

# Salahaddin University - Erbil 

College of Engineering

Mechanical engineering Department

Third year Students
Electrical DC Machine

## DIRCET CURRENT MACHINE (D.C M/C)

The Construction of D.C machine
A simplified sketch of a D.C machine is shown in Fig (1.1). The physical structure


Fig (1.1) A simplified diagram of a D.C. Machine
of the machine consists of Two parts:
1 - The Stator or stationary part.
2 - The Rotor (Armature) or rotating part.
The stationary part of the machine consists of:

- Frame, which provides physical support.
- Pole pieces, which project inward and provide a path for the magnetic flux in the machine.
The ends of the pole pieces that are near the Rotor spread out over the Rotor surface to distribute its flux evenly over the Rotor surface, these ends are called POLE SHOES, see Fig (1.2), the schematic representation of D.C machine, the field
coils are located on the poles, and the field windings is symmetrical about the centerline of the field poles. This axis (axis of field poles) is called the field axis or direct axis.


Fig (1.2) Schematic representations of D.C. machine.
The rotating part or the rotor consists of:

- A shaft machined from a steel Bar with a core built up over it.
- Iron core, is composed of slotted iron laminations that are stacked to form a soled cylindrical core, see Fig (1.3).
- A set of coils called ARMATURE WINDING.
- The commutator which is composed of an assembly of tapered copper segments insulated from each other by mica sheets, and mounted on the shaft of the machine, Fig (1.4).
Great care is taken in building the commutator because any eccentricity will cause the brushes to bounce, producing unacceptable sparking.
The sparks burns the brushes and over heat and carbonize the commutator.


Fig (1.3) Armature lamination with tapered slots
Fig (1.4) Commutator of a D.C. machine.

## BRUSHES AND ROCKER:

Multipole machine possess many brush sets as they have poles. A two pole generator has two brushes fixed diametrically opposite to each other, Fig (1.5).


Fig (1.5) a) Brushes of $\mathbf{2}$-pole generator b) Brushes and connections of 6-pole generator. The brushes slide on the commutator and ensure good electrical contact between the revolving armature and the stationary external load.
The brushes are supported by movable brush yoke (rocker arm) or sometimes called holder. The rocker arm can be moved to rotate through an angle and then locked in the neutral position.
A round commutator, the successive brush sets have (+ ve) and (-ve) polarities.

Brushes having the same polarity are connected together and leads are brought out to one (+ ve) and one (-ve) terminal as shown in Fig (1.5b).

The brushes are made of Carbon because it has good electrical Conductivity. The Brush pressure is set by means of adjustable springs, the imperfect contact may produce sparking.

The D.C. Machine Windings
There are TWO principal winding on a D.C. machine:
1 - The armature winding: which are defined as the windings in which a voltage is induced. In a normal D.C. machine, the armature winding are located on the Rotor.
Because the armature winding are located on the Rotor, a D.C. machine's Rotor It self is sometimes called an ARMATURE.
2 - The Field Windings: are defined as the windings that produce the main magnetic flux in the machine, and the field windings are located on the stator.

## Terminology

1 - Turn:
When the two conductors lying in a magnetic field are connected in series to build a single Turn, so that the e.m.f. induced in them help each other.
2 -Coil:
One or more turns of wire grouped together and mounted on the drum - wound armature.
3 -Coil Side:
Any side of the Coil that cuts lines of flux.

## 4 - Back Pitch ( $\mathbf{Y}_{\mathrm{b}}$ ):

The number of Coil sides slots spanned by the back end connection, see Fig (1.6).
5 - Front Pitch ( $\mathbf{Y}_{\mathrm{F}}$ ):
May be defined as the distance (in terms of armature conductors) between the second conductor of one Coil and the first conductor of the next coil which are connected together.


Fig (1.6) Connection Type (a) Lab winding

wave winding
(b) Wave winding

6 - Resultant Pitch ( $\mathbf{Y o r} \mathbf{Y}_{\mathbf{R}}$ ):
It is distance between the beginning of one coil and the beginning of the next coil.

## 7 - Commutator Pitch ( $\mathbf{Y}_{\mathrm{c}}$ ):

It is the distance between the Segments to which the two ends of coil are connected, $\left(\mathbf{Y}_{\mathrm{c}}\right)$ measured in commutator bars or segments.

8 - Single - Layer Winding:
It is that Winding in which one conductor or one coil side is placed in each armature slot.

$$
9 \text { - Two - Layer Winding: }
$$

For this type of Winding, there are two conductors or coil side per slot arranged in two layers, usually, one side of every coil lies in the upper of the slot and other side lies in the lower half of some other slot at a distance of approximately one pitch away.

10 - Average Pitch ( $\mathrm{Y}_{\mathrm{av}}$ ):

$$
\mathbf{Y}_{\mathrm{av}}=\left(\mathbf{Y}_{\mathrm{b}}+\mathbf{Y}_{\mathrm{F}}\right) / 2
$$

## Types of Armature Windings

In general there are two types of drum Armature Windings:
1 - Lap Windings:
In the Lap Winding the finish of each Coil is connected to the start of the next Coil, so that the Winding or Commutator Pitch is:

Commutator Pitch $\left(\mathbf{Y}_{\mathrm{c}}\right)= \pm 1 \quad \rightarrow \rightarrow$ Unity
A Lap Winding has armature coils shaped as shown in Fig (1.7).


Fig (1.7). Lap Winding


Fig (1.8) 4 pole lap winding: $Y_{F}=7, Y_{b}=9$

A Lap Winding is so called because it alternately progresses forward and then Laps back upon itself, see Fig (1.8).
A simplex (simple) Lap Winding, 4 pole machine with 36 face conductors or coil is shown in Fig (1.8).

Referring to Fig (1.8), we find that:
Back Pitch $\left(\mathbf{Y}_{\mathrm{b}}\right)=10-1=9$
Front Pitch $\left(\mathbf{Y}_{\mathrm{F}}\right)=10-3=7$
Average Pitch $\left(\mathbf{Y}_{\mathrm{av}}\right)=\left(\mathbf{Y}_{\mathrm{b}}+\mathbf{Y}_{\mathrm{F}}\right) / 2=(9+7) / 2=8$

## Commutator Pitch $\left(\mathbf{Y}_{\mathrm{c}}\right)=\mathbf{1}$

In a simple Lap Winding, front and back pitches are always ODD and differ by (2) while the average pitch is an even number.

## 2 - Wave Winding:

The shape of the armature coils is shown in Fig (1.9). A wave winding made up of such coils is called wave because it has the appearance of a wave, as may readily be observed in Fig (1.10).


Fig (1.9) One Turn Wave Winding


Fig (1.10) 4 pole, double layer, simplex Wave Winding with 30 coils side.
The following points are noting in case of a Simplex Wave winding:

1) The front and back pitches are always ODD, they may be equal or differ by 2 , and nearly equal to the Pole Pitch, in which case, they are one more or one less than the Average Pitch ( $\mathbf{Y a v}_{\mathrm{av}}$ ),
2) The Average Pitch $\left(\mathbf{Y}_{\mathrm{av}}\right)$ is always very nearly equal to $(=Z / P)$, where:

Z - Is the number of armature Conductors.
P - Number of Poles.
3) The product of the average pitch and number of poles must be:

$$
\begin{equation*}
Z=\left(P \times Y_{a v}\right) \pm 2 \rightarrow \rightarrow \quad Y_{a v}=(Z \pm 2) / P \tag{1.1}
\end{equation*}
$$

Since $\mathbf{P}$ is even and $\mathbf{Z}=\left(\mathbf{P} \times \mathbf{Y a v}_{\mathrm{av}}\right) \pm 2 \rightarrow \rightarrow \mathbf{Y a v}=(\mathbf{Z} \pm 2) / \mathbf{P}$, therefore $\mathbf{Y}_{\mathrm{av}}$ must be even number.

The ( + ) sign will give a progressive winding and ( - ) sign will give a retrogressive winding.
a) Both Pitch's $\mathbf{Y}_{\mathbf{b}}$ and $\mathbf{Y}_{\mathbf{F}}$ are ODD and of same sign.
b) Resultant Pitch $\mathbf{Y}_{\mathbf{R}}=\mathbf{Y}_{\mathbf{F}}+\mathbf{Y}_{\mathbf{b}}$
c) Commutator Pitch $\mathbf{Y}_{\mathrm{c}}=\mathbf{Y a v}^{\mathrm{a}}$
d) The Average Pitch $\left(Y_{\text {av }}\right)=(Z \pm 2) / \mathbf{P}$, which must be an even integer.
e) The number of coils $\left(\mathbf{N}_{\mathbf{c}}\right)$ can be found from the relation:

$$
\left(N_{c}\right)=\left(P \times Y_{a v}\right) / 2
$$

f) The number of armature conductors with 2 added or subtracted must be a multiple of the number of poles.

The following points should be carefully noted in Simplex Lap winding:
i) The back and front pitches are ODD and of opposite sign. But they cannot be equal, they differ by 2.
ii) Both $\mathbf{Y}_{b}$ and $\mathbf{Y}_{\mathbf{F}}$ should be nearly equal to Pole Pitch.
iii) $\quad$ The Average $\operatorname{Pitch}(\mathbf{Y a v})=\left(\mathbf{Y}_{\mathrm{b}}+\mathbf{Y}_{\mathrm{F}}\right) / 2$
iv) Commutator Pitch $\left(Y_{c}\right)= \pm 1$
v) Resultant Pitch $\mathbf{Y}_{\mathrm{R}}$ is EVEN, being the arithmetical difference of two odd numbers
vi) The number of slots for a 2 - Layer winding is equal to $=$ number of Coils.(i.e. half the number of Coil sides). The number of Commutator Segments is also the same.

Taking the first condition, we have:

$$
Y_{b}=Y_{F} \pm 2
$$

a) If $Y_{b}>\mathbf{Y}_{F} \rightarrow \mathbf{Y}_{b}=\mathbf{Y}_{\mathrm{F}}+\mathbf{2}$ then we get progressive winding, and $\left(\mathbf{Y}_{\mathrm{c}}\right)=\mathbf{+ 1}$.
b) If $\mathbf{Y}_{b}<Y_{F} \rightarrow Y_{b}=Y_{F}-\mathbf{2}$ to get retrogressive winding, in this case $\left(Y_{c}\right)=\mathbf{- 1}$.
c) For progressive winding: $\quad \mathbf{Y}_{\mathrm{F}}=(\mathbf{Z} / \mathbf{P}) \mathbf{- 1} \quad$ and $\quad \mathbf{Y}_{\mathrm{b}}=(\mathbf{Z} / \mathbf{P})+\mathbf{1}$

For retrogressive winding $\quad \mathbf{Y}_{\mathrm{F}}=(\mathbf{Z} / \mathrm{P})+\mathbf{1} \quad$ and $\quad \mathbf{Y}_{\mathrm{b}}=(Z / P)-\mathbf{1}$
Note that: ( $\mathbf{Z} / \mathbf{P}$ ) must be even number.
Example 1:
Consider a six - pole simplex wave winding machine having 70 conductors.
Find $\mathbf{Y}_{\mathrm{av}}, \mathbf{Y}_{\mathrm{b}}, \mathbf{Y}_{\mathrm{F}}$, and $\mathbf{Y}_{\mathrm{c}}$.
Solution:
The Average Pitch $\left(\mathbf{Y}_{\mathrm{av}}\right)=(\mathbf{Z}+2) / \mathbf{P}=(70+2) / 6=12$
Front Pitch $\left(\mathbf{Y}_{\mathrm{F}}\right)=\mathbf{Y a v}^{\mathrm{av}} \mathbf{- 1 = 1 2 - 1 = 1 1}$
Back Pitch $\left(\mathbf{Y}_{\mathrm{b}}\right)=\mathbf{Y}_{\mathrm{av}}+1=12+1=13$
Commutator Segments $=(\boldsymbol{Z} / \mathbf{2})=70 / 2=35$
Commutator Pitch $\mathbf{Y}_{\mathrm{c}}=$ Average Pitch $(\mathbf{Y a v})=12$
Example2:
A progressive Lap winding, 4 pole, 24 slots, with one coil side per slots, single layer. Find:
a) Front Pitch $\left(\mathbf{Y}_{\mathbf{F}}\right)$.
b) Back Pitch $\left(\mathbf{Y}_{\mathrm{b}}\right)$.

Solution: $\quad \mathbf{P}=4$ and $\quad \mathbf{Z}=24$
Average $\operatorname{Pitch}\left(\mathbf{Y}_{\mathrm{av}}\right)=\left(\mathbf{Y}_{\mathrm{b}}+\mathbf{Y}_{\mathrm{F}}\right) / 2=(\mathbf{Z} / \mathbf{P})=24 / 4=6$
i.e $\quad \mathbf{Y}_{b}+\mathbf{Y}_{\mathbf{F}}=2 \times 6=12$

For progressive Simplex winding:

$$
\begin{equation*}
\mathbf{Y}_{b}=\mathbf{Y}_{\mathrm{F}}+2 \tag{2}
\end{equation*}
$$

Solve the two equations:

$$
\mathbf{Y}_{\mathrm{b}}=7 \quad \text { and } \quad \mathbf{Y}_{\mathrm{F}}=5
$$

Example3:
A D.C. machine of double Layer Lap winding, 6 - poles, 18 slots, with two coil sides per slot, find:
a) Average Pitch ( $\mathbf{Y}_{\mathrm{av}}$ ).
b) Front Pitch $\left(\mathbf{Y}_{\mathbf{F}}\right)$.
c) Back Pitch ( $\mathbf{Y}_{\mathrm{b}}$ ).

Hint:
Number of conductors $(\boldsymbol{Z})=2 \times \mathbf{N}_{\mathbf{c}}=2 \times$ Number of coils
Also Number of coils $=$ Number of coil sides/2
And $\quad \boldsymbol{Z}=$ Number of coil sides
Solution: Number of poles =6 and Number of slots $=18$
Number of coil sides $=$ Number of slots $\times$ Number of coil sides per slots

$$
=18 \times 2=36
$$

Since $(\boldsymbol{Z})=2 \times \mathbf{N}_{\mathbf{c}}=36$

Average $\operatorname{Pitch}\left(\mathbf{Y}_{\mathrm{av}}\right)=\left(\mathbf{Y}_{\mathrm{b}}+\mathbf{Y}_{\mathrm{F}}\right) / 2=(\mathbf{Z} / \mathbf{P})=(36 / 6)=6$

$$
\begin{equation*}
\mathbf{Y}_{\mathrm{b}}=\mathbf{Y}_{\mathrm{F}}=6 \times 2=12 \tag{1}
\end{equation*}
$$

For progressive Simplex Lap windings:

$$
\begin{equation*}
Y_{b}=Y_{F}+2 \tag{2}
\end{equation*}
$$

Solving equation (1) and (2):
$\mathbf{Y}_{\mathrm{b}}=7$ and $\quad \mathbf{Y}_{\mathrm{F}}=5$
Example4:
A 4 pole simplex wave winding DC. Machine, the armature have 30 coil side. Find:
$\mathbf{Y}_{\mathrm{av}}, \mathbf{Y}_{\mathrm{b}}, \mathbf{Y}_{\mathrm{F}}$, and $\mathbf{Y}_{\mathrm{c}}$ ?
Solution: $\quad$ Number of poles P =4 ; Number of coil sides $=30$
Average Pitch $\left(\mathbf{Y a v}_{\mathrm{av}}\right)=(\boldsymbol{Z}+2) / \mathbf{P}=(30+2) / 4=8$
Front Pitch $\left(\mathbf{Y}_{\mathrm{F}}\right)=\mathbf{Y a v}_{\mathrm{av}}-1=8-1=7$
Back Pitch $\left(\mathbf{Y}_{\mathrm{b}}\right)=\mathbf{Y a v}^{\mathrm{av}} \mathbf{1}=8+1=9$
Commutator segments $=\boldsymbol{Z} / 2=30 / 2=15$
Commutator Pitch $\mathbf{Y}_{\mathrm{c}}=$ Average Pitch $\left(\mathbf{Y}_{\mathrm{av}}\right)=8$
What happen if you take $\left(\mathbf{Y}_{\mathrm{av}}\right)=(\mathbf{Z}-2) / \mathbf{P} \boldsymbol{?} \rightarrow \rightarrow \mathbf{Y}_{\mathrm{av}}=7$, If $\mathbf{Y}_{\mathrm{av}}$ is taken odd then $\left(\mathbf{Y}_{\mathrm{b}}\right)$ and $\left(\mathbf{Y}_{\mathrm{F}}\right)$ will be equal to $\mathbf{Y}_{\mathrm{av}}=7$

## Example5:

Develop simplex Lap winding for a 12 slots, 4 pole machine, with armature Commutator segments equal to 12 .

Solution:
Number of poles $P=4$.

Number of slots $=12$.
Number of coil sides $=$ Number of slots $\times$ Number of coil sides per slots

$$
=12 \times 2=24
$$

OR Number of coils = Number of Commutator segments

$$
\text { Hence } \quad \mathbf{N}_{\mathrm{C}}=12
$$

And the number of Conductors $(\mathbf{Z})=2 \times \mathbf{N}_{\mathbf{C}}=2 \times 12=24$
The Average Pitch ( $\mathrm{Y}_{\mathrm{av}}$ ) is nearly equal to:

$$
Y_{a v}=Z / P=24 / 4=6
$$

For progressive winding: $\quad \mathbf{Y}_{\mathbf{F}}=(\mathbf{Z} / \mathbf{P})-\mathbf{1}=6-1=5$

$$
\text { And } \quad Y_{b}=(Z / P)+1=6+1=7
$$

Example6:
Develop a simplex wave winding for a 13 slots, 4 pole machine, with armature Commutator segments equal to 13.

Solution: Number of poles $\mathrm{P}=4 . \quad$; Number of slots $=13$
For a simplex wave winding: commutator segments $=(\mathbf{Z} / \mathbf{2})$
Hence $\rightarrow \rightarrow \quad Z=2 \times$ commutator segments $=2 \times 13=26$.

$$
\begin{aligned}
& \left(\mathbf{Y}_{\mathrm{av}}\right)=(\mathbf{Z} \pm 2) / \mathrm{P}=(\mathbf{2 6} \pm 2) / \mathbf{4} \rightarrow=(\mathbf{2 6}+2) / \mathbf{4}=\mathbf{2 8 / 4}=\mathbf{7} \quad \text { OR } \quad \mathbf{Y}_{\mathrm{av}}=(\mathbf{2 6 - 2}) / \mathbf{4}=\mathbf{6} \\
& \quad \text { Average Pitch }(\mathbf{Y a v})=6 \\
& \text { Front Pitch }\left(\mathbf{Y}_{\mathrm{F}}\right)=\mathbf{Y}_{\mathrm{av}}-1=6-1=5 \\
& \text { Back Pitch }\left(\mathbf{Y}_{\mathrm{b}}\right)=\mathbf{Y}_{\mathrm{av}}+\mathbf{1}=6+1=7 \\
& \text { Resultant Pitch } \mathbf{Y}_{\mathrm{R}}=\mathbf{Y}_{\mathrm{F}}+\mathbf{Y}_{\mathrm{b}}=5+7=12 \\
& \text { Commutator Pitch } \mathbf{Y}_{\mathrm{c}}=\text { Average Pitch }\left(\mathbf{Y}_{\mathrm{av}}\right)=6
\end{aligned}
$$

## The Magnetization Curve of DC. Machine

The internal generated Voltage $\left(\mathrm{E}_{\mathrm{a}}\right)$ of a DC. Motor or Generator is given by:

$$
\begin{align*}
& E_{a}=K \phi \omega  \tag{1.2}\\
& K \text {-constant }=(P \times Z) / 2 \pi a
\end{align*}
$$

P-Number of pole.

Z-Number of conductors
$\boldsymbol{\phi}$ - Flux per pole (weber)
a - Number of current parallel path.
$\boldsymbol{\omega}$ - angular speed (rad. per. sec.)

Therefore, $\mathbf{E}_{\mathbf{a}}$ is directly proportional to the flux in the machine and the speed of rotation.

The Field Current in a DC. Machine produces a field magnetomotive Force ( $\mathcal{F}$ ) $\mathcal{F}=N_{F} \times I_{F}$
$I_{F}$ - Field Current.
$\mathrm{N}_{\mathrm{F}}$ - Number of the Field turns.
This magnetomotive Force produces a flux in the machine in accordance with its Magnetization curve, Fig (1.11).


Fig (1.11). The Magnetization curve of a ferromagnetic material $\{\boldsymbol{\phi}$ versus $\mathscr{F}\}$

Since the field Current is directly proportional to magnetomotive Force and since ( $\mathrm{E}_{\mathrm{a}}$ ) is directly proportional to the flux, it is customary to present the Magnetization curve as a plot of $\left(\mathbf{E}_{\mathbf{a}}\right)$ versus Field Current for a given speed ( $\boldsymbol{\omega}_{\mathbf{o}}$ ), Fig (1.12). It is noting that, to get the maximum possible power of a machine, most motors and generators are designed to operate near saturation point on the Magnetization curve (at the knee of the curve), this implies that a fairly large increase in field Current is often necessary to get a small increase in ( $\mathbf{E}_{\mathrm{a}}$ ), when operation is near full load.


Fig (1.12) The Magnetization curve of DC. Machine, a plot of $\mathbf{E}_{\mathbf{a}}$ versus $\mathbf{I}_{\mathbf{F}}$.

## Armature Reaction

We have assumed the magnetomotive Force (m.m.F) acting in a DC. Generators is that due to the field. However, the current flowing in the armature coils also creates a powerful magnetomotive Force that distorts and weakens the flux coming from the poles.

This distortion and field weaking takes place in both motors and generators. The effect produced by the armature (m.m.F) is called ARMATURE REACTION.

To understand the impact of the armature (m.m.F), we return to generator under the load, Fig (1.13).


Fig (1.13) the energy conversion in DC. Generator
If we consider the armature alone (no load), it will produce a magnetic field as shown in Fig (1.14).


Fig (1.14) Armature flux at no load


Fig (1.15) Armature reaction effect

This field acts at right angles to the field produced by the ( $\mathrm{N}-\mathrm{S}$ ).
The intensity of the armature flux depends upon its m.m.F, which in turn depends upon the current carried by the armature. Thus, contrary to the field flux, the armature flux is not constant but varies with load.

We can immediately foresee a problem which the armature flux will produce. Fig (1.15) shows that flux in the neutral zone is no longer ZERO and, consequently, a voltage will be induced in the coils that are short - circuited (S.C) by the brush's.

As a result, severe sparking may occur, the intensity of sparking will depend Upon armature flux and hence upon the load current delivered by the generator.

The second problem created by the armature m.m.F is that it distorts the flux produced by the poles.

The combination of the armature m.m.F and the field m.m.F produce a magnetic field whose shape is illustrated in Fig (1.15).

The neutral zones have shifted in the direction of armature rotation, this occurs In all DC. Generators. The flux distortion produces still another effect:

The higher flux density in pole tips 2,3 , causes saturation to set in. consequently, the increase in flux under pole tips 2,3 , is less than the decrease in flux under pole tips 1, 4.

As a result, the total flux produced by the ( $\mathrm{N}-\mathrm{S}$ ) poles is less than it was when the generator was running at no - load. This causes a corresponding reduction in the induced voltage given by:

$$
\begin{equation*}
E_{0}=(z \times n \times \phi) / 60 \tag{1.4}
\end{equation*}
$$

$\mathrm{E}_{\mathbf{o}}$ - The induced voltage in a DC. Generator having a lap winding.
Z - Total Number of conductor on the armature.
$\boldsymbol{\phi}$ - Flux per pole (weber) $\quad n-$ Speed of rotation ( $r / \mathrm{min}$.)
Equation (1.4) shows $\mathbf{E}_{\mathbf{o}}$ which is directly proportional to the flux per pole:

$$
E_{0} \alpha \phi
$$

For large machine, the decrease in flux may be as much as $10 \%$.
It is important to note that the orientation of the armature flux remains fixed in space; it does not rotate with armature.

The Internal Generated Voltage and Induced Torque of DC. Machines
The induced voltage in any given machine depends on three factors:
1 - The flux $(\boldsymbol{\phi})$ in the machine.
2 - The speed ( $\boldsymbol{\omega}$ ) of the machine's Rotor.
3 - A constant depending on the construction of the machine.
The voltage in any Single Conductor under the pole faces is:
$\mathrm{e}_{\text {ind }}=\mathrm{e}=\mathbf{V} \times \mathrm{B} \times \mathrm{L}$
Where:
V-Tangential velocity. ; B - Magnetic field density. ; L-Length of conductor.
The voltage out of a machine is thus:

$$
E_{a}=(Z \times V \times B \times L) / a
$$

$Z$ - Total Number of conductor on the armature
a - Number of current parallel path
The velocity of each conductor in the Rotor can be expressed as:

$$
\begin{equation*}
\mathbf{V}=\mathbf{r} \times \boldsymbol{\omega} \tag{1.6}
\end{equation*}
$$

$\boldsymbol{\omega}$ - Angular speed (rad. per. sec.) $\quad \mathbf{r}$-is the radius of the Rotor.
So:
$E_{a}=(z \times r \times \omega \times B \times L) / a$
This voltage can be re- expressed from noting that:
The flux of the pole $(\boldsymbol{\phi})=$ The flux density under the Pole $\times$ The Pole's area (A) Hence:

$$
\begin{equation*}
\phi=B \times A_{p} \tag{1.8a}
\end{equation*}
$$

The Rotor of the machine ( $\mathrm{m} / \mathrm{c}$ ) is shaped like a cylinder, so its area is:

$$
\begin{equation*}
A=2 \pi r L \tag{1.8b}
\end{equation*}
$$

If there are number of poles equal to $(\mathbf{P})$ on the machine, then the portion of the associated with each pole is:

$$
\begin{equation*}
\mathbf{A}_{\mathbf{p}}=(\text { the total area } \mathbf{A}) / \text { Number of poles }=\mathbf{A} / \mathbf{P} \tag{1.8c}
\end{equation*}
$$

Therefore:

$$
\begin{equation*}
A_{p}=(2 \pi r L) / P \tag{1.8d}
\end{equation*}
$$

The total flux per pole in the machine is:

$$
\begin{equation*}
\phi=B \times A_{p}=[B \times(2 \pi r L)] / P=(2 \pi r L \times B) / P \tag{1.8e}
\end{equation*}
$$

Therefore, the internal generated voltage in the machine can be expressed as:

$$
\begin{aligned}
E_{a} & =(Z \times r \times \omega \times B \times L) / a \\
E_{a} & =\{(Z \times r \times L) / a\} \times(B) \times \omega=\{(Z \times r \times L) / a\} \times\left(\phi / A_{p}\right) \times \omega \\
& =\{(Z \times r \times L) / a\} \times(P / 2 \pi r L) \times\{(2 \pi r L \times B) / P\} \times \omega \\
& =(Z \times P / 2 \pi a) \times\{(2 \pi r L \times B) / P\} \times \omega
\end{aligned}
$$

$$
\begin{equation*}
E_{a}=(Z \times P / 2 \pi a) \times \phi \times \omega \tag{1.9}
\end{equation*}
$$

Finally: $\quad \mathbf{E}_{\mathbf{a}}=\boldsymbol{K} \times \boldsymbol{\phi} \times \boldsymbol{\omega}$
Where: $\quad K=(Z \times P / 2 \pi a)$
It is common to express the speed of a machine in revolutions per minute instead of radians per second. The conversion from revolutions per minute to radians per second is:

$$
\begin{equation*}
\omega=2 \pi n / 60 \tag{1.11}
\end{equation*}
$$

So the voltage equation in terms of revolutions per minute is:

$$
\begin{align*}
& E_{a}=K^{\prime} \times \phi \times n  \tag{1.12}\\
& K^{\prime}=(Z \times P) / 60 a \tag{1.13}
\end{align*}
$$

Torque equation of DC. Machine
The Torque in any DC. Machine depend on three factors:
1 - The flux ( $\phi$ ) in the machine.
2 - The armature (or ROTOR) current $I_{a}$ in the machine.
3 - A constant depending on the construction of machine.
The Torque on the armature of a machine:
$\boldsymbol{\tau}_{\text {ind }}=$ Number of conductors $\boldsymbol{Z} \times$ The Torque on each conductor.
The Torque in any single conductor is given by:

$$
\begin{equation*}
\boldsymbol{\tau}_{\text {cond }}=\mathbf{r} \times \mathrm{I}_{\text {cond }} \times \mathbf{L} \times \mathbf{B} \tag{1.14}
\end{equation*}
$$

If there are (a) current paths in the machine, then the total armature current $I_{a}$ is split among the (a) current paths, so the current in a single conductor is given by:

$$
\begin{equation*}
I_{\text {cond }}=I_{a} / a \tag{1.15}
\end{equation*}
$$

And the Torque in a single conductor on the motor may be expressed as:

$$
\begin{equation*}
\tau_{\text {cond }}=\left(r \times I_{a} \times L \times B\right) / a \tag{1.16}
\end{equation*}
$$

Since there are $(\mathbf{Z})$ conductors, the total induced Torque in a DC. Machine rotor is:

$$
\begin{equation*}
\boldsymbol{\tau}_{\text {ind }}=\left\{\boldsymbol{Z} \times\left(\mathbf{r} \times \mathrm{I}_{\mathrm{a}} \times \mathbf{L} \times \mathbf{B}\right)\right\} / \mathbf{a} \tag{1.17}
\end{equation*}
$$

The flux per pole in this machine can be expressed as:

$$
\begin{equation*}
\phi=B \times A_{p}=[B \times(2 \pi r L)] / P=(2 \pi r L \times B) / P \tag{1.18}
\end{equation*}
$$

So the total induced Torque can be re - expressed as:

$$
\begin{gather*}
\tau_{\text {ind }}=(Z \times P / 2 \pi a) \times \phi \times I_{a}  \tag{1.19}\\
\tau_{\text {ind }}=K \times \phi \times I_{a} \tag{1.10}
\end{gather*}
$$

Where: $\quad K=(Z \times P / 2 \pi a)$
Both the internal generated voltage and the induced Torque equations given above are only approximations, because not all the conductors in the machine are under the pole faces at any time and also because the surfaces of each pole do not cover an entire(1/P) of the rotor surface.

## Example1:

A duplex Lap -wound armature is used in a 6 - pole DC. Machine with six brush sets, each spanning two commutator segments. There are 72 coils on the armature, each containing 12 turns. The flux per pole in the machine is 0.039 wb , and the machine spins at 400 r.p.m.
(a)How many current paths are in this machine?
(b)What is its induced voltage $\mathbf{E}_{a}$ ?

Solution:
(a)The number of current paths in this machine is:
$a=m \times p=2(6)=12$ current paths.
(b)The induced voltage in the machine is:

$$
E_{a}=K^{\prime} \times \phi \times n \quad \text { and } \quad K^{\prime}=(Z \times P) / 60 a
$$

The Number of conductors in this machine is:
$Z=2 \mathbf{C} \times \mathbf{N}_{\mathrm{C}}=2 \times(72) \times 12=1728 \quad$ conductors.
Therefore, the constant $\mathbf{K}^{\prime}$ is:
$K^{\prime}=(Z \times P) / 60 a=(1728 \times 6) / 60 \times 12=14.4$
And the voltage

$$
E_{a}=K^{\prime} \times \phi \times n=14.4 \times 0.039 \mathrm{wb} \times 400 \text { r.p.m. }=224.6 \text { volt. }
$$

Example2:
A 12-pole DC. Generator has a simplex wave - wound armature containing 144 coils of 10 turns each. The resistance of each turn is $0.011 \Omega$. Its flux per pole is 0.05 wb ., and it is turning at a speed of 200 r.p.m.
(a)How many current paths there is in this machine?
(b)What is the induced armature voltage of this machine?
(c)What is the effective armature resistance?
(d) If a $1 \mathrm{~K} \Omega$ load Resistance is connected to the terminals of this generator, what is the resulting induced counter - Torque on the machine shaft? Ignore the internal armature resistance of the machine.

Solution:
(a) There are $\mathbf{a}=\mathbf{2 \times m}=\mathbf{2}$ current paths.
(b) $Z=\mathbf{2 C} \times \mathbf{N}_{\mathrm{C}}=2 \times 144 \times 10=2880$ conductors.

$$
K^{\prime}=(Z \times P) / 60 a=(2880 \times 12) / 60 \times 2=288
$$

Therefore:
The induced voltage $\mathbf{E}_{\mathbf{a}}$ is:

$$
E_{a}=K^{\prime} \times \phi \times n=288 \times 0.05 \mathrm{wb} \times 200 \text { r.p.m. }=2880 \text { volt }
$$

(c)There are two parallel paths through the rotor of this machine, each one consisting of:
$z / \mathbf{2 = 1 4 4 0}$ conductors $O R \rightarrow$ Number of Turns=Number of conductors/2=720. Therefore, the resistance in each current paths is:

$$
\text { Resistance/path }=720 \text { turns } \times 0.011 \Omega \text { per turn }=7.92 \Omega \text {. }
$$

Since there are two parallel paths, the effective armature resistance is:

$$
\mathbf{R a}_{\mathbf{a}}=7.92 \Omega / 2=3.96 \Omega .
$$

(d) If a $1000 \Omega$ load is connected to the generator terminals, and if $\mathbf{R}_{a}$ is ignored. Than a current I: $\quad \mathrm{I}=\mathrm{V} / \mathrm{R}=2880 \mathrm{~V} / 1000 \Omega=2.88 \quad$ Amp.
The constant $\mathbf{K}$ is given by:

$$
K=(Z \times P / 2 \pi a)=(2880 \times 12) /(2 \pi \times 2)=2750.2
$$

Therefore, the counter - Torque of the Generator is:

$$
\boldsymbol{\tau}_{\text {ind }}=K \times \phi \times \mathrm{I}_{\mathrm{a}}=2750.2 \times 0.05 \mathrm{wb} . \times 2.88 \mathrm{~A}=396 \quad \text { N.m }
$$

## Power Flow and Losses in DC. Machine:

DC. Generators take in mechanical power and produce electric power, while DC.

Motor take in electric power and produce mechanical power. In either case, not all the power input to the machine appears in useful form at the end, there is always some loss associated with the process.
The efficiency ( $\eta$ ) of a DC. Machine defined by the equation:

$$
\begin{equation*}
\eta=(\text { output power/ input power }) \times 100 \%=\left(P_{o / p} / P_{i / p}\right) \times 100 \% \tag{1.11}
\end{equation*}
$$

The difference between the input power and the output power of a machine is the losses that occur inside it. Therefore:

$$
\begin{equation*}
\eta=\left\{\left(P_{i / p}-\text { Loss }\right) / P_{i / p}\right\} \times 100 \% \tag{1.12}
\end{equation*}
$$

The losses in DC. Machines
The losses that occur in DC. Machine are

1 - Electrical or Copper losses ( $I^{2} \mathrm{R}$ losses).
2 - Brush losses.
3 - Core losses or Iron losses.
4 - Mechanical losses.
5 - Stray load losses.


## 1 - Electrical or Copper losses ( $I^{2} \mathrm{R}$ losses)

These losses are also known as winding losses as the copper loss occurs in the armature windings and field windings of the machine.

The copper losses are given by:
Armature copper losses: $\mathbf{P}_{\mathrm{A}}=\mathbf{I}_{\mathbf{a}}{ }^{\mathbf{2}} \mathbf{R a}$
Field losses: $\mathbf{P}_{\mathbf{F}}=\mathbf{I F}_{\mathbf{F}}{ }^{\mathbf{2}} \mathbf{R}_{\mathbf{F}}$
Where:
$\mathbf{I}_{\mathbf{a}}$ is armature current.
$\mathbf{R a}$ is the armature resistance. ; $\quad \mathbf{R}_{\mathbf{F}}$ is the field resistance.
The resistance used in these calculations is usually the winding resistance at normal operating temperature.

The following equations enable us to determine the resistance at any Temperature and for any material:

$$
\begin{array}{lc}
\mathbf{R}=\boldsymbol{\rho}(\mathbf{L} / \mathbf{A}) & \ldots \ldots . .(1.14) \\
\boldsymbol{\rho}=\boldsymbol{\rho}_{\mathbf{0}}(1+\boldsymbol{\alpha} \mathbf{t}) & \ldots \ldots . .(1.15)  \tag{1.15}\\
\mathbf{R} \text { - Resistance of conductor } & {[\Omega] .} \\
\mathbf{L} \text { - Length of conductor } & {[\mathrm{m}] .} \\
\mathbf{A} \text { - Cross section of conductor } & {\left[\mathrm{m}^{2}\right] .} \\
\boldsymbol{\rho} \text { - Resistivity of conductor at Temperature (t) } & {[\Omega . \mathrm{m}] .} \\
\boldsymbol{\rho}_{\mathbf{o}} \text { - Resistivity of conductor at Temperature } & 0^{\circ} \mathrm{C}[\Omega . \mathrm{m}] . \\
\boldsymbol{\alpha} \text {-Temperature Coefficient of Resistance at } 0^{\circ} \mathrm{C} . \\
\mathbf{t} \text {-Temperature of conductor. }
\end{array}
$$

- In DC. Motors and generators, copper Losses occur in the armature, the series field, the shunt field, the commutating poles, and the compensating winding.
- $\mathbf{I}^{\mathbf{2}} \mathbf{R}$ losses show up as heat, causing the conductor Temperature to rise above ambient Temperature.
- Instead of using the $\mathbf{I}^{\mathbf{2}} \mathbf{R}$ equation, we sometimes prefer to express the losses in terms of the number of Watt per Kilogram of conductor material.

The losses are given by the equation:

$$
\begin{equation*}
\mathbf{P}_{\mathbf{c}}=\left[1000 \mathbf{J}^{2} \mathbf{\rho}\right] / \zeta \tag{1.16}
\end{equation*}
$$

Where:

$$
\mathbf{P}_{\mathbf{c}}-\text { Specific Conductor power Losses } \quad[\mathrm{W} / \mathrm{Kg}]
$$

J- Current density
[A/(m.m) $\left.{ }^{2}\right]$
$\boldsymbol{\rho}$ - Resistivity of the conductor
$\zeta$ - Density of the conductor
1000 - Constant.
[n.m]
$\left[\mathrm{Kg} / \mathrm{m}^{3}\right]$

According to this equation, the losses per unit mass is proportional to the square of the current density.

## Example 3:

A DC. Machine turning at 875 r.p.m carries an armature winding whose total weight is 40 Kg . The current density is $5\left[\mathrm{~A} /(\mathrm{m} . \mathrm{m})^{2}\right]$ and the operating Temperature is $80^{\circ} \mathrm{C}$. The total Losses in the armature amount to 1100 Watt.

## Calculate:

(a)The Copper Losses. (b)The Mechanical Drag [N.m] due to the iron Losses.

Solution:
(a) Referring to the table (AX2), the Resistivity of Copper at $80^{\circ} \mathrm{C}$ is:

$$
\boldsymbol{\rho}=\boldsymbol{\rho}_{\mathbf{0}}(1+\boldsymbol{\alpha} \mathbf{t})=15.88[1+(0.00427 \times 80)]=21.3 \quad[\mathrm{n} \Omega . \mathrm{m}] .
$$

The density of Copper is 8890
$\left[\mathrm{Kg} / \mathrm{m}^{3}\right]$
The Specific Power Losses is:

$$
\mathbf{P}_{\mathbf{c}}=\left[1000 \mathbf{J}^{2} \boldsymbol{\rho}\right] / \zeta=\left[1000 \times 5^{2} \times 21.3\right] / 8890=60 \quad[\mathrm{~W} / \mathrm{Kg}] .
$$

Total Copper Losses is:
$\mathrm{P}=60 \times 40=2400$ Watt.
(b)The braking Torque due to iron Losses can be calculated from:

$$
\begin{equation*}
P=(n \times T) / 9.55 \tag{1.1}
\end{equation*}
$$

$$
1100=(\mathbf{8 7 5} \times \mathbf{T}) / 9.55
$$

$$
\mathbf{T}=\mathbf{1 2}[\mathrm{N} . \mathrm{m}] \quad \text { or approximately } 8.85 \mathrm{ft} .1 \mathrm{bf} .
$$

## 2 - Brush losses.

Brush drop losses are the losses taking place between commutator and carbon brushes. It is the power loss at brush contact point. Its depends upon brush contact voltage drop and armature current Ia. It is given by the equation shown below:

$$
\begin{align*}
& \mathbf{P}_{\mathrm{BD}}=\mathbf{V}_{\mathrm{BD}} \times \mathbf{I a}  \tag{1.18}\\
& \mathbf{P}_{\mathrm{BD}}-\text { Brush drop losses. ; } \mathbf{V}_{\mathrm{BD}}-\text { Brush Voltage drop. (1.18) } ; \mathbf{I a}-\text { Armature Current. }
\end{align*}
$$

The reason that Brush losses are calculated in this manner is that Voltage drop across a set of Brush's is approximately constant over a large range of Armature Current.

Unless otherwise specified, Brush Voltage drop is usually to be about (2 volt).

## 3 - Core losses or Iron losses.

The core losses are hysteresis and eddy current losses occurring in the metal of the motor. These losses vary as the flux density square ( $\mathrm{B}^{2}$ ), and for the Rotor as the $\left(1.5^{\text {th }}\right)$ power of the rotation speed $\left(\mathrm{n}^{1.5}\right)$,these losses almost considered constant as the machines are usually operated at constant flux density and constant speed.

## 4 - Mechanical losses.

Mechanical losses are due to bearing friction, Brush friction, and Windage.
The friction losses depend upon the machine speed and upon the bearing design, Brush, Commutator, and Slip Rings.

Windage losses depend on the speed and design of the cooling fan and on the turbulence produced by the revolving parts.

In the absence of prior information, we usually conduct tests on the machine itself to determine the value of these mechanical losses.

Rotating machine are usually cooled by an internal fan mounted on the motor shaft, it draws in cool air from the surroundings.

In hostile environment, special cooling methods are sometimes used.

## 5 - Stray load losses or (Miscellaneous losses):

Stray losses are losses that cannot be placed in one of the pervious categories. No matter how carefully losses are accounted for, some always escape inclusion in one of the above categories. All such losses are lumped into Stray losses. For most machine, stray losses are taken by convention to be one per cent ( $1 \%$ ) of the full load output power.


The Power - Flow Diagram of DC. Machine
The Power - Flow diagram is one of the most convenient techniques for accounting the power losses in a machine.
A Power - Flow Diagram for a DC. Generator is shown in Fig (1.16a)

(b)

Fig (1.16) Power - Flow Diagram of DC. Machine: (a) Generator: (b) Motor.
In Fig (1.16a), mechanical power is input into the machine, and then the stray losses, mechanical losses, and core losses are subtracted, after they have been subtracted, the remaining power is ideally converted from mechanical to electrical form at the point labeled $\mathbf{P}_{\text {conv. }}$

The mechanical power that is converted given by:

$$
\begin{equation*}
\mathbf{P}_{\text {conv }}=\boldsymbol{\tau}_{\text {ind }} \times \omega \tag{1.19}
\end{equation*}
$$

And the resulting electric power is:

$$
\begin{equation*}
\mathbf{P}_{\text {conv }}=\mathbf{E}_{\mathbf{a}} \times \mathbf{I a} \tag{1.20}
\end{equation*}
$$

However, this is not the power that appears at the machine terminals, before the terminal are reached, electrical ( $\left.\mathbf{I}^{\mathbf{2}} \mathbf{R}\right)$ losses and Bruch losses must be subtracted.

In the case of DC. Motors, this Power - Flow Diagram is simply reversed. The power flow Diagram for a motor is shown in Fig (1.16b).

Voltage Regulation in DC. Machine
DC. Generator are compared by their voltages power ratings, efficiencies, and VOLTAGE REGULATION.

Voltage Regulation $\left(\mathbf{V}_{\mathbf{R}}\right)$ is defined by the equation:

$$
\begin{equation*}
\mathbf{V}_{\mathbf{R}}=\left[\left(\mathbf{V}_{\mathbf{n L}}-\mathbf{V}_{\mathbf{F L}}\right) / \mathbf{V}_{\mathbf{F L}}\right] \times 100 \% \tag{1.21}
\end{equation*}
$$

Where:
$\mathbf{V}_{\mathbf{n L}}$-The No-Load Terminal Voltage.
$\mathbf{V}_{\text {FL- }}$ The Full- Load Terminal Voltage.
It is a rough measure of the shape of the generators Voltage - Current characteristic.

- A positive Voltage Regulation mean a drooping characteristic.
-A negative Voltage Regulation means a rising characteristic.

TABLE AX2 ELECTRICAL, MECHANICAL AND THERMAL PROPERTIES OF SOME COMMON CONDUCTORS (AND INSULATORS)

| Material | Chemical syanbol or composition | Electrical properties |  |  | Mechanical properties |  |  | Teernal propetirs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | reststivity p |  | $\begin{gathered} \text { temp } \\ \text { coeff } \end{gathered}$ | density | $\begin{gathered} \text { yield } \\ \text { strengith } \end{gathered}$ | ultimate strength |  | $\begin{gathered} \text { thernal } \\ \text { conduc- } \\ \text { livity } \end{gathered}$ | $\begin{aligned} & \text { rucluing } \\ & \text { point } \end{aligned}$ |
|  |  | n 0 m | nflan | ( $\times 10^{-3}$ | $8 / \mathrm{dm}^{3}$ | MPa | MPa | J/kg. ${ }^{\circ} \mathrm{C}$ | W/m. ${ }^{6} \mathrm{C}$ |  |
| aluminum | Al | 26.0 | 28.3 | 4.39 | 2703 | 21 | 62 | 960 | 218 | 660 |
| brass | $\infty 70 \% \mathrm{Cu}, \mathrm{Zn}$ | 60.2 | 62.0 | 1.55 | $\sim 8300$ | 124 | 370 | 370 | 143 | 960 |
| carbonl graphite | C | 8000 to <br> 30000 | - | - -0.3 | $\sim 2500$ | - | . | 710 | 5.0 | 3600 |
| constantan | $\begin{aligned} & 54 \% \mathrm{Cu}, \\ & 45 \% \mathrm{Ni}, 1 \% \mathrm{Mn} \end{aligned}$ | 500 | 500 | $-0.03$ | 8900 | - | - | 410 | 22.6 | 1190 |
|  |  |  |  |  |  |  |  |  |  |  |
| copper | Cu | 15.88 | 17.24 | 4.27 | 8890 | 35 | 220 | 380 | $\begin{aligned} & 394 \\ & 296 \end{aligned}$ | $\begin{aligned} & 1083 \\ & 1063 \end{aligned}$ |
| gold | Au | 22.7 | 24.4 | 3.65 | 19300 | - | 69 | 130 |  |  |
| iron | Fe | 88.1 | 101 | 7.34 | 7900 | 131 | 290 | 420 | 79.4 | 1535 |
| manganin | Pb | 482 | 220 | 4.19 | $\begin{array}{r} 11300 \\ 8410 \end{array}$ | - | 15 | 130 | 35 |  |
|  | $84 \% \mathrm{Cu}, 4 \% \mathrm{Ni}$, <br> $12 \% \mathrm{Mn}$ |  | 482 | $\pm 0.015$ |  |  | 15 | - | 35 20 | $\begin{aligned} & 327 \\ & 1020 \end{aligned}$ |
| mercury | Hg | 951 | 968 | 0.91 | 13600 | . | . |  |  |  |
| molybdenum | Mo | 49.6 | 52.9 | 3.3 | 10200 | . | 690 | 246 | 138 |  |
| monel | $30 \% \mathrm{Cu}$, | 418 | 434 | 1.97 | 8800 | 530 | 690 | 530 | 25 | 1360 |
|  | $69 \% \mathrm{Ni}, 1 \% \mathrm{Fe}$ |  |  |  |  |  |  |  |  |  |
| nichrome <br> nickel <br> platinum | $80 \% \mathrm{Ni}, 20 \% \mathrm{Cr}$ | 1080 | 1082 | 0.11 | 8400 | 200 | $\begin{aligned} & 690 \\ & 300 \end{aligned}$ | $\begin{aligned} & 430 \\ & 460 \\ & 131 \end{aligned}$ | $\begin{gathered} 11.2 \\ 90 \\ 71 \end{gathered}$ | $\begin{aligned} & 1400 \\ & 1455 \\ & 1773 \end{aligned}$ |
|  | Ni | 78.4 | 85.4 | 4.47 | 8900 |  |  |  |  |  |
|  | Pt | 9.7 | 10.4 | 3.4 | 21400 |  |  |  |  |  |
| sitver | Ag | 15.0 | 16.2 | 4.11 | 10500 | - | - | 230 | 408 | 960 |
| tungsten | W | 49.6 | 55.1 | 5.5 | 19300 | - | 3376 | 140 | 20 | 3410 |
| zinc | Zn | 55.3 | 59.7 | 4.0 | 7100 | - | 70 | 380 | 110 | 420 |
| airhydrogenpure water | $78 \% \mathrm{~N}_{2}$ | $\cdot{ }_{-}$ | . |  | 1.29 | - |  | 994 | 0.024 | - |
|  | $21 \% \mathrm{O}_{2}$ |  |  |  | 1.2 |  |  |  |  |  |
|  | $\mathrm{H}_{2}$ |  | - | - | 0.690 | - | - 1 | 14200 | 0.17 |  |
|  | $\mathrm{H}_{2} \mathrm{O}$ |  | 2.5 | - | 1000 | - | - | 4180 | 0.58 | 0.0 |
|  |  |  |  |  |  |  |  |  |  |  |

