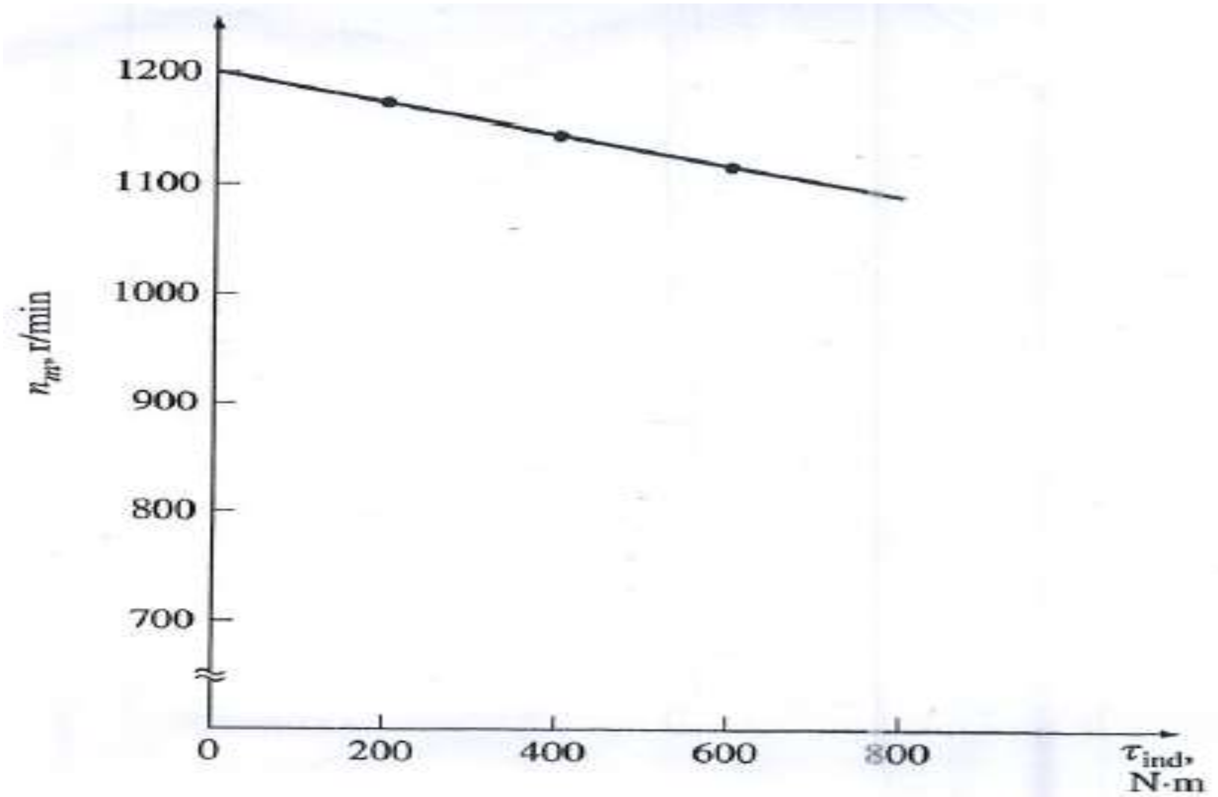


Fig (3.6) The Torque – speed characteristic of the motor in Example 3.3

Speed control of Shunt DC. Motor

There are two common methods and one less common in use.

Two common ways in which shunt DC. Machine speed can be controlled are by:

1) Adjusting the field resistance R_F (and thus the field flux).

2) Adjusting the terminal voltage applied to the armature.

The less common method of speed control is by:

3) Inserting a resistor in series with armature circuit.

Shunt DC. Motor under Load

Consider a DC. Motor running at No – load. If a mechanical load is suddenly applied to the shaft, the small No – load current does not produce enough torque to carry the load and the motor begins to slow down. This causes e.m.f.to diminish, resulting in a higher current and a corresponding higher Torque.

When the motor developed torque is exactly equal to the Torque imposed by the mechanical load, then, and only then, will the speed remain constant. To sum up, as the mechanical load increase, the armature current rises and the speed drops.

The speed of a shunt motors stays relatively constant from No – load to full load

In small motors it only drops by; 10% to 15% when full – load is applied in big machines, the drop is even less.

Example3.4:

A shunt motor rotating at 1500 RPM. is fed by a 120 V source, see Fig (3.7a).The line current is 51 A and the shunt – field resistance is 120 Ω; if the armature resistance is 0.1 Ω, calculate the following:

a) The current in the armature   b) The Counter – e.m.f.

c) The mechanical Power developed by the motor.

Solution:

$\mathbf{I_F} = 120V/120\ \Omega = 1\ A$

The armature current is: $\mathbf{I_a} = 51 - 1 = 50\ A$

b) The voltage across the armature is: $\mathbf{E_a} = 120\ V$.

Voltage drop due to armature resistance is:

$\mathbf{I_a} \times \mathbf{R_a} = 50 \times 0.1 = 5\ V$

The Counter – e.m.f. generated by the armature is: $\mathbf{E_a} = 120 - 5 = 115\ V$

c) The total Power supplied to the motor is: $\mathbf{P_{i/p} = E \times I_L} = 120 \times 51 = 6120$ Watt.

Power absorbed by the armature is: $\mathbf{P_a = E \times I_a} = 120\times \mathbf{50 = 6000}$ Watt.

Power dissipated in the armature is: $\mathbf{P = I_a^2 \times R} = 50^2 \times 0.1 = 250$ Watt.

Mechanical power developed by the armature is:

P = 6000 – 250 = 5750 Watt. **Equivalent to 5750/746 = 7.7 h.p.**

Actual mechanical output is slightly less than **5750** Watt because some of mechanical Power is dissipated in (bearing friction, windage, and armature iron) losses

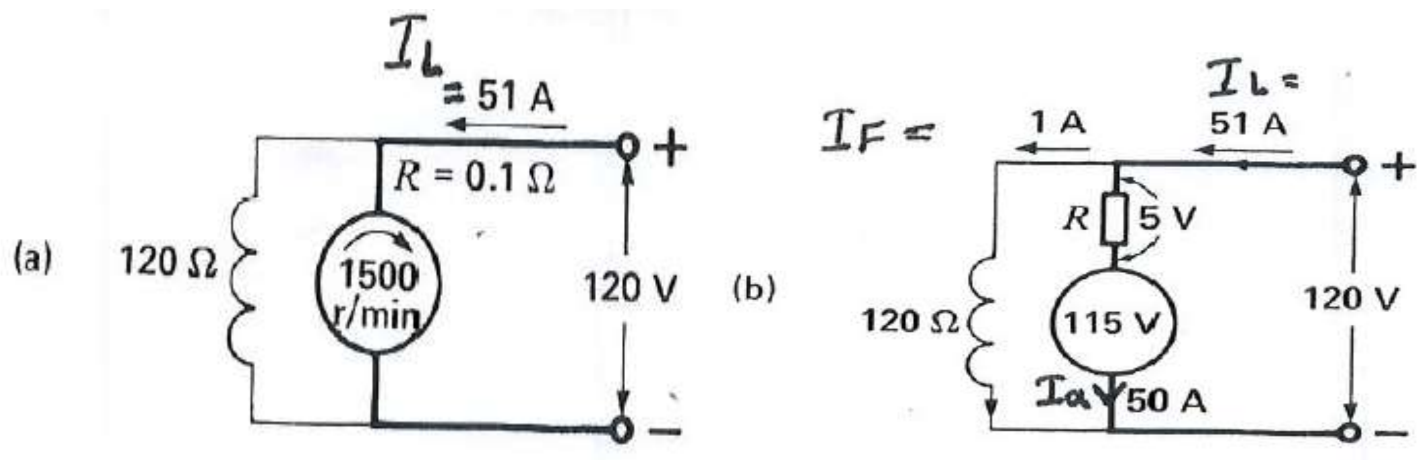


Fig (3.7).see example 3.4:

The Permanent – Magnet DC. Motor

A Permanent – Magnet DC. (PMDC) Motor is a DC motor whose poles made of Permanent – Magnet.

PMDC Motors offer a number of benefits compared with shunt DC motors in some circuit applications. Since these motors do not require an external field, they do not have the field copper losses associated with shunt DC motors. Because no field are required, they can be smaller than corresponding shunt DC motors.

PMDC Motors are especially common in smaller fractional and sub -fractional horse Power sizes, where the expense and space of a separate field circuit cannot justified.

However PMDC Motors also have disadvantage, Permanent – Magnet cannot produce as high a flux density as an externally supplied shunt field, so a PMDC Motors will have a lower induced torque (τ_{ind}) per ampere of armature current(I_a) than a shunt motor of same size and construction.

In addition, PMDC Motors runs with the risk of demagnetization. Also in a PMDC machine the pole flux is just the residual flux in the permanent magnets

If the armature current becomes very large, there is some risk that the armature **m.m.F** may demagnetize the poles, permanently reducing and reorienting the residual flux in them. Demagnetization may also be caused by the excessive heating which can occur during prolonged periods of over load. Fig (3.8) shows a magnetization curve for a typical ferromagnetic material, it is the plot of flux

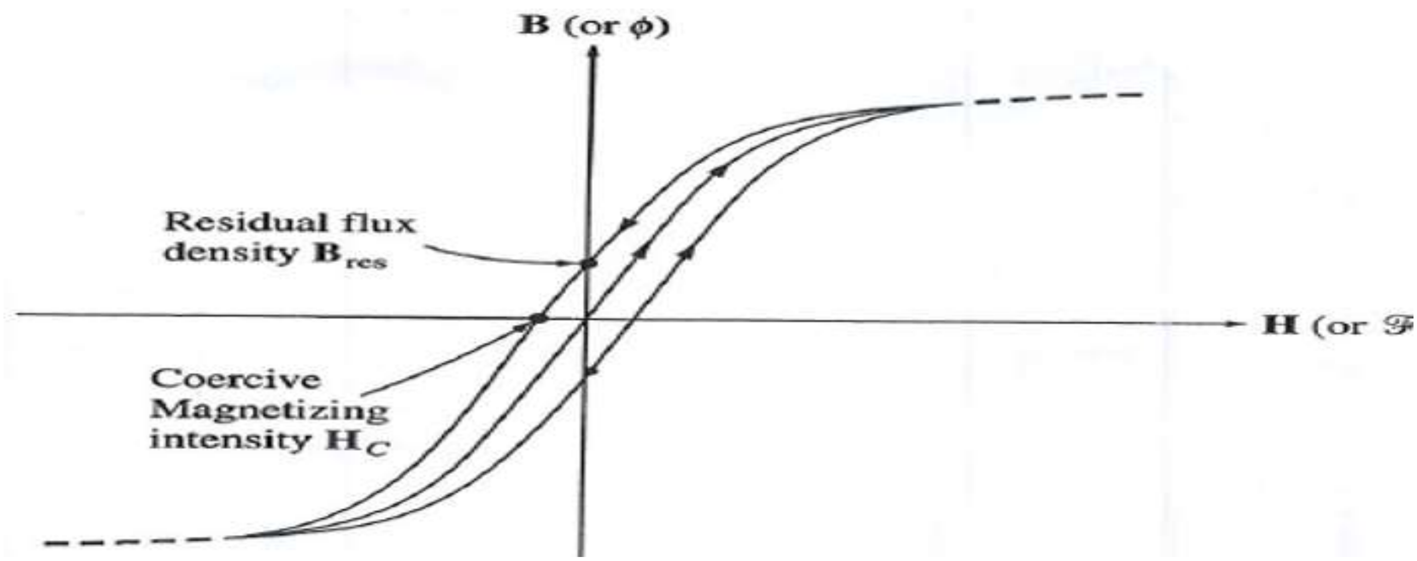


Fig (3.8) a magnetization curve for a typical ferromagnetic material. Note the hysteresis loop. After a large magnetizing intensity H is applied to core and then removed, a residual flux density B_{res} remains be behind in the core, this flux can be brought to zero if a coercive magnetizing intensity H_C is to the core with the opposite polarity, in this case, a relatively small value of it will demagnetize the core.

Density (**B**) versus magnetizing intensity **H** (or equivalently a plot of **ϕ** versus **m.m.F**$\mathcal{F}$). When a strong external **magnetomotive** force is applied to this material and then removed, a residual flux (**B_{res}**) will remain in the material.

To force the residual flux to ZERO, it is necessary to apply a coercive magnetizing intensity $\mathbf{H_c}$ with polarity opposite to the polarity of magnetizing intensity (**H**) that originally established the magnetic field.

For normal machine application such as rotors and stators, a ferromagnetic material should be picked which has as small a ($\mathbf{B_{res}}$) and ($\mathbf{H_c}$) as possible.

On the other hand, a good material for the poles of a PMDC Motors should have as large a residual flux density ($\mathbf{B_{res}}$) as possible.

A number of new magnetic materials have been developed which have desirable characteristics for making Permanent -Magnets. The major types of materials are the ceramic (ferrite) magnetic materials. The best rare – earth magnets can produce the same residual flux as the best conventional ferromagnetic alloys, while simultaneously being largely immune to demagnetization problems due to armature reaction.

The Series DC. Motor

It is a motor whose field windings consist of a relatively few turns connected in series with the armature circuit. The equivalent circuit of a DC Motor is shown in Fig (3.9).

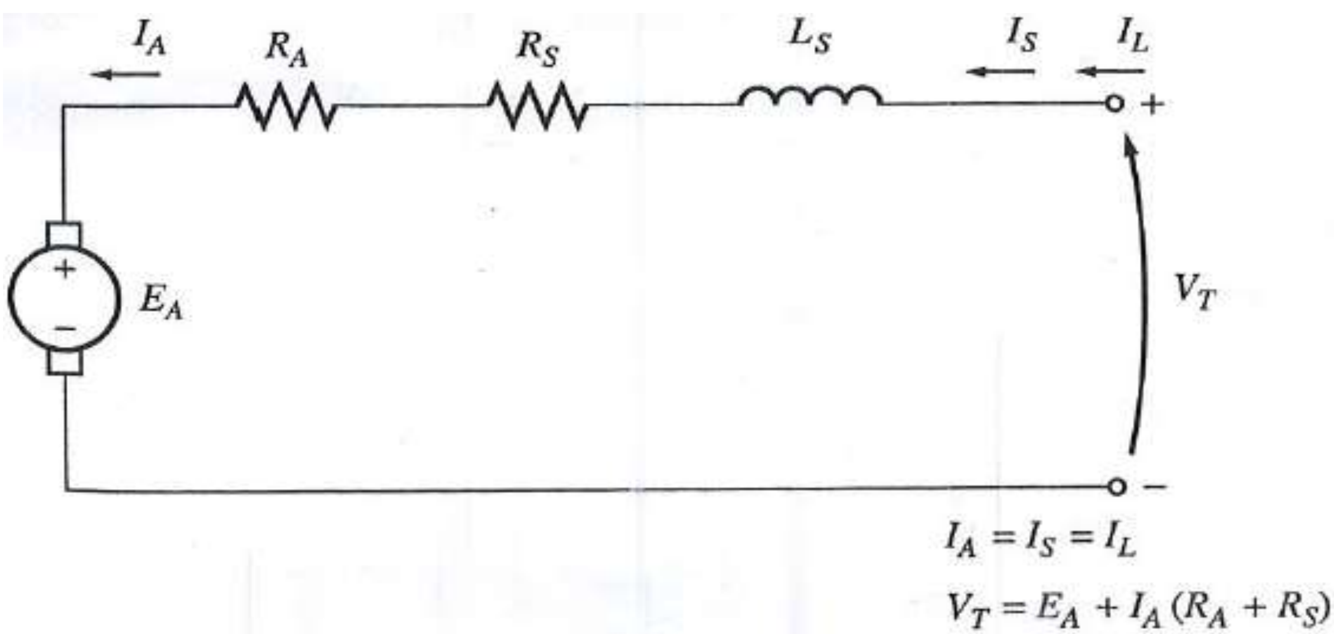


Fig (3.9). The equivalent circuit of a Series DC. Motor

In a series motor, armature current, field current, and Line current are all the same:

$$I_a = I_{se} = I_L \quad \text{......... (3.15)}$$

$$V_T = E_a + I_a(R_a + R_{se}) \quad \text{...........(3.16)}$$

Induced Torque in series DC. Motor

The terminal characteristic of a series DC. Motor is very different from that of the Shunt Motor.

The basic behavior of a series DC. Motor is due to the fact that the flux is directly proportional to the armature current (I_a), at least until saturation is reached.

As the load on the motor increases, its flux increases too, an increases in the motor flux causes a decrease in the speed. The result is that a series DC. Motor has a sharply drooping.

Torque – Speed characteristic of a series DC. Motor

The induced Torque in this machine is given by equation (3.4)

$$\tau_{ind} = K \times \phi \times I_a \quad \text{.........(3.4)}$$

The flux in this machine is directly proportional to its armature current I_a (until the saturation point), therefore, the flux in the machine can be given by:

$$\phi = C \times I_a \quad \text{........... (3.17)}$$

Where:

C – is a constant of proportionality.

The induced Torque in this machine is thus given by:

$$\tau_{ind} = K \times \phi \times I_a = K \times C \times I_a^2 \quad \text{......... (3.18)}$$

So that, the torque in the motor is directly proportional to the square of its armature current (I_a).

As a result, the series motor gives more torque per ampere than any other DC motor. It is therefore used in applications requiring very high Torque. Examples:

-Starter motors in cars. –Elevator (Lift) motors. – And Tractor motors.

The Terminal Characteristic of Series DC. Motor

To determine the Terminal Characteristic of Series DC. Motor, analysis will be based on the assumption of a linear magnetization curve, and the effects of saturation will be considered in a graphical analysis. The assumption of linear magnetization curve implies that the flux in the motor will be given by equation:

$$\phi = C \times I_a \quad \text{.......... (3.17)}$$

This equation will be used to derive the Torque – Speed Characteristic for the Series DC. Motor. The derivation starts with the Kirchhoff's Voltage Law equation:

$$V_T = E_a + I_a (R_a + R_{se}) \quad \text{.......... (3.16)}$$

From equation (3.18): The armature current can be expressed as:

$$I_a = \sqrt{\tau_{ind}/kc}$$

Also: $E_a = K \times \phi \times n$

Substituting these expressions in equation (3.16)

$$V_T = K \times \phi \times n + [\sqrt{\tau_{ind}/kc}\,] (R_a + R_{se}) \quad \text{...... (3.19)}$$

If the flux can be eliminated from this expression, it will directly relate the torque of a motor to its speed.

To eliminate the flux from the expression, notice that:

$$I_a = \phi / C$$

And the induced Torque equation (3.18)

$$\tau_{ind} = (K / C) \times \phi^2$$

Therefore, the flux in the motor can be rewritten as

$$\phi = \sqrt{C/K} \times \sqrt{\tau_{ind}} \quad .....(3.20)$$

Substituting equation (3.20) into equation (3.19) and solving for speed yields:

$$V_T = K \times \sqrt{C/K} \times \sqrt{\tau_{ind}} \times \omega + [\sqrt{\tau_{ind}/kc}] \times (R_a + R_{se})$$

$$\sqrt{kc} \times \sqrt{\tau_{ind}} \times \omega = V_T - [(R_a + R_{se})/\sqrt{kc}] \times \sqrt{\tau_{ind}}$$

$$\omega = [V_T/(\sqrt{kc} \times \sqrt{\tau_{ind}})] - (R_a + R_{se})/KC]$$

The resulting Torque – Speed relationship is:

$$\omega = [V_T/(\sqrt{kc} \times \sqrt{\tau_{ind}})] - (R_a + R_{se})/KC] \quad ....(3.21)$$

Notice that:

For an unsaturated series motor the motor speed varies as the reciprocal of the Torque square root. That is quite an unusual relationship!

This ideal Torque – Speed Characteristic is shown in Fig (3.10).

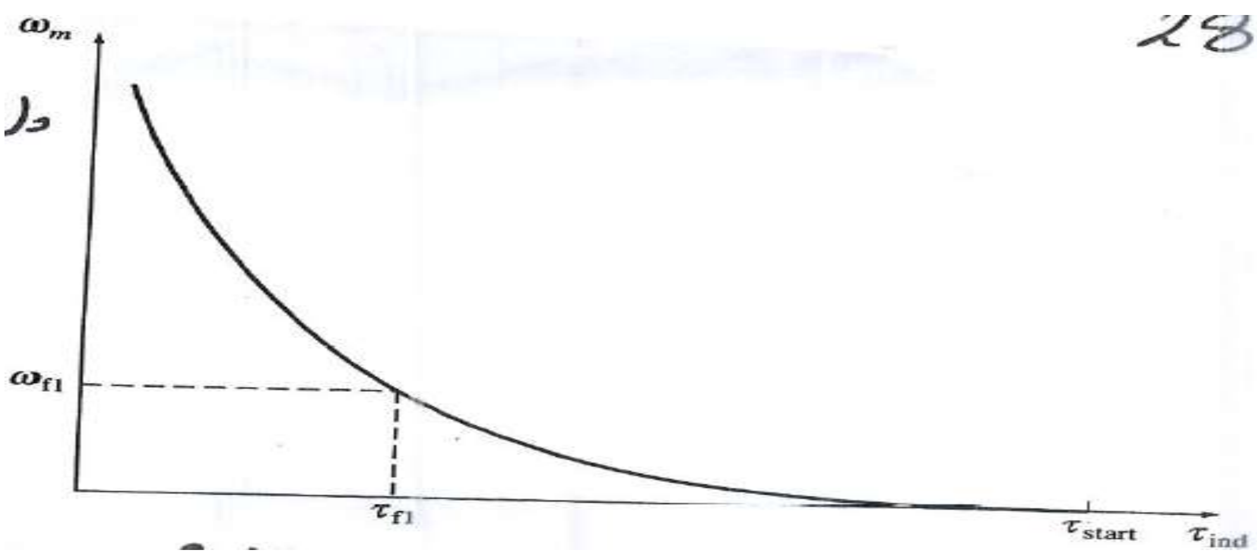


Fig (3.10) Torque – Speed Characteristic of Series DC. Motor

One disadvantage of series motor, see equation (3.21), when the motor torque goes to ZERO, its speed goes to INFINITY. In practice, the torque can never go entirely to zero, because of the mechanical, core, and stray losses that must be overcome.

However, if no other load is connected to the motor, it can turn fact enough to seriously damage itself.

Example 3.5:

Fig (3.9), shows a 250 V series DC motor with compensating windings, and a total series resistance ($R_a + R_{se}$) of 0.08Ω.The series field consist of 25 turns per pole, with the magnetization curve shown in Fig (3.11):

a) Find the speed and (τ_{ind}), when its armature current is = 50 Amp.

b) Calculate and plot the Torque – speed characteristics.

Solution:

a) To analyze the behaver of series motor with saturation, pick points along the operating curve and find the torque and speed for each point. Notice that the magnetization curve is given in units of **magnetomotive** force (Ampere -turns) versus E_a for a speed of 1200 R.P.M., so calculated E_a **values must be compared to the equivalent values at** 1200 R.P.M., to determine the actual speed for I_a = 50 Amp.

$$E_a = V_T - I_a (R_a + R_{se}) = 250\ V - (50\ A \times 0.08\ \Omega) = 246\ V.$$

Since $I_a = I_F$ = 50 Amp.

The magnetomotive force is:

$$\mathcal{F} = N \times I = 25\ \text{turns} \times 50\ A = 1250\quad A.\ Turns.$$

From the magnetization Curve: at $\mathcal{F}$ = 1250 A. Turns →→ E_{ao} = 80 V.

To get the correct speed of the motor, recall equation:

$$n = (E_a / E_{ao})\, n_o \quad \rightarrow\rightarrow \quad n = (246V / 80\ V) \times 1200\ r.p.m. = 3690\ r.p.m.$$

To find τ_{ind} supplied by the motor at that speed:

$$P_{conv} = E_a \times I_a = \tau_{ind} \times \omega \quad \rightarrow\rightarrow \quad \tau_{ind} = (E_a \times I_a)/\ \omega$$

$$\tau_{ind} = (246V \times 50A)/\ 3690\ r.p.m. \times (1minute\ /\ 60) \times (2\ \pi\ rad/r) = 31.8\ N.m$$

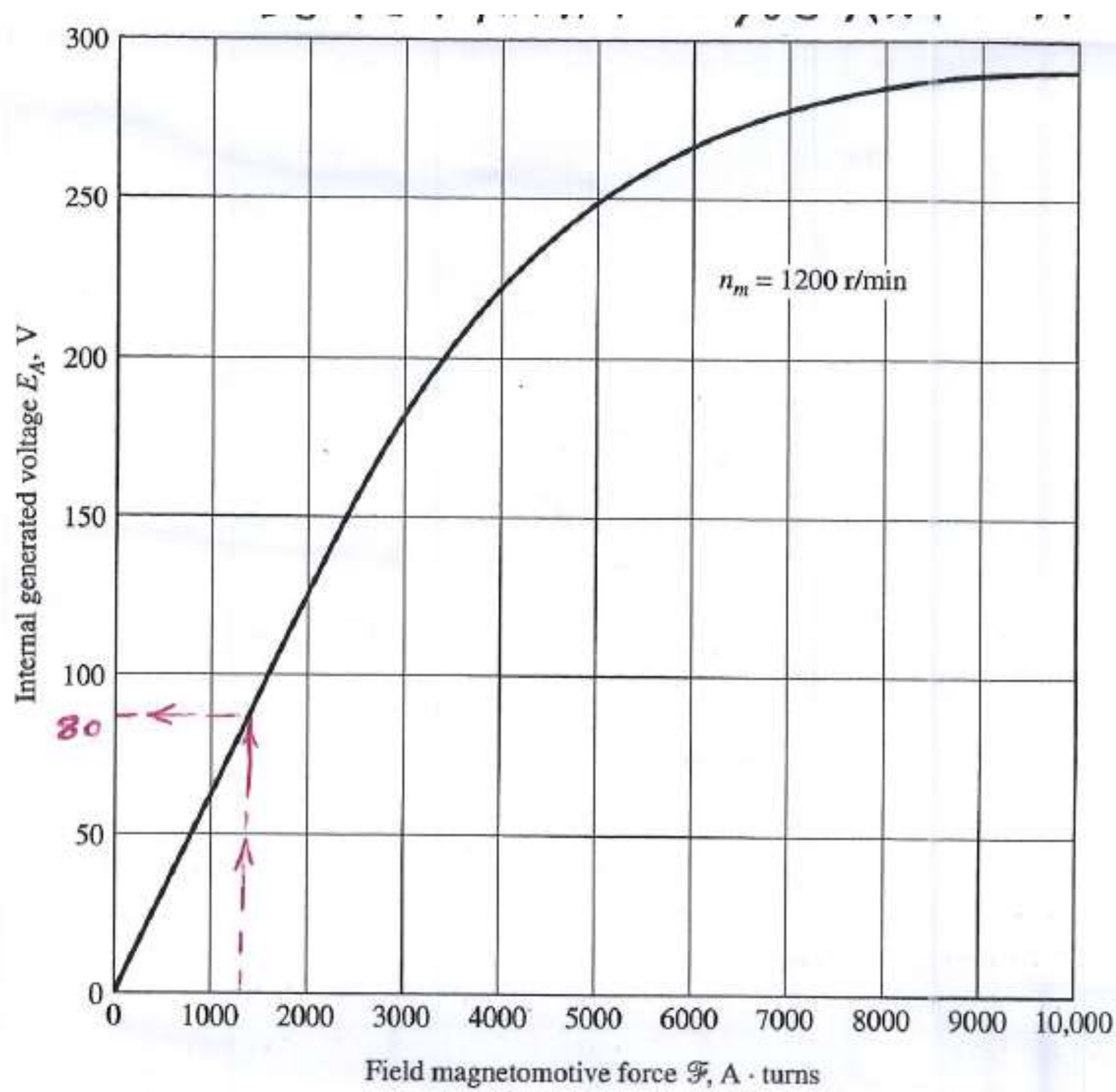


Fig (3.11) the magnetization Curve at speed of 1200 r.p.m.

b) To calculate the complete torque – speed characteristic, we must repeat the steps in (a) for many values of armature current. A MATLAP m- file that calculates the complete torque – speed characteristic can be used. The resulting motor torque – speed characteristic is shown in Fig (3.12).

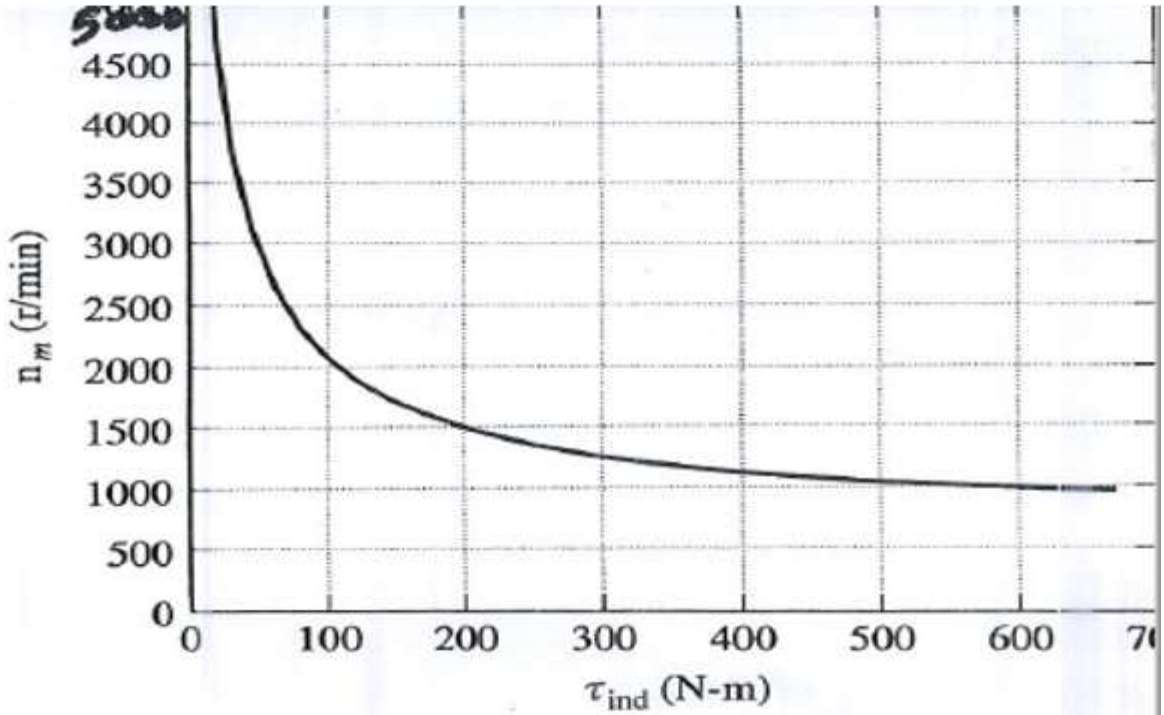


Fig (3.12) the torque – speed characteristic of the DC motor in Example 3.5.

The resulting torque – speed characteristic is shown in Fig (3.12).Notice the severe overspending at very small Torque.

Speed Control of Series DC. Motor

1) There is only one efficient way to change the Series DC Motor speed. That method is to change motor terminal voltage. If the voltage is increased, the first term of equation (3.21) is increased, resulting in a higher speed for any given torque.

2) The speed of Series DC Motor can also be controlled by the insertion of a series resistor into the motor circuit, but this technique is very wasteful of power and is used only for intermittent periods during the start – up of some motors.

Now:

There is a convenient way to change (V_T) using (**SCR**) control circuits for obtaining variable terminal voltages.

*The insertion of a series external resistor may be lowered the series DC. Motor speed, because the total (**IR**) drop across the resistor and field reduces the armature supply Voltage, and so the speed must fall.

Typical Torque- Speed and Torque -current characteristics are shown in Fig (3.13), they are quite different from the shunt Motor.

3) Another mothed is to control the speed of loaded series Motor, when a series Motor carries a load, its speed may have to be adjusted slightly. Thus, the speed can be increased by placing a low resistance in parallel with the series field.

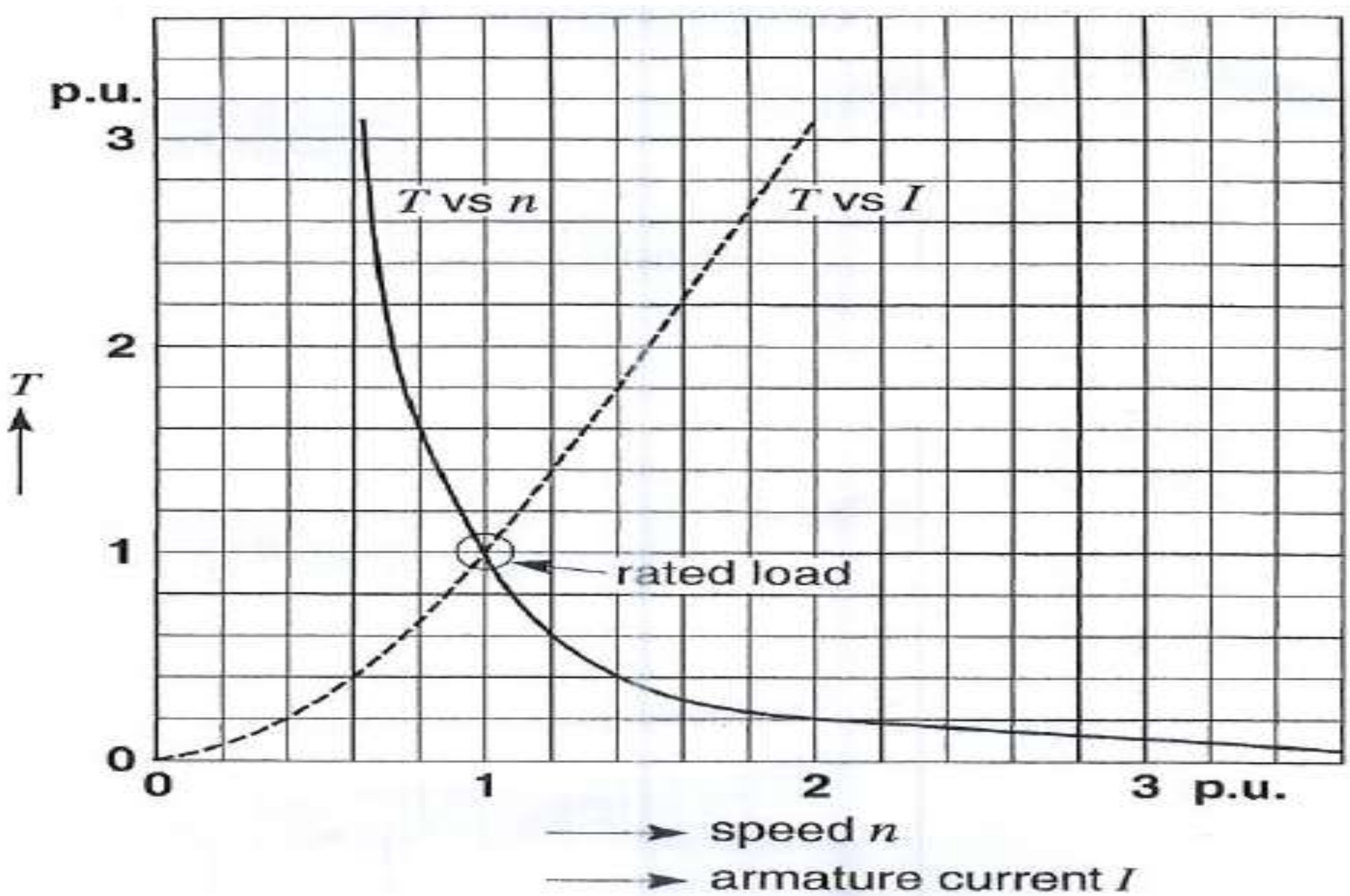


Fig (3.13) Typical Torque- Speed and Torque -current characteristics.

Example 3.6:

A 15 hp., 240V, 1780 RPM ,DC series Motor has a full – load rated current of 54A . Its operating characteristic are given by the per – unit curves Fig (3.13). Calculate:

a) The current and speed when the load Torque is 24 N.m.

b) The efficiency under these conditions.

Solution:

a) Establish the base power, base speed, and base current, correspond to the full – load ratings as:

$\mathbf{P_B}$ = 15 h.p. = 15×746 = 11190 Watt.

$\mathbf{n_B}$ = 1780 RPM : $\mathbf{I_B}$ = 54 Amp.

The base Torque is; therefore:

$\mathbf{T_B} = (9.55\times \mathbf{P_B}) / \mathbf{n_B}$ = = (9.55× **11190**)/ 1780 = 60 N.m.

A load Torque of 24 N.m. corresponds to a per unit Torque of:

$\mathbf{T_{P.U} = T_{Actual} / T_{Base}}$ = 24/60 = 0.4

Referring to Fig (3.13), a Torque of 0.4 per unit attained at a speed of 1.4 per unit.

Thus, the speed is:

$\mathbf{n = n_{per\ unit} \times n_{Base}}$ = 1.4 ×1780 = 2492 RPM.

From the torque versus current curve Fig (3.13):

A Torque of 0.4 p.u. →→ requires → a current of 0.6 Amp

Consequently, the load current is:

$\mathbf{I = I_{per\ unit} \times I_{Base}}$ = 0.6 ×54 = 32.4 Amp.

b) To calculate the efficiency (η), we have to know $\mathbf{P_O}$ and $\mathbf{P_i}$:

P_i = E×I = 240×32.4 = 7776 Watt.

P_o = (**n × T)/9.55** = (2492×24)/9.55 = 6293 Watt.

Efficiency (η) = **P_o** / **P_i** = 6293/ 7776 = 0.805 OR 80.5 %

The Compounded DC Motor

A Compounded DC Motor is a motor with both a Shunt and a Series Field, Such a Motor is shown in Fig (3.14).

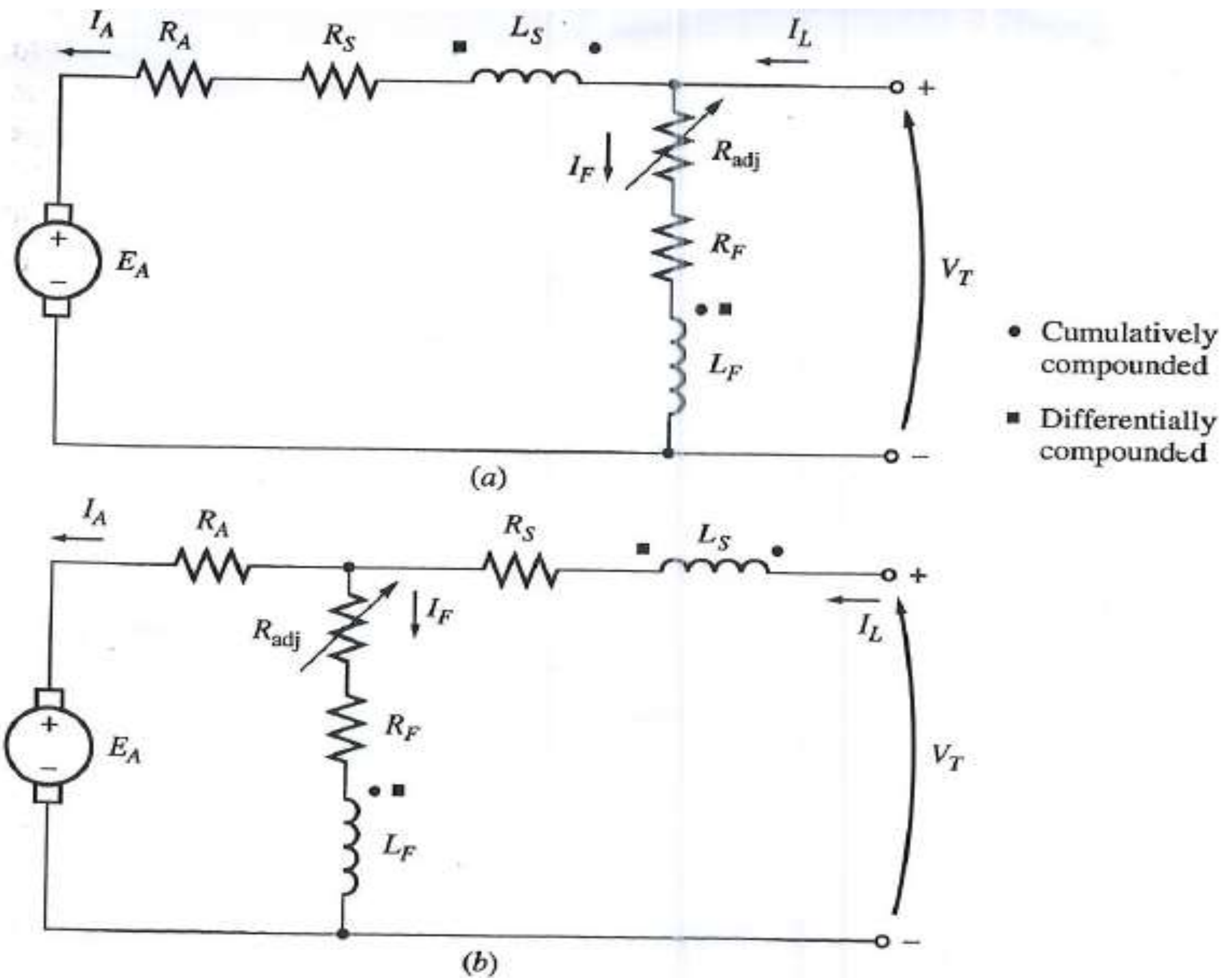


Fig (3.14). **Equivalent circuit of compounded DC Motor a) long- shunt b) short - shunt.**

The dots that appear on the two field coils have the same meaning us the dots on a transformer. Current flowing into a dot produces a (+ ve) magnetomotive force.

If current flows into the dots on both field coils, the resulting magnetomotive forces ADD to produce alarger total magnetomotive force. This situation is known as CUMULATIVE COMPOUNDING.

If current flows into the dot on one field coil and out of the dot on other field coil the resulting magnetomotive forces SUBTRACT each other.

In Fig (3.14), the round dots correspond to cumulative compounding of the motor, and the squares correspond to differential compounding.

The Kirchhoff's Voltage Law equation for a Compound DC motor is:

$$V_T = E_a + I_a (R_a + R_{se}) \quad \text{........ (3.22)}$$

The current in the Compound DC motor are related by:

$$I_a = I_L - I_F \quad \text{........ (3.23)}$$

$$I_F = V_T/R_F \quad \text{........ (3.24)}$$

The net magnetomotive force and the effective shunt field current in the Compound DC motor are given by:

$$\mathcal{F}_{net} = \mathcal{F}_F \pm \mathcal{F}_{se} - \mathcal{F}_{AR} \quad \text{........ (3.25)}$$

And:

$$I_F^* = I_F \pm (N_{se}/ N_F) \times I_a - (\mathcal{F}_{AR} / N_F) \quad \text{........ (3.26)}$$

Where: + ve sign in equations is associated with a cumulatively compounded motor

– ve sign in equations is associated with a differentially compounded motor.

The Torque – speed characteristic of Cumulatively Compounded DC Motor

In the Cumulatively Compounded DC Motor, there is flux component which is constant and another component which is proportional to its armature current (and thus to its load), therefore, cumulatively Compounded DC Motor has a higher starting Torque than a shunt Motor [whose flux is constant], but a lower starting Torque than a Series Motor [whose entire flux is proportional to armature current].

Cumulatively Compounded DC Motor combines best features of shunt and series Motors:

*Like a Series Motor, it has extra Torque for starting.

*Like a Shunt Motor, it does not over speed at No load.

At light loads, the series field has a very small effect, so the Motor behaves approximately as a Shunt DC.

As the load gets very large, the series flux becomes quite important and the Torque – speed curve begins to look like a series motor's characteristics. A comparison of the Torque– speed curve of each of these types of machine is shown in Fig (3.15).

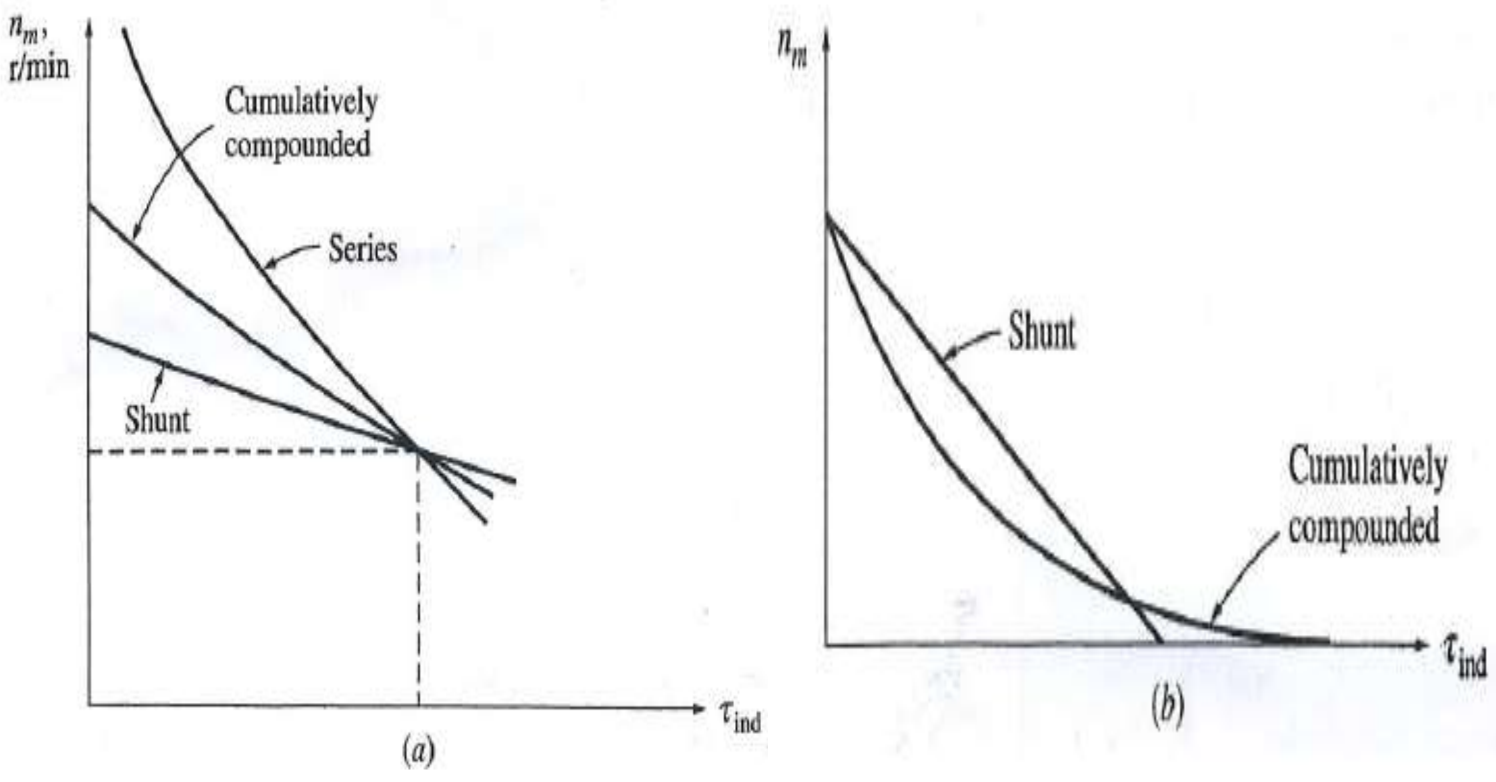


Fig (3.15). **The Torque – speed characteristic of Cumulatively Compounded DC Motor.**

To determine the characteristic of Cumulatively Compounded DC Motor by nonlinear analysis, the approach is similar to that for the shunt and series motors.

The Torque – speed characteristic of Differentially Compounded DC Motor

In a Differentially Compounded DC Motor, the shunt magnetomotive force and series magnetomotive force subtract from each other, this means that as the load on the motor increase **I_a** increases and the flux in the motor decrease.

As the flux decrease , the speed of the motor increase. This speed increase causes another increase in load, which further increase in **I_a**. further decreasing the flux, and increasing the speed again.

The result is that a differentially Compounded DC Motor is unstable and tends to run a way. This instability is much worse than of a shunt motor with armature reaction.

It is so bad that a differentially compounded motor is unsuitable for any application.

A typical terminal characteristic for a differentially Compounded DC Motor is shown in Fig (3.16).

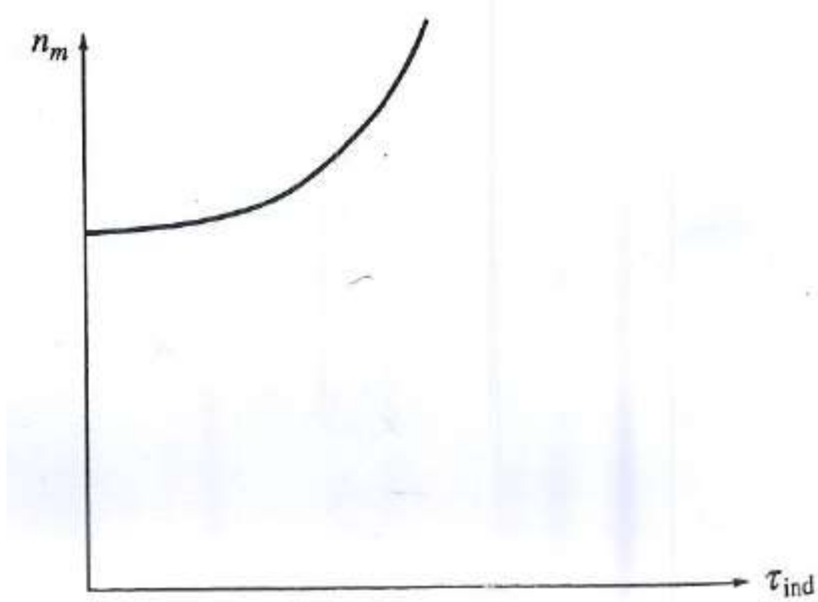


Fig (3.16) **Torque – speed characteristic of Differentially Compounded DC Motor**

Reversing the Direction of Rotation of DC Motor

To reverse the rotation direction of DC Motor, we must reverse:

1) The armature connections.

2) Both the Shunt and Series field connection. The inter poles are considered to form part of the armature. The change in connection is shown in Fig (3.16).

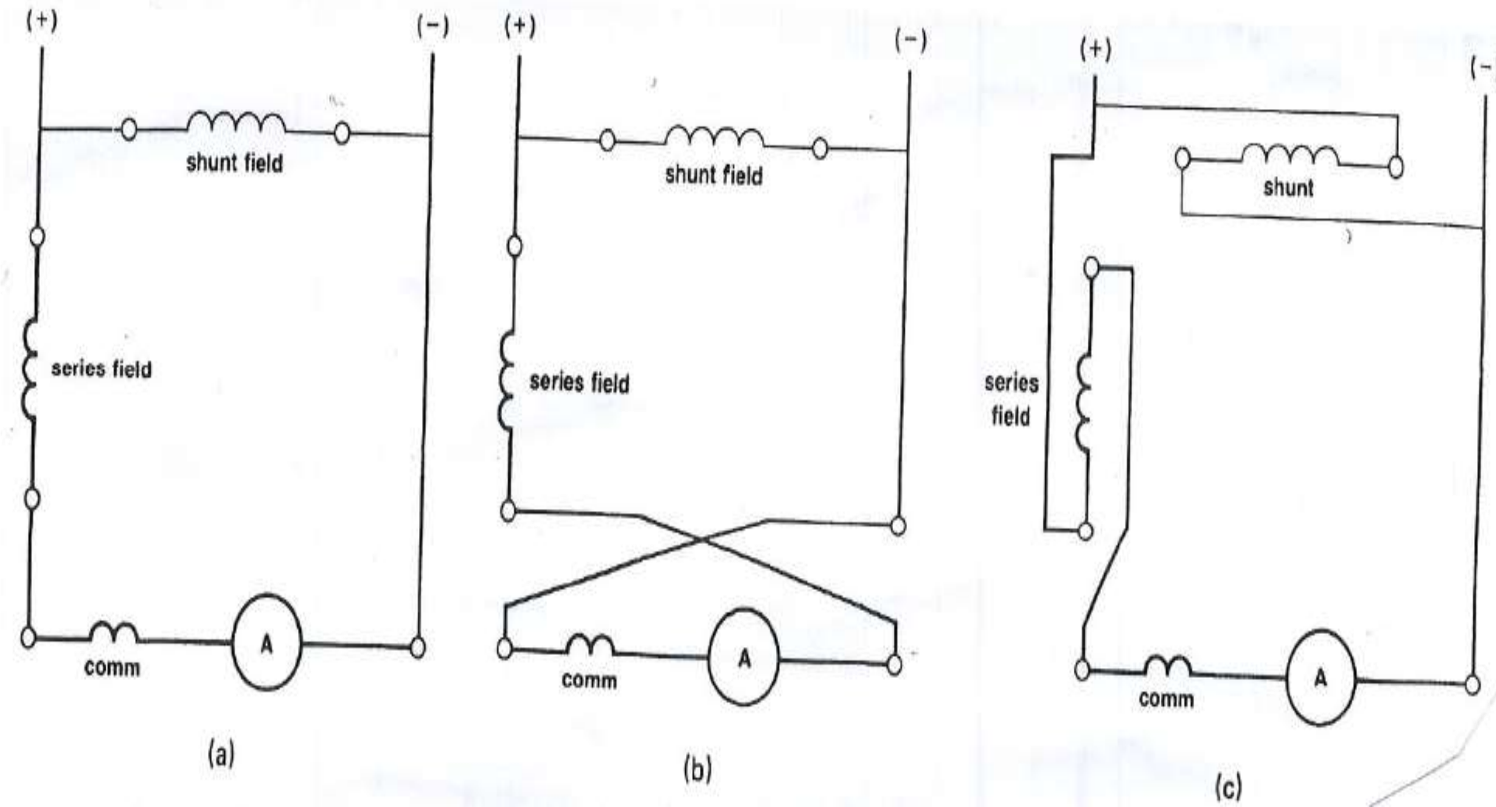


Fig (3.17).a) Original connection of a Compound Motor. b) Reversing the armature connections to reverse the rotation direction. c) Reversing the field connections to reverse rotation direction.

Example 3.7:

A 100 H.P, 250 V, compounded DC Motor with compensating windings has an internal resistance, including the series winding of 0.04Ω. There are 1000 turns per pole on shunt field and 3 turns per pole on series winding. The machine is shown **Fig (3.18),** and its magnetization curve is shown in **Fig (3.19),** at no load, the field

Resistor has been adjusted to make the motor run at 1200 RPM. The core, mechanical, and stray losses may be neglected.

a) What is the shunt field current at no Load?

b) If the motor is cumulatively Compounded, find its speed when $\mathbf{I_a}$ = 200 Amp.

c) If the motor is differentially Compounded, find its speed when $\mathbf{I_a}$ = 200 Amp.

Solution:

a) At no load, the armature current is ZERO, so the internal voltage $\mathbf{E_a = V_T}$. From magnetization curve, a field current of 5 Amp. Will produce a voltage $\mathbf{E_a}$ = 250 V at 1200 RPM. Therefore, the field current must be 5 Amp.

b) When an armature current of 200 Amp. Flows in the motor, the machine's internal generated voltage is:

$$\mathbf{E_a} = \mathbf{V_T} - \mathbf{I_a}(\mathbf{R_a} + \mathbf{R_{se}}) = 250\text{ V} - (200\text{ A} \times 0.04\ \Omega) = 242\text{V}$$

The effective field current of this cumulatively Compounded Motor is:

$$I_F^* = I_F + (N_{se}/N_F) \times \mathbf{I_a} - (\mathcal{F}_{AR}/N_F) = 5\text{ A} + (3/1000) \times 200 - \text{ZERO} = 5.6\text{ Amp.}$$

From the magnetization curve, $\mathbf{E_{ao}}$ = 262 V at speed n_o= 1200 RPM. Therefore the Motor's speed will be:

$$n = (E_a/E_{ao})\, n_o \rightarrow\rightarrow n = (242\text{V}/262\text{V}) \times 1200\text{ RPM} = 1108\text{ RPM.}$$

c) If the machine is differentially Compounded the effective field current is:

$$I_F^* = I_F - (N_{se}/N_F) \times \mathbf{I_a} - (\mathcal{F}_{AR}/N_F) = 5\text{ A} - (3/1000) \times 200\text{ A} - \text{ZERO} = 4.4\text{ Amp.}$$

From the magnetization curve, $\mathbf{E_{ao}}$ = 236 V at speed n_o= 1200 RPM. Therefore the Motor's speed will be: $n = (E_a/E_{ao})\, n_o \rightarrow n = (242\text{V}/236\text{V}) \times 1200\text{ RPM} = 1230\text{ RPM.}$

Notice that: The speed of cumulatively Compounded Motor decrease with load, while the speed of differentially Compounded Motor increases with load.

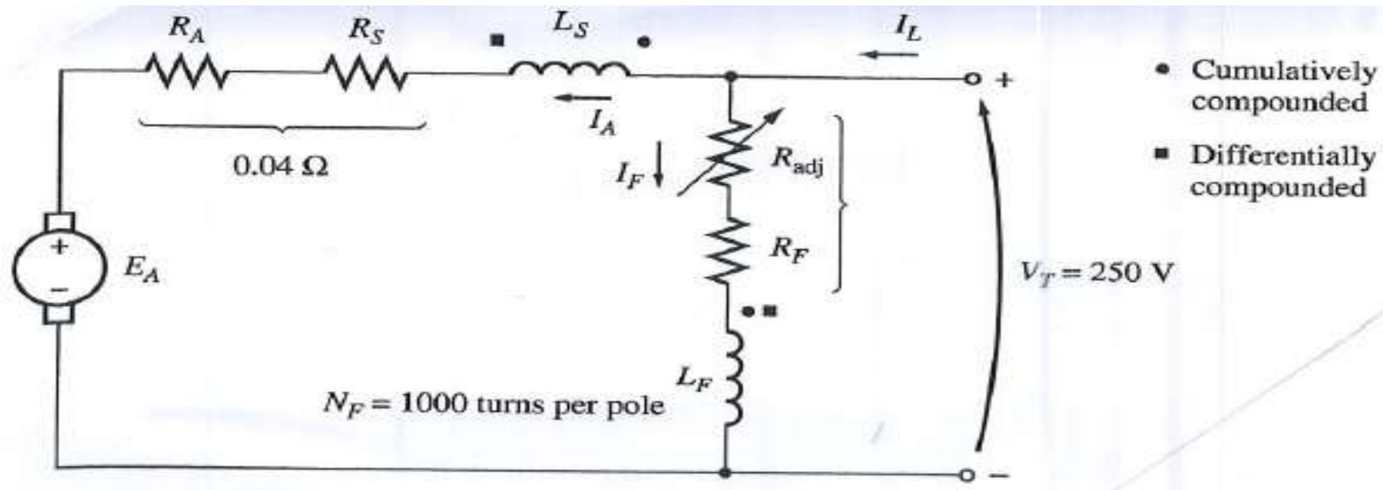


Fig (3.18) The **Compound Motor in example 3.7**

Fig (3.19) magnetization curve of a typical DC Motor, taken at a speed of 1200 RPM

Compensating Winding in DC Motor

Some DC Motors in the range of 100 KW to 10MW (134HP to 13400HP) employed in steel mills perform a series of rapid, heavy – duty operations. They accelerate, decelerate, stop, and, reverse, all in a matter of seconds.

The corresponding armature current increases, decreases, reverse in stepwise fashion, producing very sudden changes in armature reaction. For such motors the commutating poles and series stabilizing windings do not adequately neutralize the armature **m.m.F.**

Torque and speed control is difficult under such transient conditions and flashover may occur across the commutator. To eliminate this problem, special COMPENSATING WINDINGS are connected in series with the armature winding.

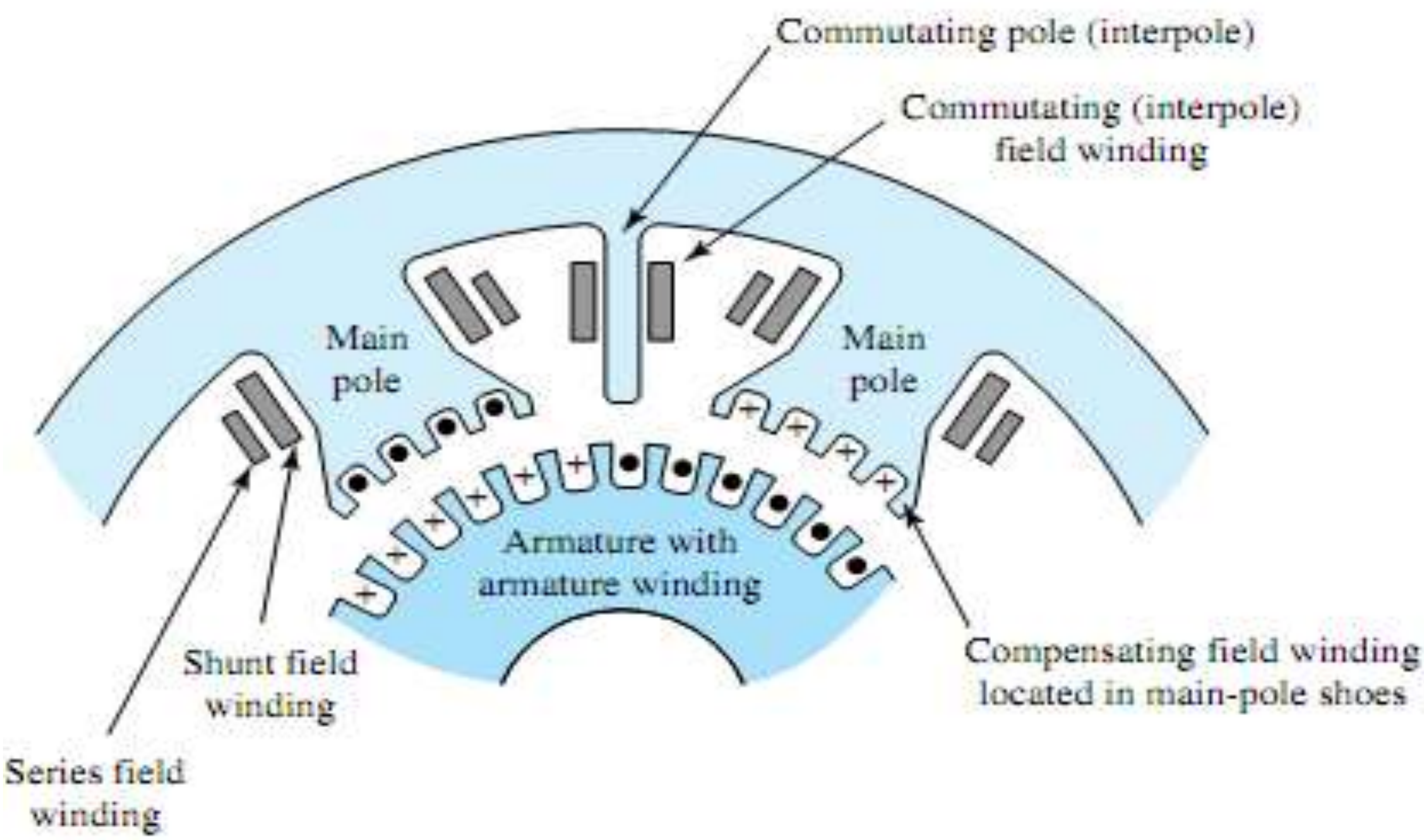


Fig (3.20). Compensating Winding are distributed in slots in the main poles.

The Compensating Winding are distributed in slots, cut into the pole faces of the main field poles, Fig (3.20).

Like commutating poles, these windings produce an **m.m.F** EQUAL AND OPPOSITE to **m.m.F** of the armature.

However, because the winding are distributed across the pole faces, the armature **m.m.F** is bucked from point to point which eliminate the field distortion.

DC Motor Protection

DC Motor must have some special control and protection equipment:

1) To protect the Motor against damage due to short – circuits in the equipment.

2) To protect the Motor against damage from long over loads

3) To protect the Motor against from excessive starting current.

4) To provide a convenient manner in which to control the operating speed of the Motor.