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Third- Year Students

Electrical Machines

(**TRANSFORMERS**)

TRANSFORMERS

1.1 Introduction:

A TRANSFORMER: is a device that changes A.C. electric power from one Voltage level to A.C. electric power at another Voltage level through the action of a magnetic field. It consists of TWO or more coils of wire wrapped around a common ferromagnetic core, these coils are (usually) not directly connected.

The only connection between the coils is the common Magnetic Flux present within the core.

One of the Transformer winding connected to the power supply is called the PRIMARY winding or input winding, and the winding which connected to the load is called the SECONDARY winding or output winding.

In a modern power system, electric power is generated at Voltages of (12 to 25) KV.

Transformers step up the Voltage to between 32KV and 400 KV (in Iraq) for transmission over long distances at very low losses, Transformers then step down the Voltage to 11KV for local distribution and finally permit the power to be used safely in homes, offices, and factories at voltages as low as 400V Line– To– Line and 230V Line – To – Neutral.

In addition to the various power Transformers, two special – purpose Transformers are used with electric machinery and power system:

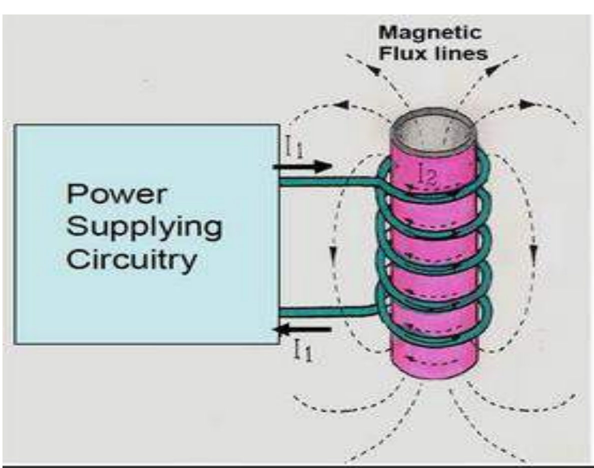
1 – Potential Transformer (PT): The primary side of the transformer is connected to the high-voltage circuit and the instrument or other circuits are connected to the secondary winding. Any number of instruments can be connected to the secondary until total impedance does not exceed the rated burden of the PT. See Fig (1).

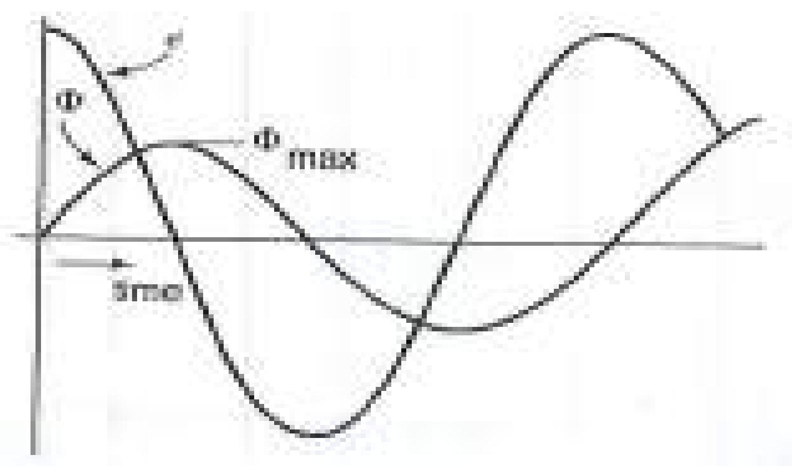
Fig (1) Potential Transformer (PT) **Fig (2) Current Transformer (C.T)**

2– Current Transformer (C.T): is a device designed to provide a secondary current much smaller than primary current, but directly proportional to its primary current. Fig (2)

1.2 Voltage induced in a coil:

Consider the coil of Fig (3) which surrounds (or links) a variable Flux (***ϕ***), the Flux alternates Sinusoidally at frequency (*f*), periodically reaching positive and negative peaks (***ϕmax***).





a) **A Voltage is induced in a coil when it links a variable flux.** **b) A sinusoidal Flux induces a sinusoidal voltage**

Fig (3) **Voltage induced in a coil**

The alternating Flux induces a Sinusoidal **A.C** Voltage in the coil, whose effective value is given by:

$$V = 4.44 \times f \times N \times \phi_{max} \quad \ldots\ldots. \quad (1.1)$$

Where:

V = effective Voltage induced [Volt].
f = frequency of the Flux [HZ].
N= Number of turns on the coil
ϕ_{max}= peak value of the Flux (WB) : *4.44*= a constant [exact value= $(2\pi/\sqrt{2}]$.

Equation (1.1) is derived from Faradays Law Equation:

$$e=N\left(\frac{\Delta\phi}{\Delta t}\right)$$

In which:

$\left(\frac{\Delta\phi}{\Delta t}\right)$ Is the rate of Flux change with the time, and

e-is the instantaneous induced Voltage.

Example (1.1):

A coil possesses 4000 turns and links an A.C Flux having of 2 mWB. If The frequency is (60 HZ), calculate the effective value of the induced voltage.

Solution:

$$V = 4.44 \times f \times N \times \phi_{max} = 4.44 \times 60 \times 4000 \times 0.002 = 2131 \text{ Volt}$$

induced Voltage has an effective or RMS value of 2131 V and frequency of (60HZ) .

The peak Voltage = $\sqrt{2}$ × RMS or effective value = $\sqrt{2} \times V$= $\sqrt{2} \times 2131$=3014V.

1.3 THE IDEAL TRANSFORMER

An ideal Transformer is a lossless device with an input winding and an output winding. The relationships between the input and output Voltages, and between the (i/p) and (o/p) currents shows in Fig (4), for in ideal Transformer, the relationships are given by:

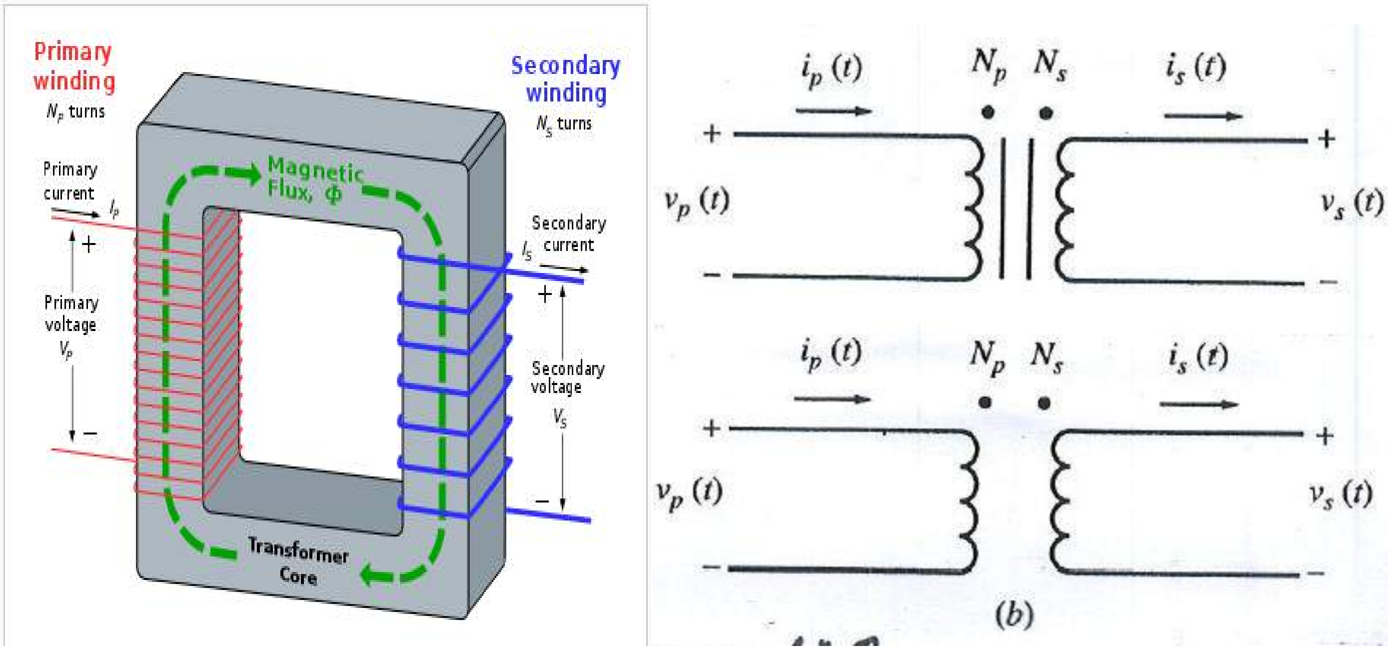


Fig (4) a) **sketch of an ideal Transformer.** b) **Schematic symbols of an ideal Transformer**.

$$\frac{Vp}{Vs} = \frac{Np}{Ns} = N \qquad \text{............ (1.2)}$$

Where:

N - Is defined to be the transformation ratio of the Transformer:

$$\boldsymbol{N = \frac{Np}{Ns}} \qquad \text{......... (1.3)}$$

The relationship between current I_P flowing into the primary side of Transformer and the current I_s flowing out of the secondary side of the Transformer is:

$$\boldsymbol{Np \times I_P = Ns \times I_s} \qquad \textbf{.......... (1.4a)}$$

$$\boldsymbol{\frac{I_P}{I_s} = \frac{Ns}{Np} = \frac{1}{N}} \qquad \textbf{.......... (1.4b)}$$

In terms of phasor quantities, these equations are:

$$\boldsymbol{\frac{\overrightarrow{V_p}}{\overrightarrow{V_S}} = N} \qquad \textbf{.......... (1.5)}$$

$$\frac{\overrightarrow{I_p}}{\overrightarrow{I_S}} = \frac{1}{N} \qquad \text{..........} \quad (1.6)$$

Notice that the phase angle of $\overrightarrow{V_p}$ is the same as the angle of $\overrightarrow{V_s}$, and the phase angle of I_P is the same as the angle of I_S.

The transformation ratio of the ideal Transformer affects the magnitudes of the Voltages and currents, but not their angles.

The dots appearing at one end of each winding in Fig (4) tell us the polarity of the Voltage and current on the Secondary side of the Transformer.

The relationship is as follows:

1-If the primary voltage is **(+ ve)** at the dotted end, then the Secondary Voltage will **(+ ve)** at the dotted end also.

Voltage polarities are same with respect to the dots on each side of the core.

2- If the Transformer primary current flows into the dotted end of primary winding, secondary current will flow out of the dotted end of Secondary winding.

Example (1.2):

A not-quite ideal Transformer having 90 turns on the primary and 2250 on the Secondary, is connected to a 120V, 60HZ source, the magnetizing current is 4 AMPS, Calculate:

a) The effective Voltage across the Secondary terminals.

b) The Peak Voltage across the Secondary terminals.

c) The instantaneous Voltage across the Secondary when the instantaneous Voltage across the primary is 37 Volt.

Solution:

a) $\frac{Vp}{Vs} = \frac{Np}{Ns} = \quad ; \frac{120}{Vs} = \frac{90}{2250} \qquad Vs = 3000\ V$

b) The peak Secondary Voltage is

$$Vs\ \text{(Peak)} = \sqrt{2} \times Vs = \sqrt{2} \times 3000 = 4242\ V.$$

c) The Secondary Voltage Vs is 25 times greater than Vp at every instant, consequently, when e_p **= 37V then** $e_s = e_p \times 25 = 37 \times 25 = 925V$**.**

Example (1.3):

An ideal Transformer having 90 Turns on the Primary and 2250 turns on the Secondary, is connected to a 200 V, 50HZ source. The load across Secondary draws a current of 2 AMPS at a power Factor of 80% lagging. Calculate:

a) The active value of the Primary current.

b) The instantaneous current in the Primary when the instantaneous current in the Secondary is 100 mAMPS.

c) The Peak Flux linked by the Secondary winding.

d) Draw the phasor diagram.

Solution:

a) The transformation ratio is:

$$N = \frac{Np}{Ns} = \frac{90}{2250} = \frac{1}{25}$$

$Np \times I_P = Ns \times I_s$ →→→ $90 \times I_P = 2250 \times 2$ I_P = 50 AMPS.

b) The instantaneous current in the Primary is always **25** times greater than instantaneous current in the Secondary winding, therefore when:

I_s = 100 mAMPS. I_P is:

$I_{P(ins)}$ = 25 × I_s instantaneous.

=25× 0.1 AMPS →→→ $I_{P(ins)}$ = 2.5 AMPS

c) In an ideal Transformer, the Flux linking Secondary winding is the same as that linking the Primary winding:

The Peak Flux in the Secondary winding is:

$Vp = 4.44 \times f \times Np \times \phi max$ →→ $\phi max = \frac{Vp}{4.44 \times f \times Np}$ →

$\phi max = \frac{200}{4.44 \times 50 \times 90}$ =0.01 →→→→→→→ =10 mWB.

d) To draw the phasor diagram, Secondary Voltage is:

$$\frac{Vp}{Vs} = \frac{1}{25} = N \quad ; \frac{120}{Vs} = \frac{90}{2250}$$

$\boldsymbol{Vs}$ =25 Vp =25 × 200= →→→→ Vs = 5000 Volt

$\boldsymbol{Vs}$ Is in phase with Vp , $\boldsymbol{I_P}$ is in phase with $\boldsymbol{I_s}$, see Fig(5),

phase angle between Vs &$\boldsymbol{I_s}$ is:

Power Factor (P.F) = cosϴ

0.8 = cosϴ ϴ=$\cos^{-1}$ 0.8 →→→ ϴ=36.9°

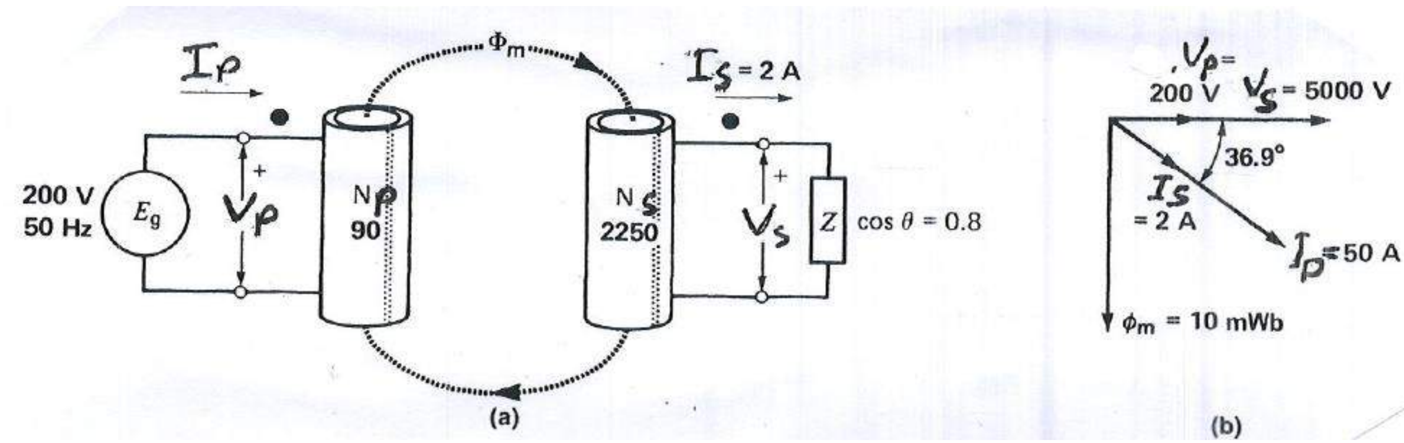


Fig (5) a- See example (1.3) b- Phasor relationships.

4.3.1 POWER IN AN IDEAL TRANSFORMER

The power supplied to the transformer by the primary circuit is given by:

Pin = Vp × Ip × COSϴp ....(1.7)

Where:

ϴp-is the angle between Primary Voltage and the Primary Current.

The power supplied by the transformer Secondary circuit to its load is given by:

Pout = Vs × Is × COSϴs .... (1.8)

Where:

ϴs-is the angle between Secondary Voltage Vs and Secondary current Is.

Since Voltage and current angles are unaffected in an ideal Transformer:

ϴp – ϴs = zero

i.e : The Primary and Secondary windings of an ideal Transformer have the same power factor (P.F)

It is possible to find out through a simple application of Voltage and current equations: The power out of a Transformer is:

$$Pout = Vs \times Is \times COS\theta_s \qquad ....(1.9)$$

Applying the transformation – ratio equation gives:

$$Vs = \frac{Vp}{N} \qquad \& \qquad Is = N \times I_P$$

So that :

$$Pout = \frac{Vp}{N} \times (N \times I_P) \times COS\theta = Vp \times Ip \times COS\theta = Pin \qquad ....(1.10)$$

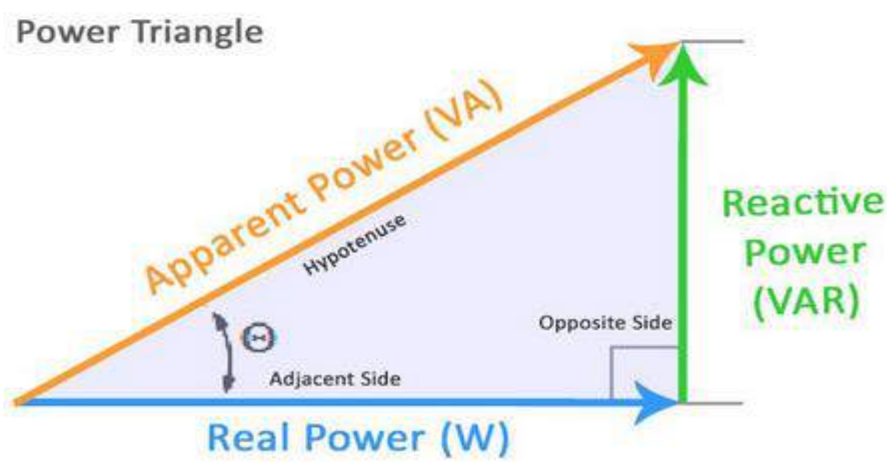
Thus (O/P) Power of an ideal Transformer is equal to its (I/P) Power.

The same relationship applies to the:

Reactive Power (Q) and Apparent Power (S);

$$Qin = Vp \times Ip \times SIN\theta = Vs \times Is \times SIN\theta = Qout \qquad ....(1.11)$$

$$Sin = Vp \times Ip = Sout \qquad ....(1.12)$$



4.3.2 IMPEDANCE TRANFORMATION through a Transformer:

The impedance of a device or an element is defined as the ratio of the phasor Voltage across it TO the phasor current flowing through it:

$$Z_L = \frac{\overrightarrow{V_L}}{\overrightarrow{I_L}} \qquad .... (1.13)$$

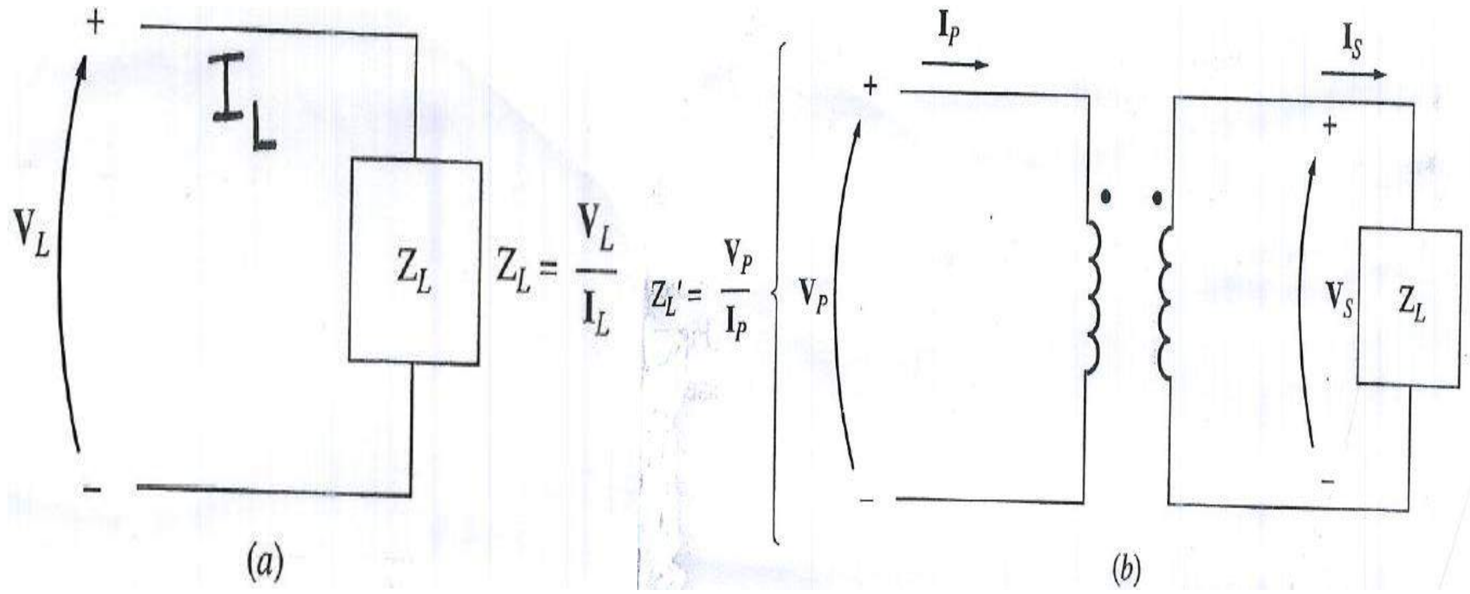


Fig (6) a) Definition of Impedance b) Impedance scaling through a Transformer.

Refer to Fig (6) if Secondary current is called $\overrightarrow{Is}$ and the Secondary Voltage $\overrightarrow{Vs}$
Then the impedance of the load is given by:

$$Z_L = \frac{\overrightarrow{V_s}}{\overrightarrow{I_s}} \qquad \text{..............} \ (1.14)$$

The apparent impedance of the primary circuit of the Transformer is:

$$Z_L^{`} = \frac{\overrightarrow{V_p}}{\overrightarrow{I_p}} \qquad \text{...........} \ (1.15)$$

Since the primary Voltage can be expressed as:

$$\overrightarrow{Vp} = N \times \overrightarrow{Vs} \qquad \text{..................} \ (1.16)$$

And the Primary current also can be expressed as:

$$\overrightarrow{Ip} = \frac{\overrightarrow{Is}}{N} \qquad \text{..................} \ (1.17)$$

The apparent Impedance of the Primary is:

$$Z_L^{`} = \frac{\overrightarrow{V_p}}{\overrightarrow{I_p}} = \frac{N \times \overrightarrow{Vs}}{\frac{\overrightarrow{Is}}{N}} = N^2 \times \frac{\overrightarrow{V_s}}{\overrightarrow{I_s}} \quad \text{Therefore:} \quad Z_L^{`} = N^2 \times Z_L \quad \text{.....} \ (1.18)$$

Example (1.4):

Calculate Voltage (E) and current (I) in the circuit of Fig (7) knowing that ideal Transformer (T) has a Primary to Secondary ratio of (1/100).

Solution:

Shaft all Impedances to the Primary side using equation (1.18);

$N^2 \times Z_L = (\frac{1}{100})^2 \times 20$ KΩ →→ To Primary side= 2Ω.

$(\frac{1}{100})^2 \times 40$ KΩ →→ To Primary side= 4Ω. , see Fig (8)

The Impedance of the circuit in Fig (8) is:

$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$ →→ $Z = \sqrt{[4^2 + (5 - 2)^2]} = \sqrt{[16 + 9]} = 5\ \Omega$

The current in the circuit is:

$$I = \frac{E_g}{Z} = \frac{10}{5} = 2 \quad AMPS.$$

The Voltage across the Resistor (4 Ω) is:

$$E \times N = E \times (\frac{1}{100}) = I \times R = 2 \times 4 = 8\ Volt$$

$$E \times (\frac{1}{100}) = 8\ \text{Volt} \rightarrow\rightarrow E = 8 \times 100 = 800\ \text{Volt.}$$

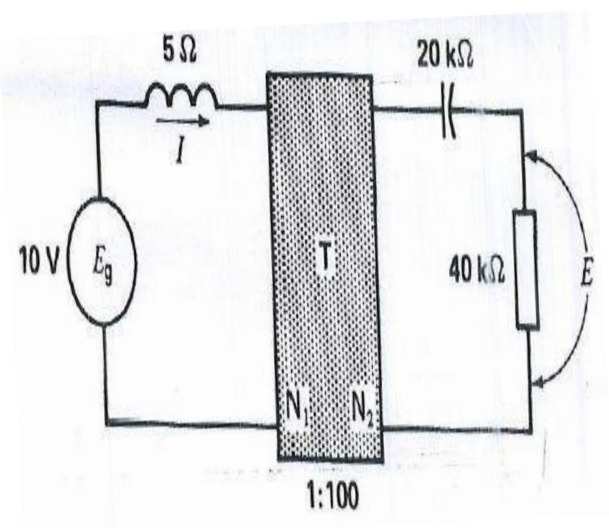


Fig (7) see example (1.4)

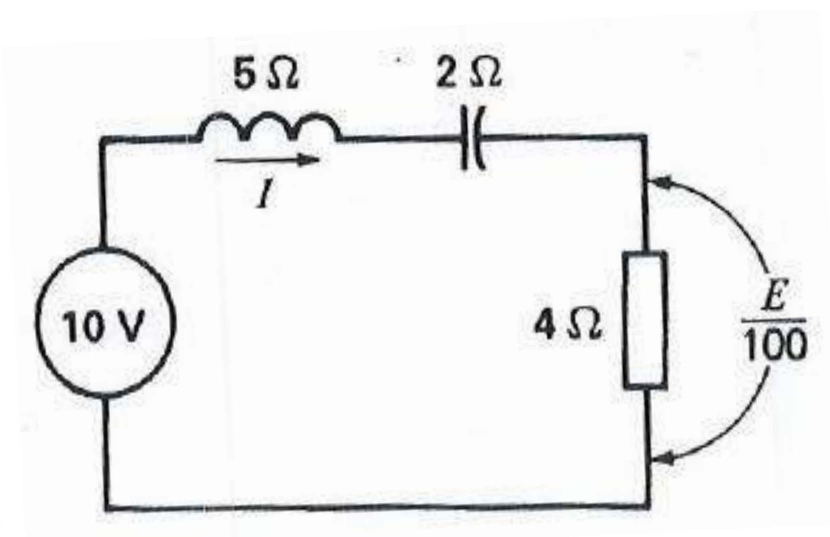


Fig (8) the equivalent circuit of Fig (7).

4.3.3 ANALYSIS OF CIRCUIT containing Ideal Transformer:

If a circuit contains an ideal Transformer, then easiest way to analyze the circuit for its voltages and currents is to replace the portion of the circuit on one side of the Transformer by an equivalent circuit with the same terminal characteristic.

After the equivalent circuit has been substituted for one side, then the NEW circuit (without Transformer present) can be solved for its voltages and currents.

The process of replacing one side of Transformer by its equivalent at other side's Voltage level is known as REFFRRING first side of Transformer to second side.

Example (1.5)

A 1-phase power system consists of a 480V, 60HZ generator supplying a load of Impedance Z_{load} = (4 + j3) Ω, through a Transmission line of Impedance

Z_{line} = (0.18 + j0.24) Ω. Answer the following;

a) For a system shown in Fig (9 a), what will the Voltage at the load? Find the Transmission line losses?

b) With 1:10 step-up Transformer is placed at the generator side, and a 10:1 step-down Transformer at the load end Fig (9 b), what will be the load Voltage now? What will be the transmission line losses now?

Solution:

a) Without Transformer Fig (9 a)

$$I_G = I_{line} = I_{load}$$

$$I_{line} = \frac{V}{Z_{line}+Z_{load}} = \frac{V\llcorner 0°}{[0.18+j0.24+4+j3]\Omega} = \frac{480\angle 0°}{[4.18+j3.24]\Omega}$$

$$= \frac{480\llcorner 0°}{5.29\angle 37.8°} = 90.8\angle -37.8° \quad \text{AMPS}$$

The load voltage:

$$V_{Load} = I_{line} \times Z_{load} = (90.8\angle -37.8° \ \text{AMPS}) \times (4 + j3)\ \Omega$$

$$= (90.8\angle -37.8° \ \text{AMPS}) \times (5\angle 36.9°) = 454\angle -0.9° \ \text{Volt.}$$

And the line losses

$P_{Loss} = (I_{Line})^2 \times R_{Line} = (90.8)^2 \times (0.18) = 1484$ Watt

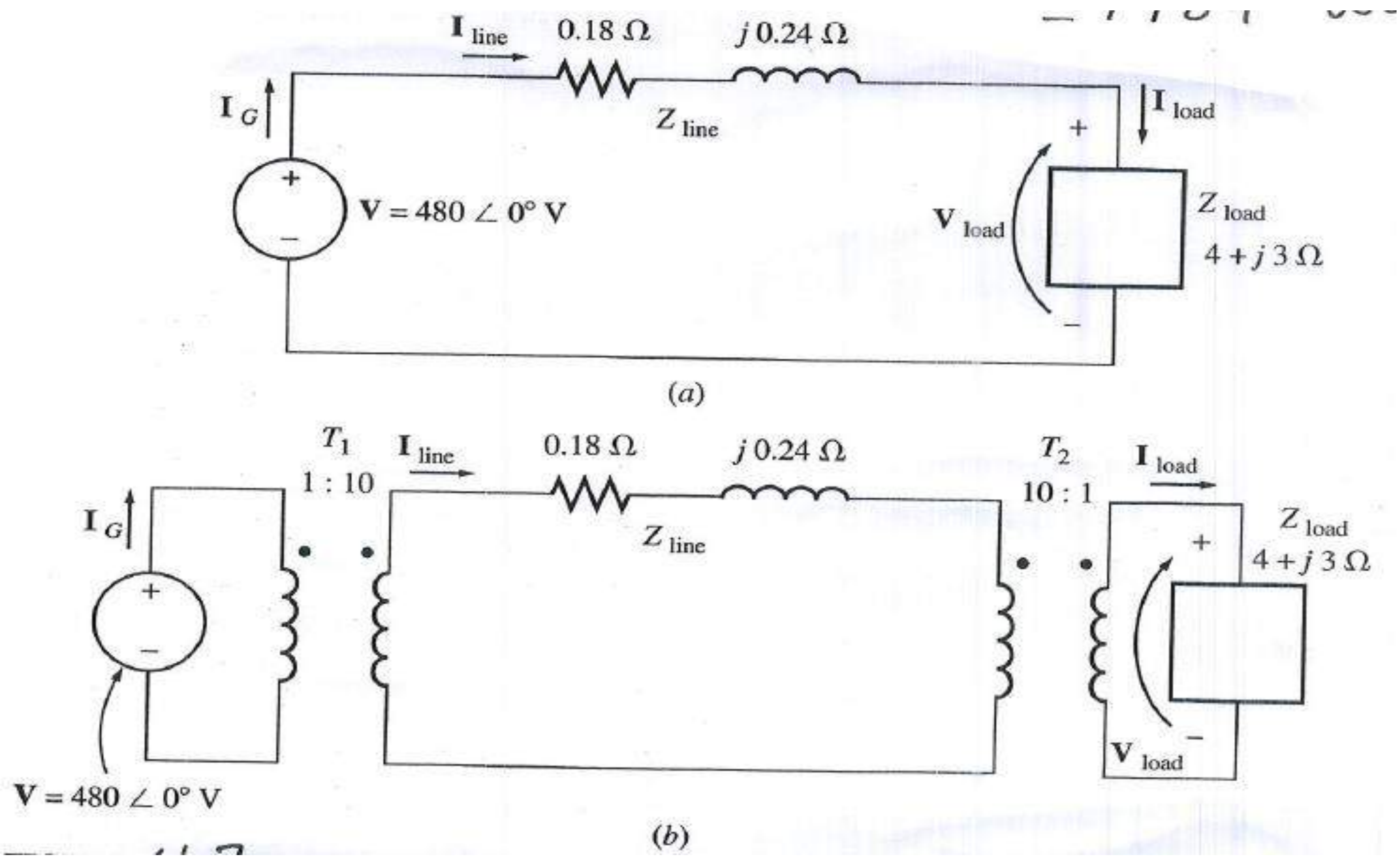


Fig (9) **The power system of example (1.5)(a)without and(b) with transmission line**

b)Fig (9 b) shows the power system with Transformers, converting to a common Voltage level. This is done in two steps:

1-Eliminate **T2** by referring the load over to the transmission line's Voltage level.

2- Eliminate **T1** by referring the transmission line's element and the equivalent load at the transmission line's Voltage over to the Source side.

The value of Z_{Load} when reflected to the transmission system's Voltage is:

$$Z'_{Load} = N^2 \times Z_{Load} = \left(\frac{10}{1}\right)^2 \times (4 + j3)\ \Omega = (400 + j300)\ \Omega.$$

The total Impedance at the transmission line level is:

$$Z_{eq} = Z_{Line} + Z'_{Load} = (400.18 + j300.24)\ \Omega = 500.3 \angle 36.88°\ \Omega.$$

The equivalent circuit Fig (9-a). Total Impedance (Z_{eq}) in new reflected across **T1:**

$$Z^{`}_{eq} = N^2 \times Z_{eq} = N^2 \times (Z_{Line} + Z^{`}_{Load}) =$$

$$= (\frac{1}{10})^2 \times (400.18 + j300.24)\ \Omega = 5.003\angle 36.88^\circ\ \Omega.$$

The resulting equivalent circuit is shown in Fig (9-b).

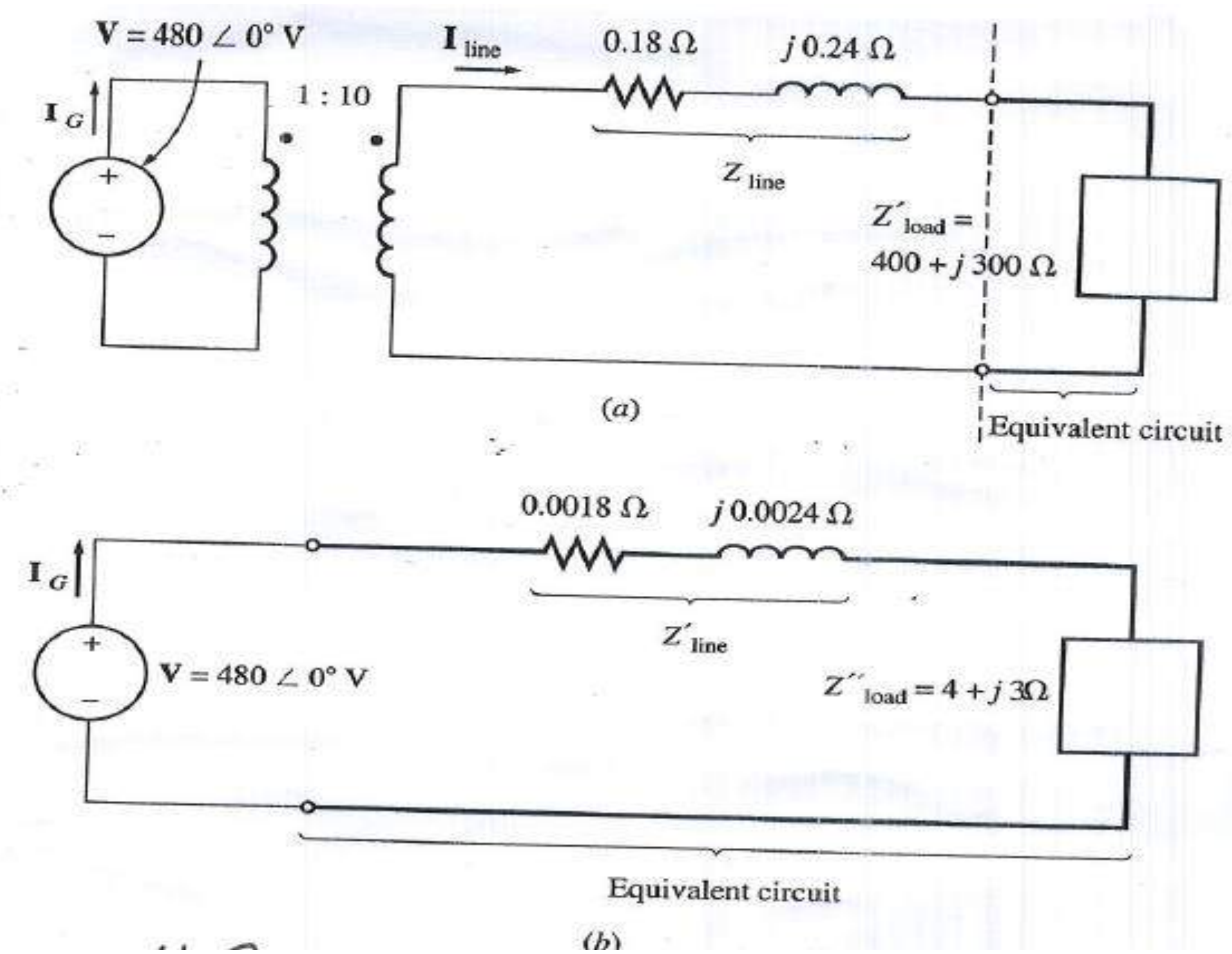


Fig (9) a) **system with the load referred to the transmission system voltage level.**

b) System with load and transmission line referred to generators voltage level.

The generator's current:

$$I_G = \frac{480\angle\, 0°}{5.003\angle\, 36.88°} = 95.94\angle - 36.88°$$

Knowing I_G , we can now work back and find I_{Line} and I_{Load} , working back through **T1**:

$$N_{P1} \times I_G = N_{S1} \times I_{Line}$$

$$I_{Line} = \frac{N_{PI}}{N_{s1}} \times I_G = \frac{1}{10}(95.94\angle - 36.88°) = 9.594\angle - 36.88°)\text{ A.}$$

Working back through **T2**:

$$N_{P2} \times I_{Line} = N_{S2} \times I_{Load}$$

$$I_{Load} = \frac{N_{P2}}{N_{s2}} \times I_{Line} = \frac{10}{1}(9.594\angle - 36.88°) = 95.594\angle - 36.88°\text{A.}$$

The load Voltage is given by:

$$V_{Load} = I_{Load} \times Z_{Load} = 95.594\angle - 36.88° \times 5\angle 36.87° = 479.7\angle - .01°$$

And the line losses:

$$P_{Loss} = (I_{Line})^2 \times R_{Line} = (9.594)^2 \times (0.18) = 16.7 \quad \text{Watt}$$

Notice that raising the transmission Voltage of power system reduced losses

5- PRACTICAL (REAL) TRANSFORMER

In the real world, Transformers are not ideal and so our simple analysis must be modified, the windings of practical Transformers have resistance and the cores not infinitely permeable, furthermore, Flux produced by Primary is not completely captured by Secondary, consequently, leakage Flux must be taken into account.

Finally, the iron cores produce Eddy-current and Hysteresis losses, which contribute to Transformer temperature rise.

5.1 THE LOSSES IN REAL TRANSFORMERS

The losses that occur in real Transformers have to be accounted for in any accurate model of Transformer, the major items are:

1-Copper (I^2 R) losses; Copper losses are the resistive heating losses in the Primary and Secondary windings of the Transformer, they are;

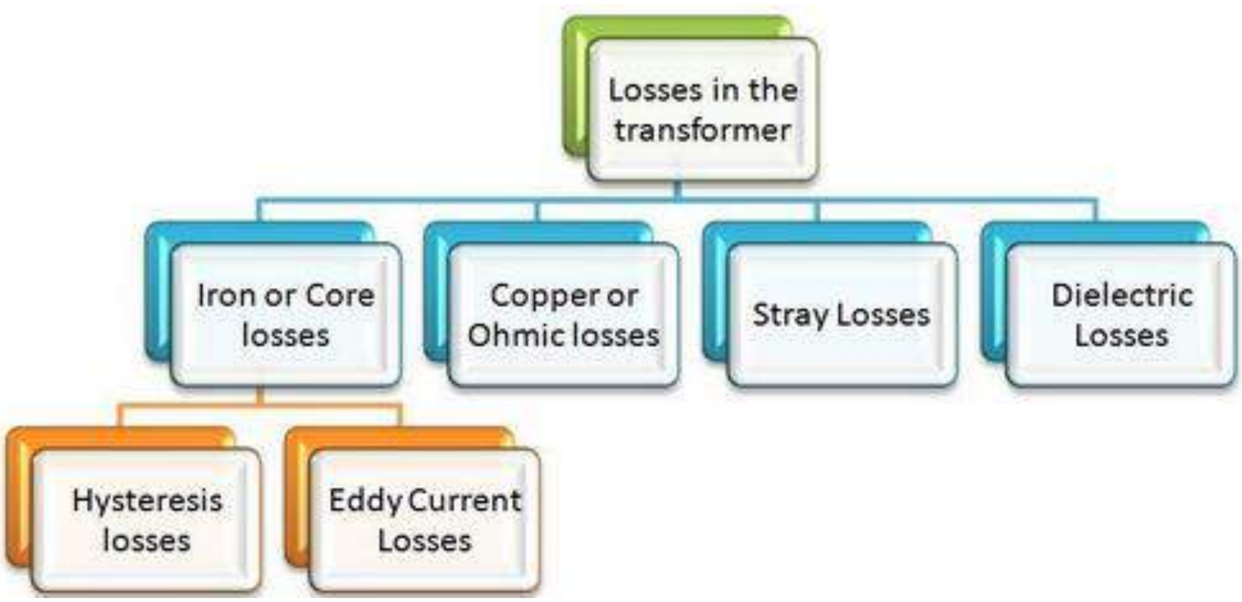
$$I^2 \text{ R } \alpha \ I^2 .$$

2-Eddy current losses; Eddy current losses are resistive losses in the Transformer core, they are proportional to square Voltage applied to Transformer:

Eddy current losses α square applied Voltage

3- Hysteresis losses; Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle ,they are a complex, non- Linear function of the Voltage applied to the Transformer.

4-Leakage Flux; Flux loss caused by the leakage of the magnetic flux from the core of Transformer, these escaped Fluxes produce a self- inductance in the Primary and Secondary coils , and the effects of this must be accounted for.



5.2 THE EXACT EQUIVALENT CIRCUIT of a real Transformer:

It is possible to construct an equivalent circuit that takes into account the major imperfections in real Transformers.

The Copper losses are resistive losses in the Primary and Secondary winding of Transformer core, they are modeled by placing a resistor (R_P) in Primary and (R_s) in the Secondary circuits.

The leakage Fluxes will be modeled by the Primary and Secondary Inductors.

The core excitation effects can be modeled by taking the relation between the magnetization current (I_m) and the Voltage applied to core (in the unsaturated region), I_m lagging the applied Voltage by $90°$, so it can modeled by a reactance X_m connected across the Primary Voltage source.

The core-loss current I_{he} is a current proportional to the Voltage applied to core that is in phase with applied Voltage, so it can be modeled by (R_C) connected across the Primary Voltage source.

The resulting equivalent circuit shown in Fig (10), although Fig (10) is an accurate model of a Transformer, it is not a very useful one.

To analyze practical circuits containing Transformers, it is normally necessary to convert the entire to an equivalent circuited at a single Voltage level,

Therefore, the equivalent circuit must be referred either to its Primary side or to its Secondary side in problem solution.

Fig (11a) is the equivalent circuit referred to the Primary side.

Fig (11b) is the equivalent circuit referred to the Secondary side.

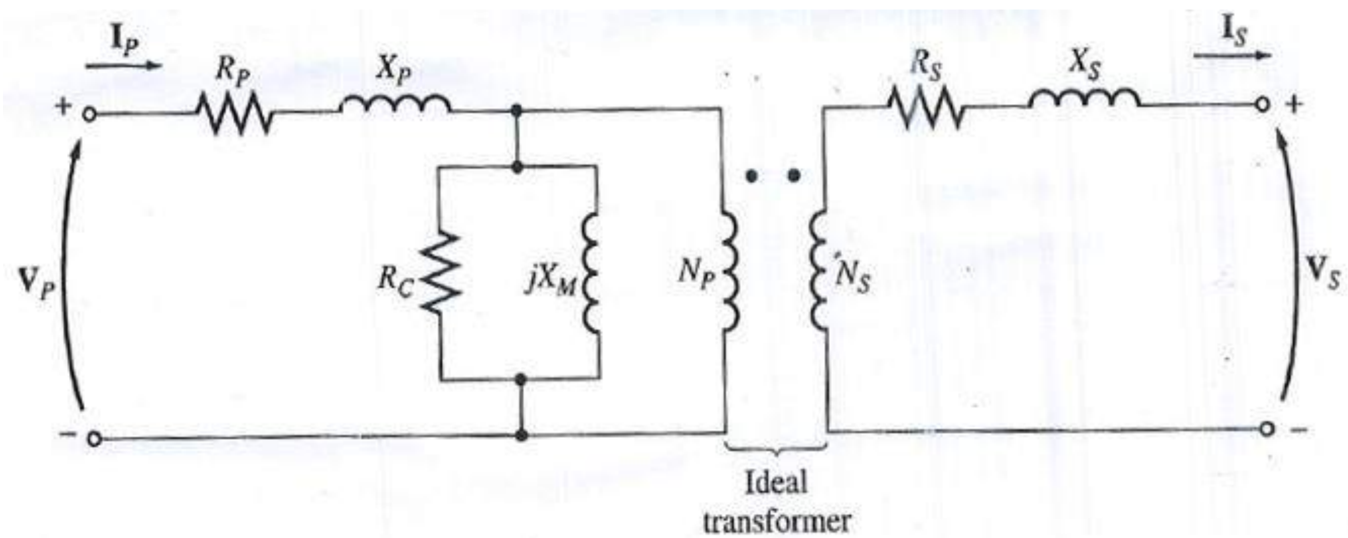


Fig (10) the model of a real Transformer.

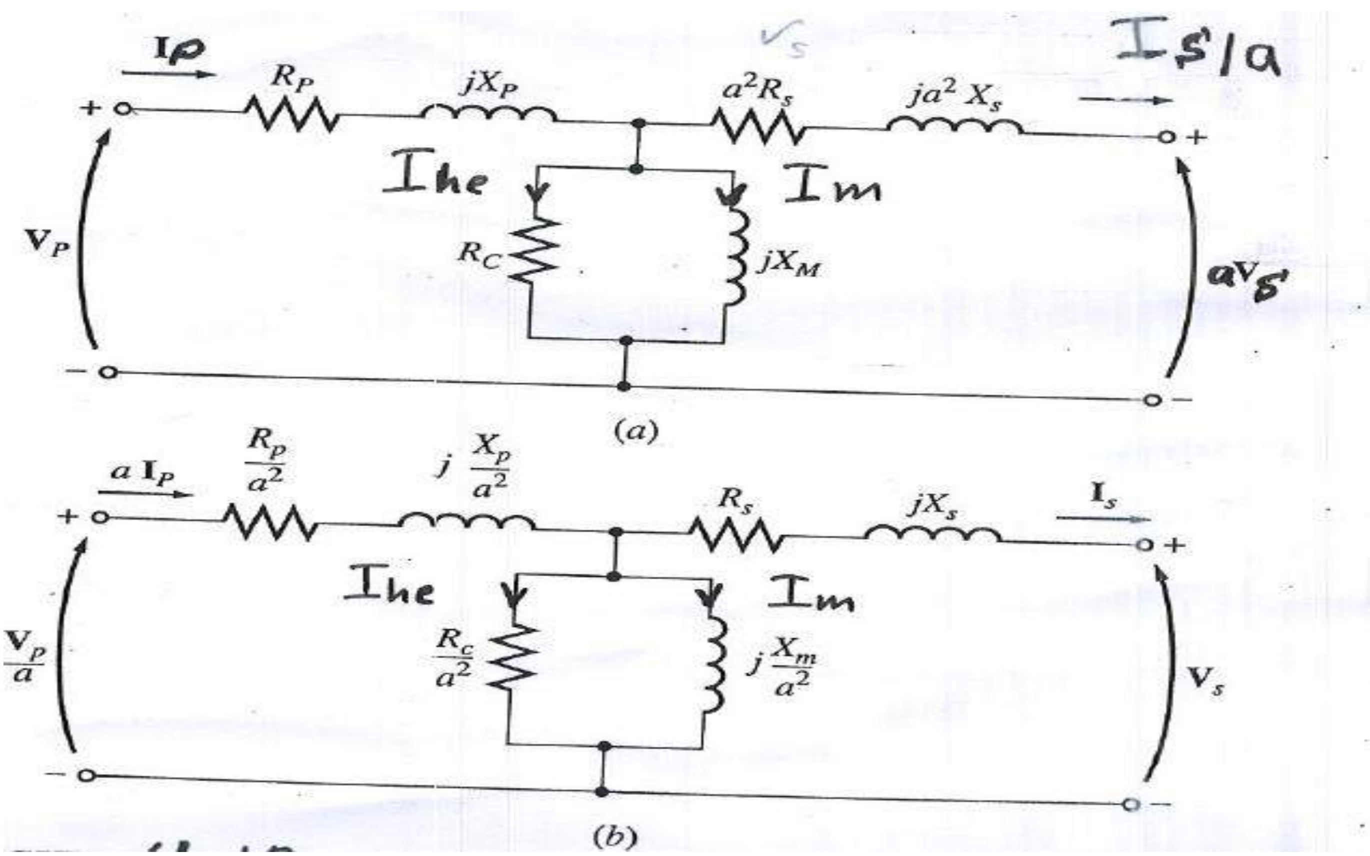


Fig (11 a) The Transformer model referred to its Primary Voltage level side.

b) The Transformer model referred to its Secondary Voltage level side.

5.3 APPROXIMATE EQUIVALENT CIRCUIT of a Transformer

The Transformer models shown above in exact equivalent circuit are often more complex than necessary. The excitation branch has a very small current compared with load current of Transformers, it is so small that under normal circumstances it causes a completely negligible Voltage drop in (R_P& X_P), a simplified equivalent circuit can be produced.

The excitation branch is simply moved to the Front of Transformer, and the Primary and Secondary Impedances are left in series with each other.

These Impedances are just added, creating the approximate equivalent circuits shown in Fig (12).

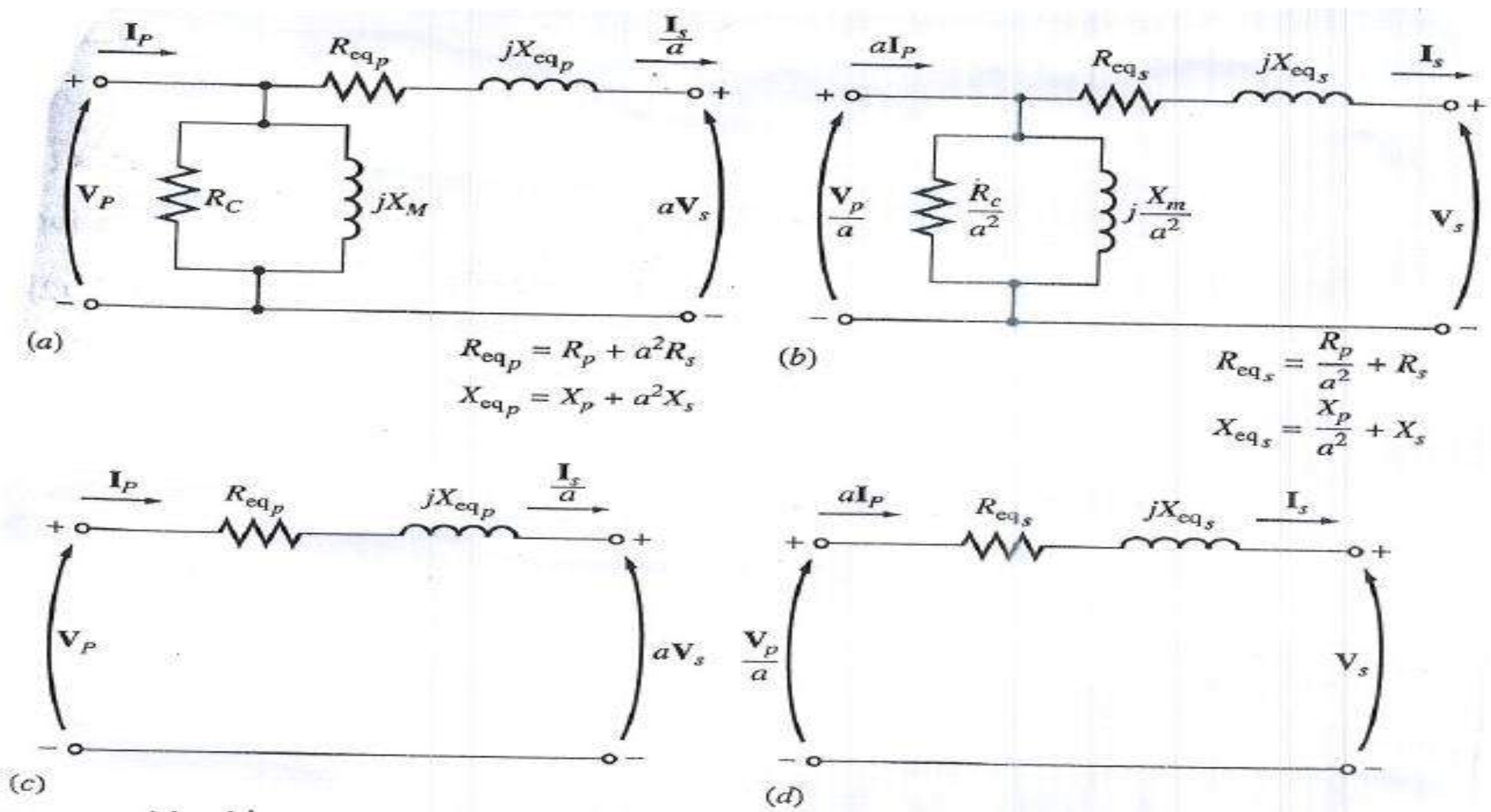


Fig (12). Approximate models (a) referred to Primary (b) referred to Secondary (c) referred to Primary, with no excitation branch (d) referred to Secondary with no excitation branch.

$$R_{eq} = R_P + N^2 R_s \quad \text{........ (1.19)}$$

$$X_{eq} = X_P + N^2 X_s \quad \text{........ (1.20)}$$

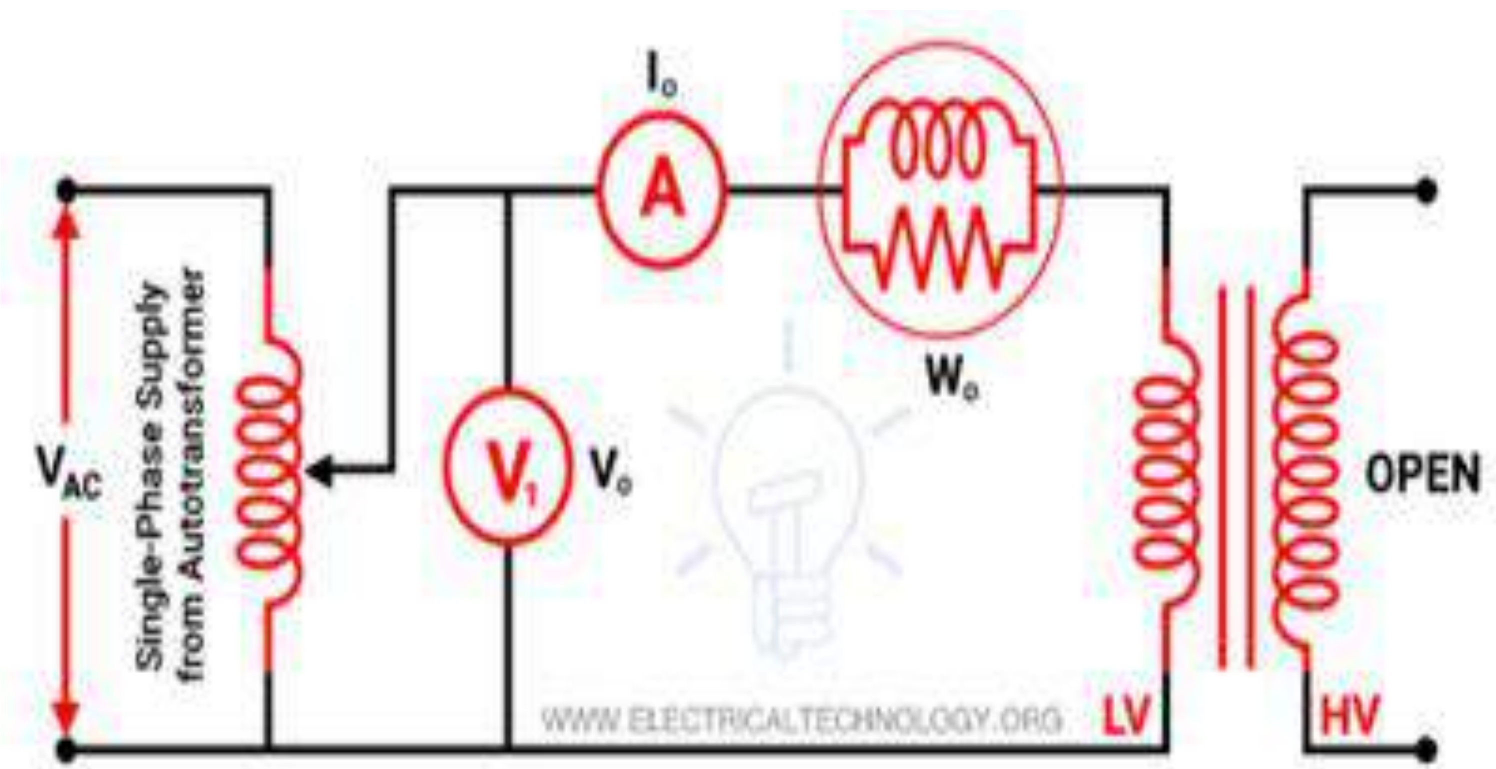
$$Z_{eq} = R_{eq} + j X_{eq} \quad \text{........ (1.21)}$$

4.6 Tests OF TRANSFORMERS

It is possible to experimentally determine the values of the inductances and resistances in the Transformer model.

4.6.1 OPEN-CIRCUIT TEST (NO- LOAD) TEST to find R_C & x_m

A Transformer's Secondary winding is open-circuited and its Primary winding connected to a Full-rated Voltage. Look at Fig (13), For the NO- Load test, and the equivalent circuit in Fig (14).



Fig(13) no-load (open circuit) test of Transformer.

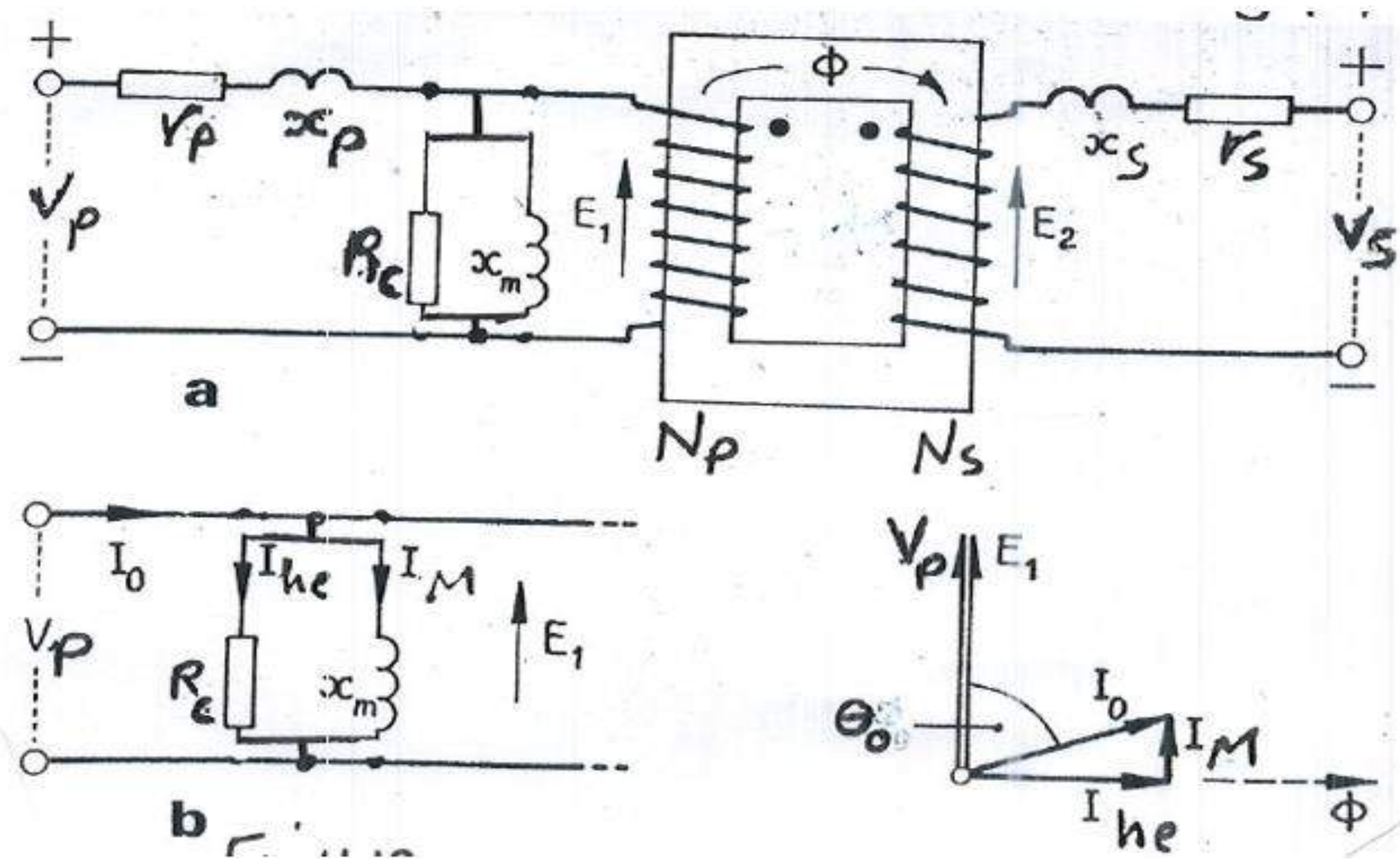


Fig (14) the equivalent circuit of no-load (open circuit) test of Transformer.

The E.M.F E_1 balances the primary applied Voltage (neglecting the very small Voltage drops due to the magnetizing and loss currents flowing through the Primary Leakage Impedance (Z_P):

$$V_P = E_1 \quad \text{.........} \ (1.22)$$

$$I_P = I_o = \sqrt{[I_m^2 + I_{he}^2]} \quad \text{.........}(1.23)$$

Active Power absorbed by the core:

$$\boldsymbol{P = Po.c.}$$

Apparent Power absorbed by the core:

$$\boldsymbol{S = So = E_1 \times I_o}$$

Reactive Power:

$$Q = \sqrt{(S^2 - P^2)}$$

Resistance R_C corresponding to the core losses is:

$$R_C = \frac{E_1^2}{P_{o.c}} \qquad \text{.......... (1.24)}$$

Magnetizing reactance ($\mathcal{X}_m$) is:

$$\mathcal{X}_m = \frac{E_1^2}{Q_{o.c}} \qquad \text{.......... (1.25)}$$

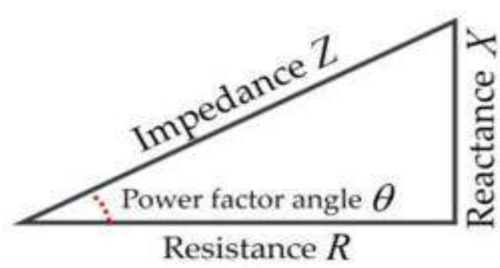
In terms of admittance:

$$G_C = \frac{1}{R_C} \qquad \& \qquad B_m = \frac{1}{\mathcal{X}_m} \qquad \text{........ (1.26)}$$

The magnitude of the excitation Impedance Z_o (referred to Primary circuit) can be found from the open – circuit Voltage $V_{o.c}$ and the current $I_{o.c}$, $V_{o.c} = E_1$ & $I_{o.c} = I_O$:

$$\left| Z_o \right| = \frac{V_{o.c}}{I_{o.c}} \qquad \text{........ (1.27)}$$

Angle of the impedance can be found from a knowledge of the circuit Power Factor. The open circuit – Power Factor (P.F) is given by:



$$P.F = \cos\theta_0 = \frac{P_{o.c}}{V_{o.c} \times I_{o.c}} = \frac{P_{o.c}}{S_{o.c}}$$

.... (1.28)

And The Power Factor (P.F) $(\boldsymbol{\theta})$ is given by:

$$\boldsymbol{\theta} = \boldsymbol{COS}^{-1} \frac{\boldsymbol{P_{o.c}}}{\boldsymbol{S_{o.c}}} \quad \text{......... (1.29)}$$

$$R_C = \boldsymbol{Z_o} \times \boldsymbol{COS}\ \boldsymbol{\theta_0} \quad \text{......... (1. 30)}$$

Or we can find R_C as:

$$P_{o.c} = \boldsymbol{I_O^2} \times R_C$$

$$R_C = \frac{P_{o.c}}{I_O^2} \quad \text{......... (1. 31)}$$

$$\mathcal{X}_m = \boldsymbol{Z_o} \times \boldsymbol{SIN}\ \boldsymbol{\theta_0} \quad \text{......... (1. 32)}$$

Or from Impedance Triangle $\mathcal{X}_m$ can be calculated as:

$$\mathcal{X}_m = \sqrt{(Z_O^2 - R_O^2)} \quad \text{......... (1. 33)}$$

The Power Factor is always Lagging for a real Transformer, so the angle of of the current always Lags the angle of the Voltage by $(\boldsymbol{\theta})$:

$$\left|\boldsymbol{Z_o}\right| = \frac{\boldsymbol{V_{o.c}}}{\boldsymbol{I_{o.c}}} \angle -\boldsymbol{\theta} = \frac{\boldsymbol{V_{o.c}}}{\boldsymbol{I_{o.c}}} \angle -\boldsymbol{COS}^{-1}\ \text{P.F} \quad \text{......... (1. 34)}$$

Example 1.6:

An open – circuit test was conducted on transformer of a 250KVA,(69/4.16)KV, 60HZ, the following results were obtained when Low- Voltage winding was excited

V_S=4160 Volt	**I_o= 2 AMPS**	**P=5000 Watt**
R_P=150 Ω	**$\mathcal{X}_p$=632 Ω**	

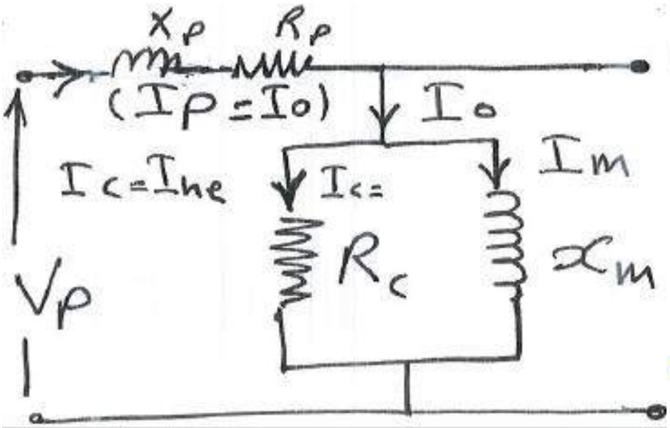
Calculate:

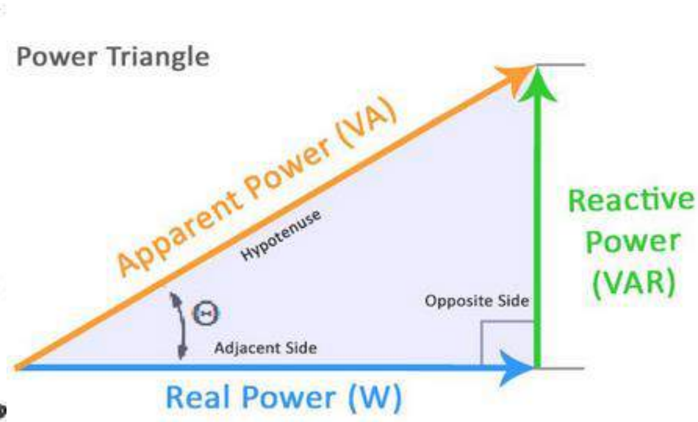
a) The value of R_C & $\mathcal{X}_m$ on the Primary side

b) The efficiency (η), if the Transformer is supplies a load of 250KVA, whose Power Factor (P.F) is 80% Lagging.

Solution:

a) Applying the equation (1.24) to the Secondary side:





$$R_c = \frac{V_S^2}{P} = \frac{(4160)^2}{5000} = 3461\ \Omega$$

S= $V_s \times I_0$ =4160× 2= 8320 VA.

$S^2 = P^2 + Q^2 \quad \rightarrow \quad$ Q = $\sqrt{(S^2 - P^2)} = \sqrt{(8320^2 - 5000^2)} = 6650\ VAR.$

$$\mathcal{X}_m = \frac{V_S^2}{Q} = \frac{(4160)^2}{6650} = 2602\ \Omega$$

The values of R_C & $\mathcal{X}_m$ referred to the Primary side will be:

$$N^2 \times R_C \qquad \& \qquad N^2 \times \mathcal{X}_m$$

$$N^2 = \left(\frac{6900}{4160}\right)^2 = 275$$

The values on the primary side are therefore:

$\mathcal{X}_m$= 275× 2602= 715.5× 10^3= 715 K Ω

R_C= 275× 3461= 952 × 10^3= 952 K Ω

b) The load current:

$$I_S = \frac{S}{V_S} = \frac{25000}{4160} = 60 \quad \text{AMP}$$

The transformation ratio is:

$$N = \frac{69\ KV}{4160\ V} = 16.59$$

The current on the primary side is:

$$N = \frac{I_S}{I_P} \quad \rightarrow\rightarrow \quad I_P = \frac{I_S}{N} \quad \rightarrow\rightarrow \quad I_P = \frac{60}{16.59} = 3.62 \quad \text{AMP.}$$

The total copper losses (Primary and Secondary) is:

$$Pcopper_{losses} = I_P^2 \times R_P = (3.62)^2 \times 150 = 1966 \text{ Watt.}$$

The iron losses is the same as the measured at rated Voltage on the Low-Voltage side: P_{iron}= 5000 Watt

The total Losses are:

$$Ptotal_{losses} = 5000 + 1966 = 6966\text{Watt} \quad \rightarrow\rightarrow \quad \simeq 7 \quad \text{KW}$$

The active Power delivered by the Transformer is:

$$P_O = S \times COS\ \theta = 250 \times 0.8 = 200 \text{ KW.}$$

The active Power received by the Transformer is:

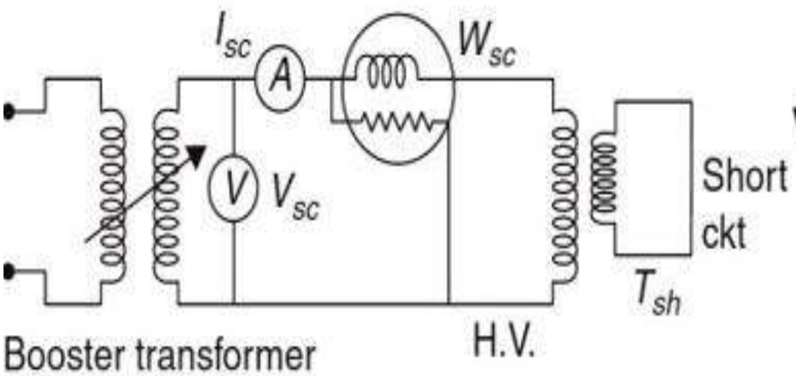
$$P_{i/p} = P_{o/p} + Ptotal_{losses} = 200 + 7 = 207 \text{ KW}$$

The efficiency is therefore:

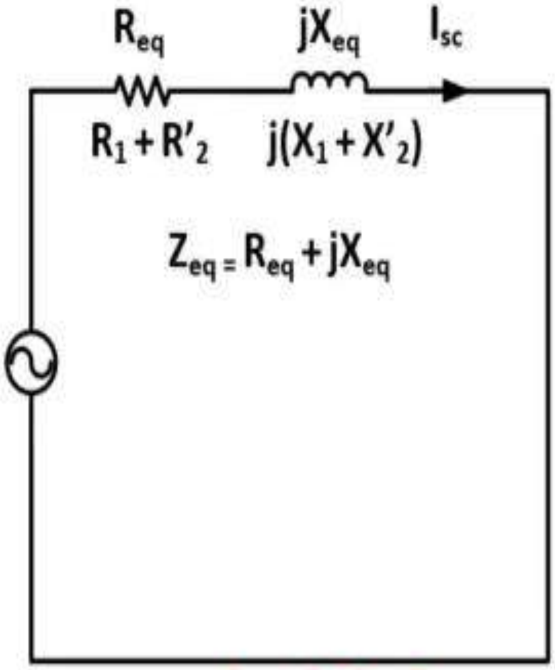
$$\eta = \frac{P_{o/p}}{P_{i/p}} = \frac{200}{207} = 0.966 \quad \text{OR} \quad 96.6\%$$

4.6.2 SHORT- CIRCUIT TEST (S.C TEST)

The Secondary terminals of transformer are short- circuited (S.C) and Primary terminals are connected to a fairly Low- Voltage source as shown in Fig (15).



S.C. test circuit diagram



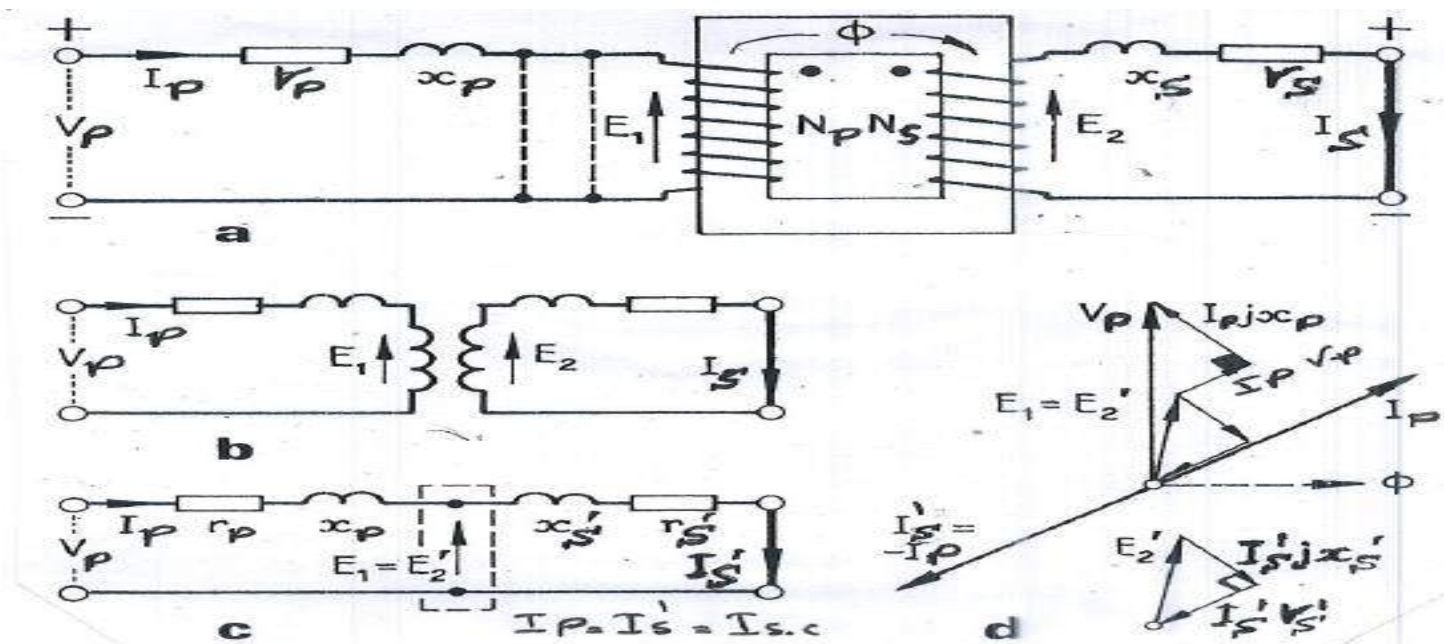


Fig (15) Transformer on a short- circuit.

The input Voltage is adjusted until the current in the short- circuited winding is equal to its rated value,(be sure to keep the Primary Voltage at safe level).

The input Voltage, Current, and Power are:

$$I_S^{`} = \frac{I_S}{N}$$

$$\mathcal{X}_S^{`} = N^2 \times \mathcal{X}_S \qquad \& \qquad \boldsymbol{R_s^{`} = N^2 \times r_s}$$

Since the input Voltage is so Low during the short- circuited test, negligible current flows through the excitation branch. If the excitation current is ignored, then all the Voltage drops in the circuit elements are in series, therefore all currents are same:

$$\boldsymbol{I_P = I_s^{`} = I_{s.c}} \qquad \ldots\ldots \qquad \textbf{(1.35)}$$

The magnitude of the series Impedance ($Z_{S.C}$) referred to the Primary Side of the Transformer is:

$$\left| \boldsymbol{Z_{s.c}} \right| = \frac{\boldsymbol{V_{s.c}}}{\boldsymbol{I_{s.c}}} \qquad \ldots\ldots \qquad \textbf{(1.36)}$$

The Power Factor of the current is given by:

$$\boldsymbol{P.F = COS\ \theta = \frac{P_{S.C}}{V_{s.c} \times I_{s.c}}} \qquad \ldots\ldots \qquad \textbf{(1.37)}$$

And is lagging. The current angle is thus negative and the overall Impedance angle ($\boldsymbol{\theta}$) is positive:

$$\boldsymbol{\theta = COS^{-1}}\ \text{P.F} = \boldsymbol{COS^{-1} \frac{P_{.S.c}}{V_{s.c} \times I_{s.c}}}$$

Therefore:

$$Z_{s.c} = \frac{V_{s.c}\angle 0}{I_{s.c}\angle -\theta} = \frac{V_{s.c}}{I_{s.c}} \angle + \theta$$

The series Impedance $Z_{s.c}$ is equal to, see Fig (15 c):

$$Z_{s.c} = R_{eq} + j\mathcal{X}_{eq} = [\, RE_P + (N^2 \times R_s)\,] + j[\, \mathcal{X}_P + (N^2 \times \mathcal{X}_s)\,] \quad ...(1.38)$$

From the Power input reading ($P_{s.c}$) which is due to the Losses:

$$P_{s.c} = I_{S.C}^2 \times R_{eq}$$

The effective resistance:

$$R_{eq} = \frac{P_{s.c}}{I_{S.C}^2} \qquad ...... \qquad (1.39)$$

OR: $$R_{eq} = Z_{s.c} \times COS\ \theta \qquad ...... \qquad \mathbf{(1.40)}$$

The total Leakage reactance is:

$$\mathcal{X}_{eq} = Z_{s.c} \times SIN\ \theta \qquad ...... \qquad (1.41)$$

OR:

$$\mathcal{X}_{eq} = \sqrt{(Z_{s.c}^2 - r_{eq}^2)} \qquad ...... \qquad (1.42)$$

Note:

1- $I_{s.c}$ need not be rated current since the equivalent circuit is Linear.

2-The supply could be fed to either winding and for the same winding currents the Power input be the same.

3-In Laboratory experiments using small Transformers, Impedance and power consumption of the measuring instruments may have to be allowed for to get the true value of Transformer terminal Voltage, (input &output) Power. The instrument positions shown usually result in minimum error for these short- circuit tests at very Low- Voltage.

Example 1.7:

During a short- circuit test on a Transformer, rated 500 KVA, (69/4.16) KVA, 60HZ, the following, Voltage, Current, and Power measurements were made, Fig (16):

$E_{S.C}$ = 2600 Volt	$I_{S.C}$ =4 AMP	$P_{S.C}$ = 2400 Watt

Calculate:

The value of the Reactance and resistance of the Transformer referred to the High Voltage side.

Solution:

Transformer Impedance referred to Primary side is:

$$\boldsymbol{Z_{Ps.c}=\frac{E_{s.c}}{I_{s.c}}=\frac{2600\ V}{4\ AMP}= 650\ \Omega} \qquad \boldsymbol{R_{eq}=\frac{P_{S.C}}{I^2_{S.C}}=\frac{2400}{16}= 150\ \Omega}$$

Leakage Reactance referred to Primary side is:

$$X_{eq} = \sqrt{(Z^2_{s.c}-R^2_{eq})} = \sqrt{[650^2-150^2]} = 632\ \Omega.$$

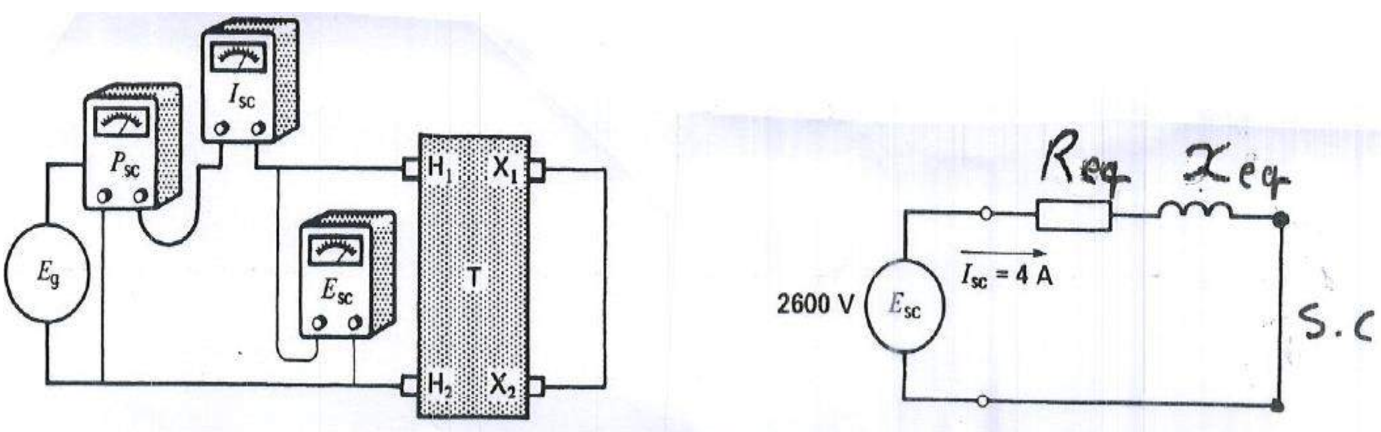


Fig (16) Short- circuit test to determine Leakage Reactance and Winding Resistance, see example (1.7).

Example 1.8:

The equivalent circuit Impedance of a 20KVA, (8000/ 240) V, 60HZ, Transformer are to be determined. The open- circuit test and the short- circuit test were performed on the Primary side, and the following data were taken:

open- circuit test on Primary side	$V_{O.C}$=8000V	$I_{O.C}$=0.214 AMP	$P_{O.C}$=400 W
short- circuit test on Primary side	$V_{S.C}$=489V	$I_{S.C}$=2.5 AMP	$P_{S.C}$=240 W

Find the Impedance of the approximate equivalent circuit referred to Primary side.

Solution:

The Power Factor during the open- circuit test is:

$$\text{P.F} = \frac{P_{O.C}}{V_{O.C} \times I_{O.C}} = \text{COS}\ \theta = \frac{400\ Watt}{8000_{Volt}\ \times 0.214\ AMP} = \quad 0.234\ \text{Lagging.}$$

$$\boldsymbol{Y_E = \frac{I_{O.C}}{V_{O.C}} \angle -COS^{-1}\ \text{P.F} = \frac{0.214}{8000} \angle -76.5 = 0.0000063 - j0.0000261 = \frac{1}{R} - j\ \frac{1}{X}}$$

$$\boldsymbol{R_C = \frac{1}{0.0000063} = 159\ K\Omega\ :\ X_m = \frac{1}{0.0000063} = 38.4\ K\Omega}$$

The Power Factor during the short- circuit test is:

$$\text{P.F} = \frac{P_{S.C}}{V_{S.C} \times I_{S.C}} = \text{COS}\ \theta = \frac{240\ \ Watt}{489 \times 2.5} = 0.196 \quad \text{Lagging}$$

The series Impedance is given by:

$$\boldsymbol{Z_{s.c} = \frac{V_{s.c}}{I_{s.c}} \angle -COS^{-1}\ \text{P.F} = \frac{489\ V}{2.5\ \ A} \angle 78.7° = 195.6 \angle 78.7° = (38.4 + j\ 192)\ \Omega}$$

Therefore, the equivalent Resistance and Reactance are

R_{eq}= 38.4 Ω and X_{eq}=192 Ω

The resulting simplified equivalent circuit is shown in Fig (17)

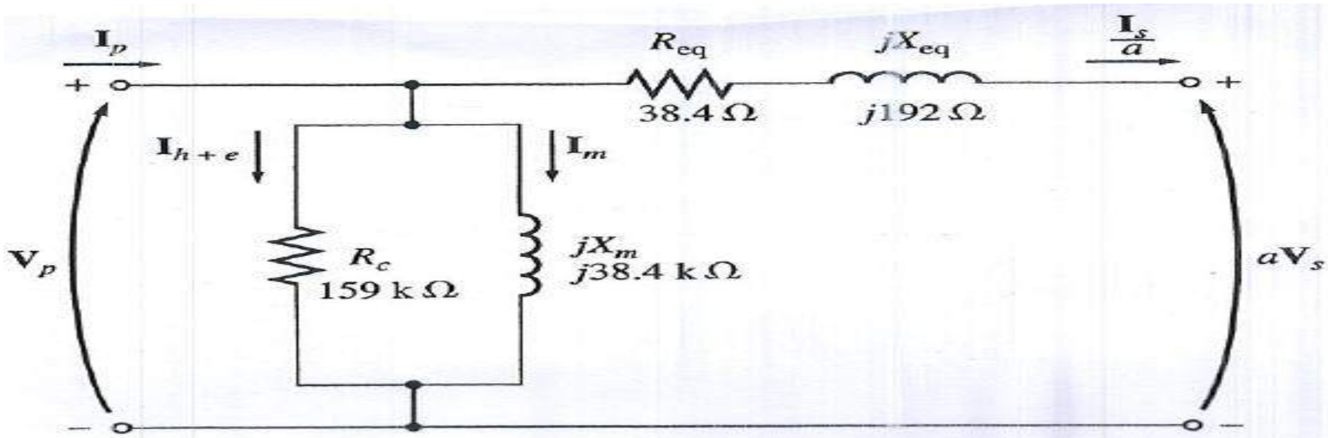


Fig (17) the equivalent circuit of example (1.8)

4.6.3 TRANSFORMER ON LOAD (LOAD TEST)

IF a load Impedance is connected across the Secondary terminals, Fig (18):

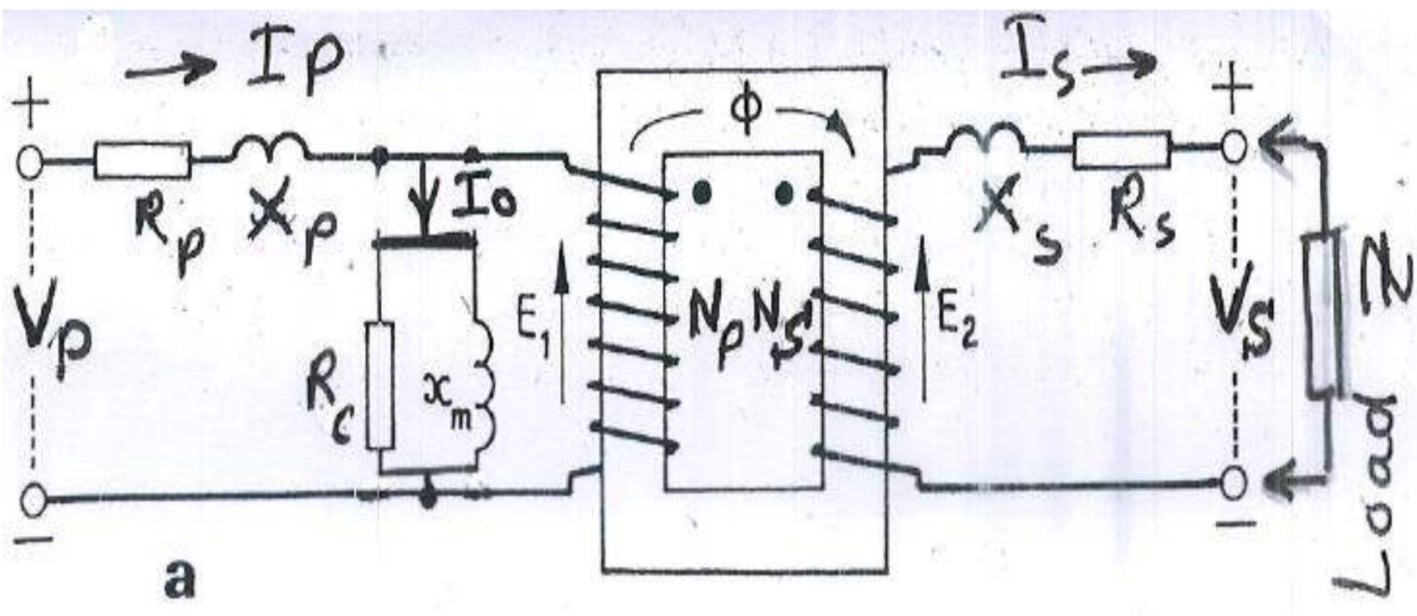


Fig (18) Transformer on load

The E.M.F (E_2) will drive an output current (I_S) through it, and the Secondary winding will impress the m.m.F (F_2):

$$F_2 = N_S \times I_S$$

1* The Primary current I_p is the phasor sum of $(-I_s^{`})$ and I_o

Where:

$I_S^{`}$: Is the Secondary current referred to Primary side.

2* The applied Primary Voltage V_P balances E_1 Provides for the Primary Resistance and Leakage Reactance $(I_P \times R_P)$ and $(I_p \times \mathcal{X}_P)$.

3* For the Secondary Leakage Impedance Voltage drop $(I_S^{`} \times R_s)$ and $(I_S^{`} \times J\mathcal{X}_s)$. The Secondary phase angle is $\emptyset_2$

4* If we adopt the equal- turns ratio condition with $(N_P = N_S)$, Primary and Secondary currents can be used to represent also the winding (m.m.Fs).

Making the phasor diagram more easily drawn.

It is shown for partially inductive Secondary load, Fig (19) ,that:

$$\overrightarrow{I_P} = -\overrightarrow{I_S} +$$

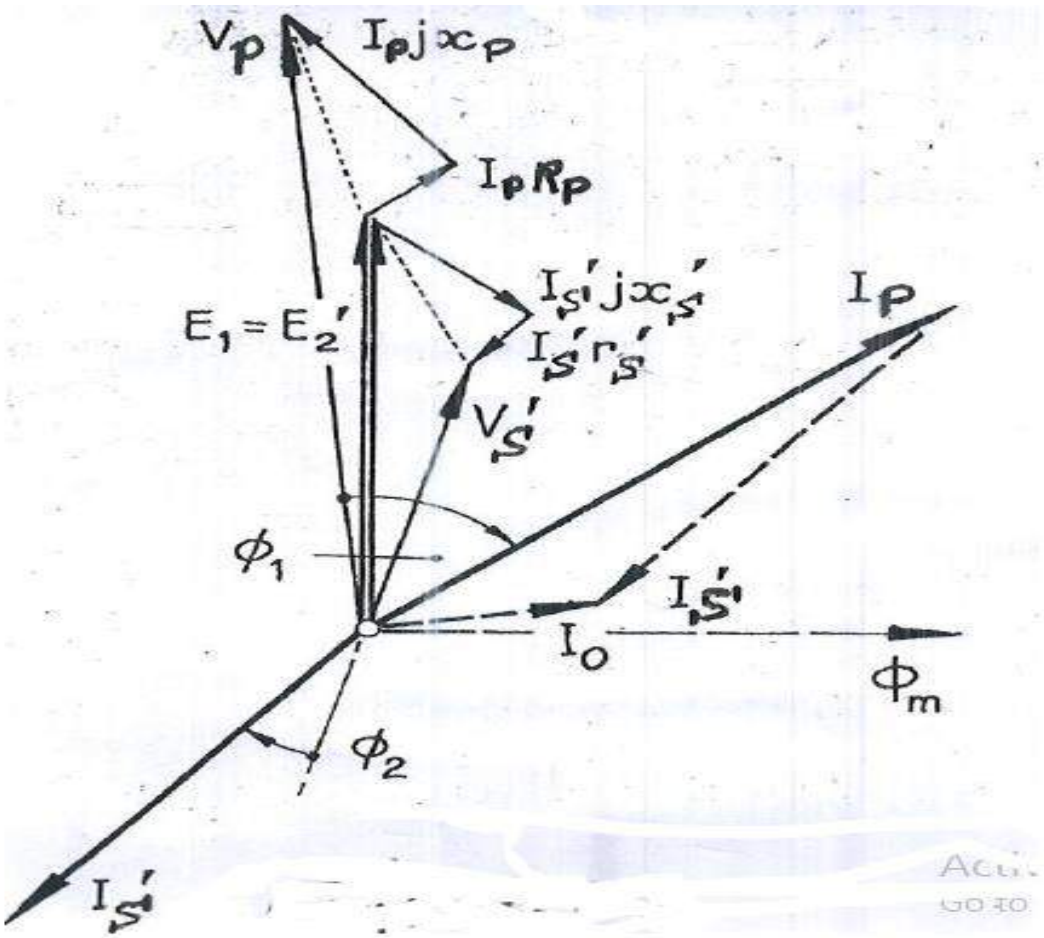


Fig (19) Transformer on load: phasor diagram

Example 1.9:

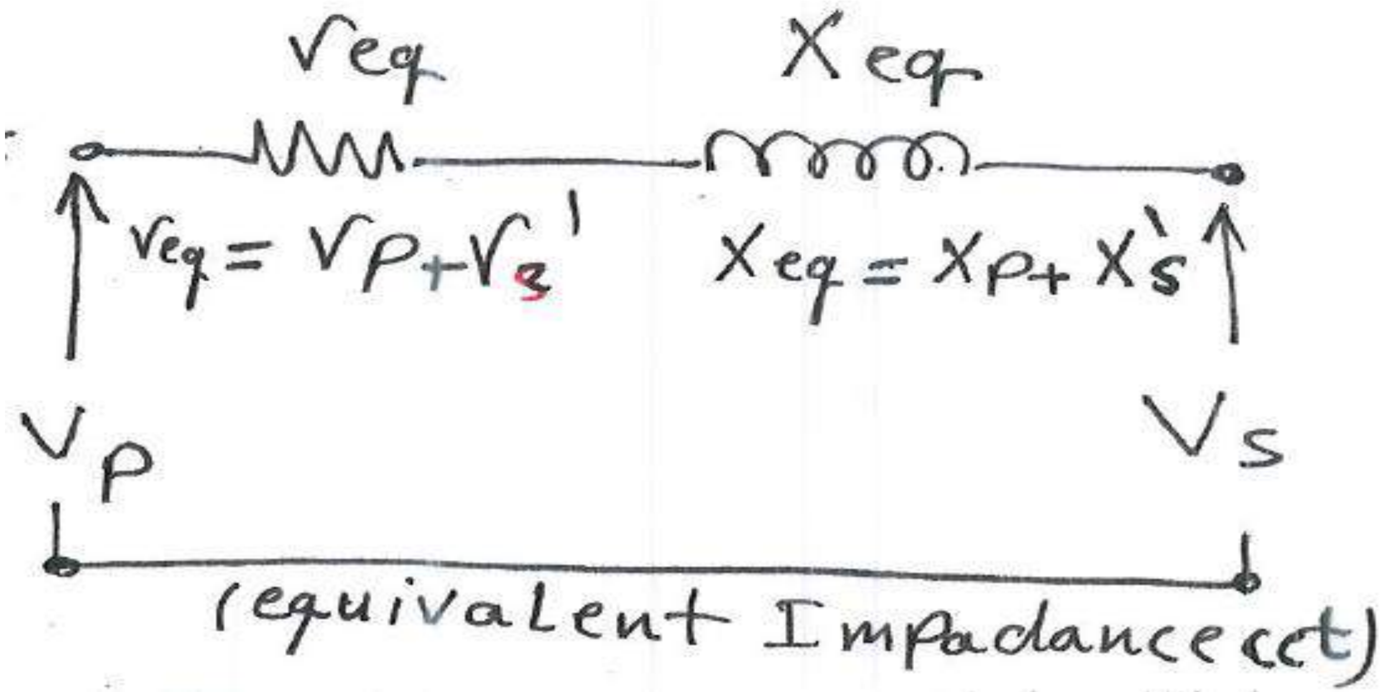
50 KVA, (2400/240) V, 60HZ, distribution Transformer has a Leakage Impedance of (0.72+ j0.92) Ω in the high Voltage side and (0.007+ j0.009) Ω in the Low Voltage side, used to step down the Voltage at the end of a feeder whose Impedance is (0.30+ j1.6) Ω.

The sending Voltage is (V_P=2400V). Find the Voltage at the Secondary side, when the load connected to its Secondary draws rated current from the Transformer and the Power Factor (P.F=0.8) Lagging.

Neglect Voltage drops in the Transformer and feeder caused by the exciting current.

Solution:

The circuit with all quantities referred to High- Voltage side (Primary) is shown;



The Transformer is represented by its equivalent Impedance:

$$\boldsymbol{R_{eq}} = \boldsymbol{R_P} + \boldsymbol{R_s'} = \boldsymbol{rR_P} + \left(\frac{N_P}{N_S}\right)^2 \boldsymbol{R_s} = 0.72 + \left(\frac{2400}{240}\right)^2 \times 0.007 = 1.42\ \Omega.$$

$$\boldsymbol{X_{eq}} = \boldsymbol{X_P} + \boldsymbol{X_s'} = \boldsymbol{X_P} + \left(\frac{N_P}{N_S}\right)^2 \times \boldsymbol{X_s} = 0.92 + \left(\frac{2400}{240}\right)^2 \times 0.009 = 1.82\ \Omega.$$

With: $\boldsymbol{X_s'} = \left(\frac{N_P}{N_S}\right)^2 \times \boldsymbol{X_s}$.

The total Impedance in series is:

$\boldsymbol{Z_{total}}$= **{0.3+1.42+j(1.82+1.6)}=(1.72+j3.42)** Ω.

The load current referred to the High- Voltage side:

$I = \frac{KVA}{V}$ **= (5000/2400) = 20.8 AMP.**

With the phasor diagram referred to the High- Voltage side is shown in Fig (20 b):

P.F= COS θ=0.8 →→ $\theta = -COS^{-1}$ 0.8 = − 36.87°

From the phasor diagram:

$(\boldsymbol{V_P})^2 = (\boldsymbol{ob})^2 + (\boldsymbol{bc})^2$ →→ $\boldsymbol{ob} = \sqrt{\{(\boldsymbol{V_P})^2 - (\boldsymbol{bc})^2\}}$

And:

$\boldsymbol{ob = V_s + ab}$

Note that:

bc =IXCOS θ– IXSIN θ

ab= ad+db = IRCOS θ+ IXSIN θ

Where**:**

X= (1.6+1.82) Ω=3.42 Ω & R=(0.3+1.42) Ω=1.72 Ω

bc= 20.8 × 3.42×0.8 **–** 20.8 × 1.72×0.6= 35.5 Volt

ab = 20.8 ×1.72×0.8+ 20.8 × 3.42×0.6= 71.4 Volt

$\boldsymbol{ob} = \sqrt{\{(\mathbf{2400})^2 - (\mathbf{35.5})^2\}}$ **=2399.73≃ 2400 Volt.**

$\boldsymbol{V_s^{`}}$ **= *ob* – *ab* =2399 – 71.4 =2329 ≃ 2330 Volt.**

The actual Voltage at the Secondary is:

$\frac{\boldsymbol{V_S^{`}}}{\boldsymbol{V_S}} = \frac{\mathbf{2400}}{\mathbf{240}}$ **= 10.** $\boldsymbol{V_S}$**=233 Volt**

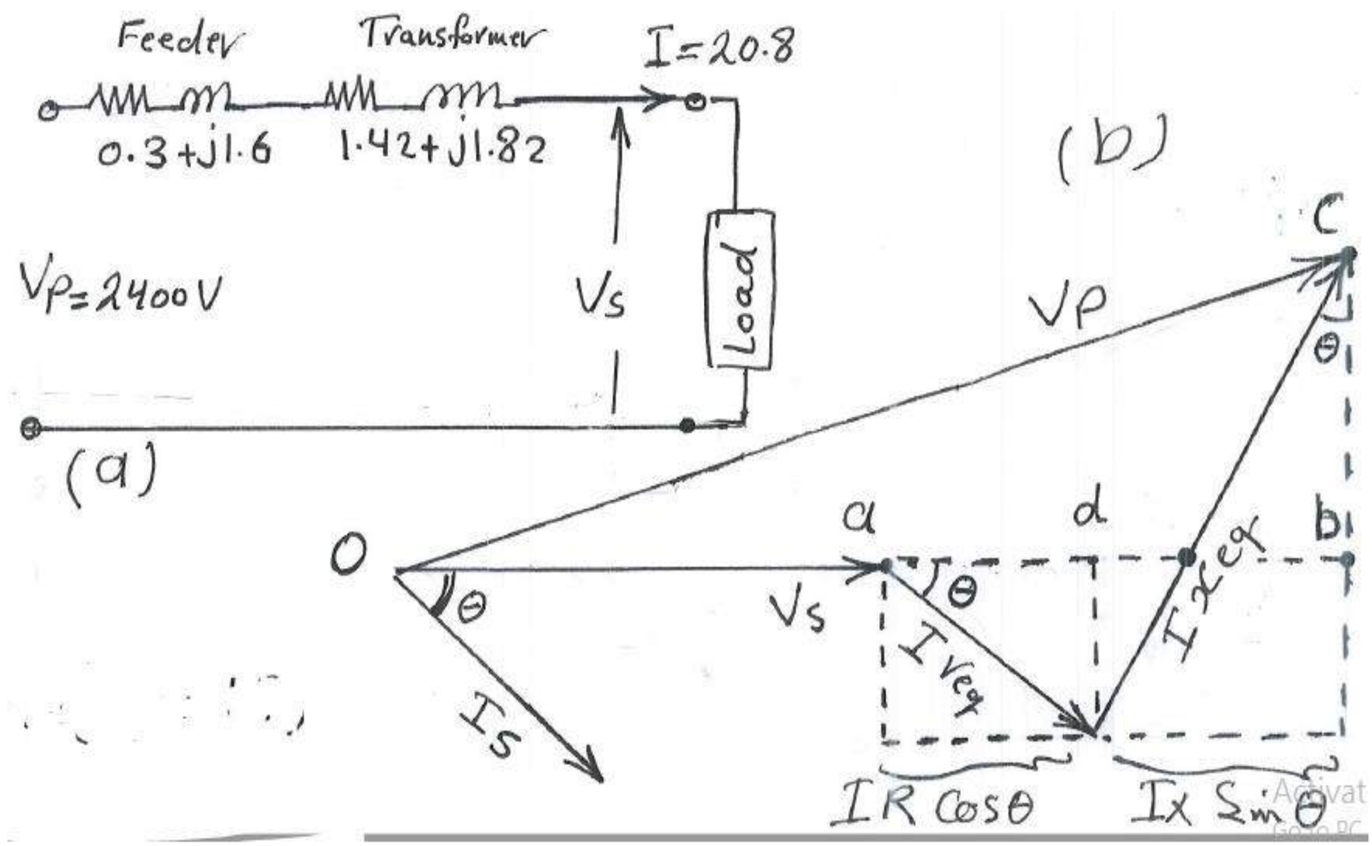
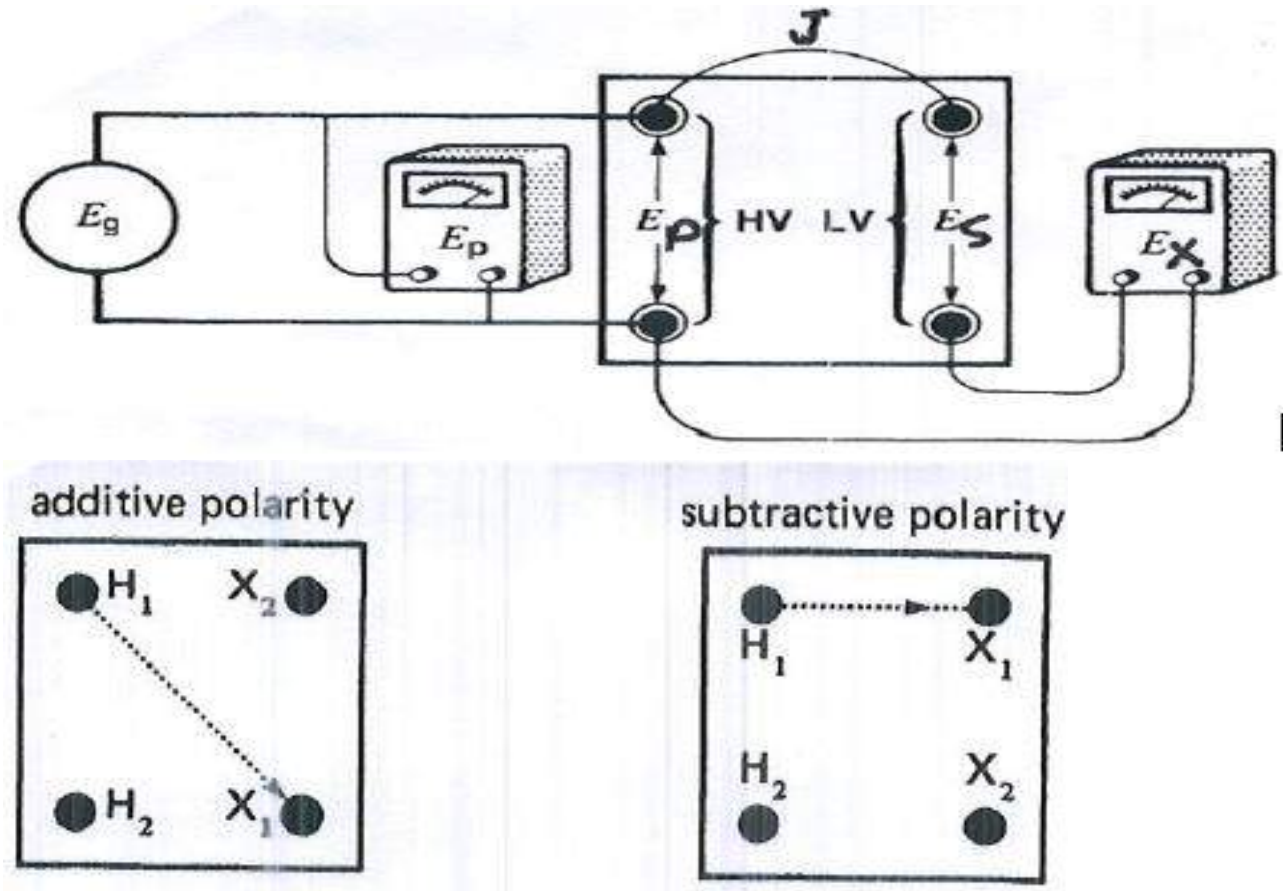


Fig (20): Example 1.9

4.6.4 POLARITY TEST

To determine whether a Transformer possesses additive or subtractive Polarity, we proceed follows, Fig (21)



a) determining the transformer polarity using an A.C source. **b) additive and subtractive Polarity, depend upon the location of H1-X1 terminals.**

Fig (21) Polarity TEST

1-Connect the High- Voltage winding to a Low Voltage (say 110 V) a.c source $\boldsymbol{E_g}$.

2- Connect a jumper **j** between any two adjacent **H.V** and **L.V** terminals.

3-Connect a Voltmeter $\boldsymbol{E_X}$ between the other two adjacent H.V and L.V terminals.

4-Connect another Voltmeter $\boldsymbol{E_P}$ across the High- Voltage winding. If $\boldsymbol{E_X}$ gives a higher reading than $\boldsymbol{E_P}$ the Polarity is additive (this tells us that **H1** and **X1** are diagonally opposite, see Fig (21 b)

On the other hand If $\boldsymbol{E_X}$ gives a Lower reading than $\boldsymbol{E_P}$ the Polarity is subtractive and terminals that **H1** and **X1** are adjacent.

In this Polarity test, jumper **j** effectively connects the Secondary Voltage $\boldsymbol{E_s}$

In series with Primary Voltage $\boldsymbol{E_p}$ **.**In other words:

$$\left(\begin{array}{ll} E_X = E_p + E_s & \textbf{Additive} \\ E_X = E_p - E_s & \textit{subtractive} \end{array} \right)$$ **depending on the Polarity .**

4.7 TRANSFORMER IN PARALLEL

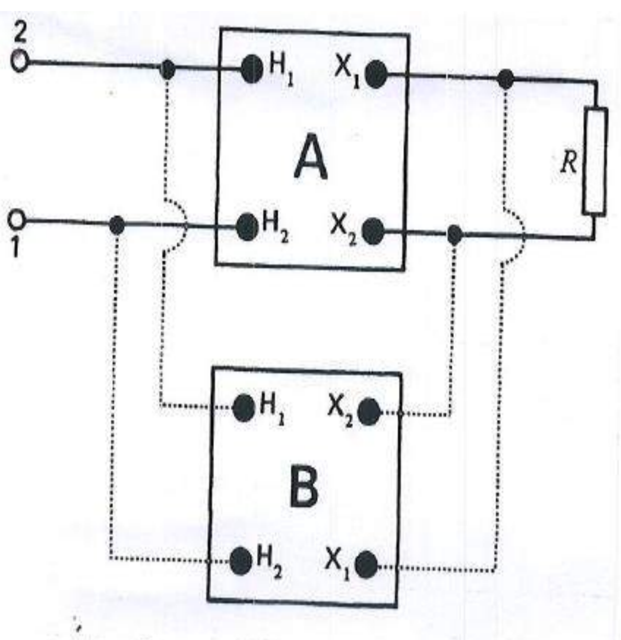
When a growing load eventually exceeds the Power rating of an installed Transformer, we connect a second Transformer in parallel with it, to ensure proper load- sharing between the two Transformers.

They must posse the following:

a- same Primary and Secondary Voltages.

b- same per-unit Impedance

Attention must be paid to the polarity of each Transformer, so that only terminals having the same polarity are connected together, Fig (22)



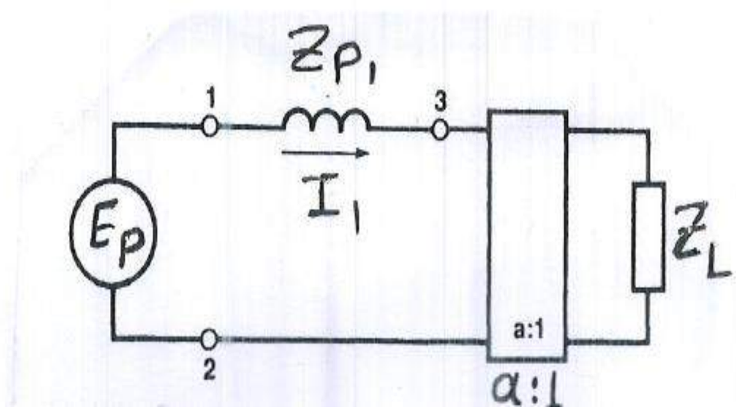


Fig (22) connecting Transformers in parallel to share a load **Fig (23 a) equivalent CCT of a Transformer feeding a load Z**

An error in Polarity produces a severe short- circuit.

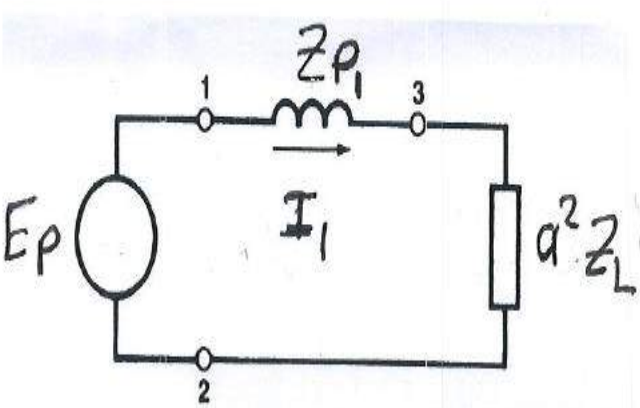
In order to calculate the currents flowing in each Transformer when they are connected in parallel. We first determine the equivalent circuit when a single Transformer feeds a load $\boldsymbol{Z_L}$ Fig (23 a):

$\boldsymbol{E_P}$**=** is the Primary Voltage.

$\boldsymbol{Z_{P1}}$ **=** Impedance of the Transformer referred to Primary.

$\boldsymbol{a}$**=** the transformation ratio.

The circuit can be simplified to that shown in Fig (23 b):



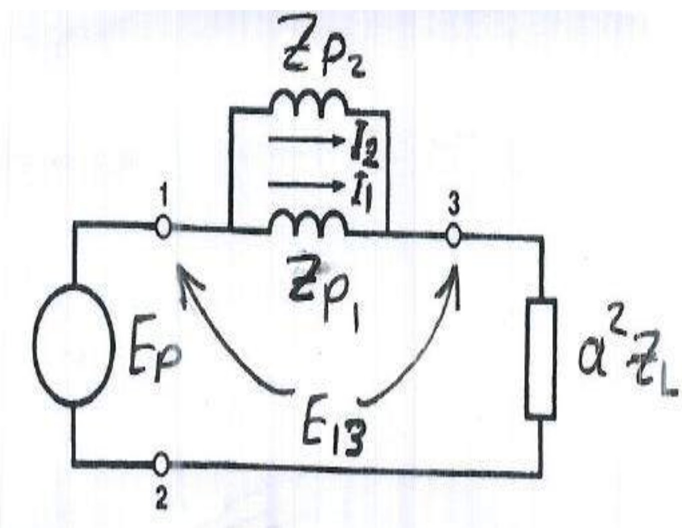


Fig (23 b) **equivalent CCT with all Impedance referred to Primary** Fig (23 c) **equivalent CCT of two transformer in parallel feeding a load Z**

If a second Transformer having an Impedance $\boldsymbol{Z_{P2}}$ is connected in parallel with the first, the equivalent circuit becomes as shown in Fig (23 c).

The Impedances of the transformers are in parallel:

$\boldsymbol{I_1}$ **&** $\boldsymbol{I_2}$**=** the primary currents.

Because the Voltage drop (1&3) is $\boldsymbol{E_{13}}$ across the Impedances is the same:

$$E_{13}= I_1 \times Z_{P1}=I_2 \times Z_{P2} \qquad \text{......... (1.43)}$$

so that:

$$\frac{I_1}{I_2}=\frac{Z_{P2}}{Z_{P1}}$$

The ratio of the Primary currents is therefore determined by the magnitude of the Primary Impedances.

In order that the temperature rise be the same for both Transformers, the currents must be proportional to the respective KVA ratings :

$$\frac{I_1}{I_2}=\frac{S_1}{S_2} \qquad \text{......... (1.44)}$$

The following example shows what happens when the per- unit Impedance are different.

Example 1.10:

A 100 KVA Transformer is connected in parallel with an existing 250 KVA transformer to supply a load of 330 KVA.

The Transformers are rated (7200/240) Volt, but the 100 KVA has an Impedance 4%, while the 250 KVA transformer has an Impedance 6%,

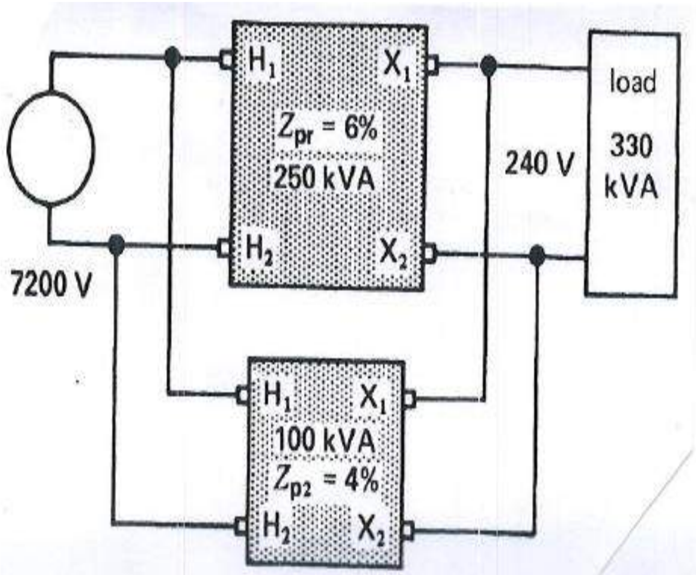
Fig (24 a). Calculate:

a-The nominal Primary current of each Transformer.

b-The Impedance of the load referred to Primary side.

c- The Impedance of each Transformer referred to Primary side.

d-The actual Primary current in each Transformer.



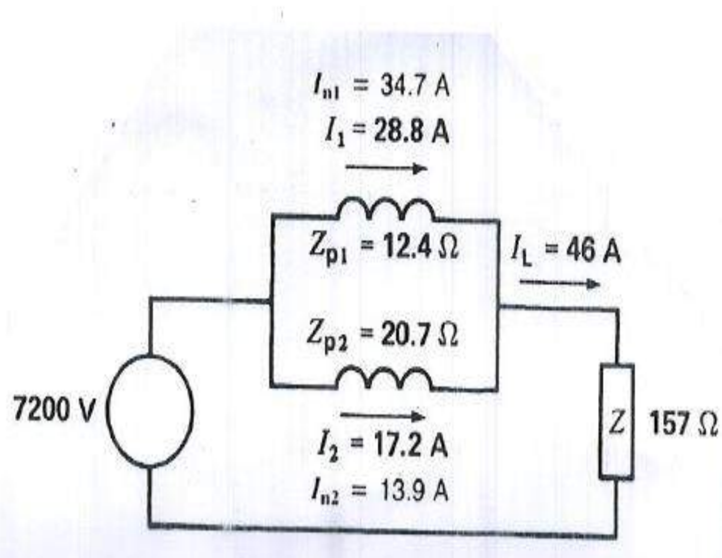


Fig (24 a) actual Transformer connections Fig (24 b) equivalent CCT. Calculations show that 100KVA is seriously over loaded.

Solution:

a- **250 KVA** →→ I_{n1}**= (250000/7200) = 34.7 AMP.**

100 KVA →→ I_{n2}**= (100000/7200) = 13.9 AMP.**

b-The equivalent circuit of the Transformers and the load, referred to the Primary side, is given in Fig (24 b), so that load Impedance referred to the Primary side is:

$$Z = \frac{E_P^2}{S_{load}} = \frac{(7200)^2}{330000} = 157\ \Omega$$

The approximate load current is:

$$I_L = \frac{S_{load}}{E_P} = (330000/7200) = 46\ \text{AMP.}$$

C-The base Impedance of 250 KVA unit is:

$$Z_{np1} = \frac{(7200)^2}{250000} = 207\ \Omega$$

Transformer Impedance referred to the Primary side is:

$$Z_{actual} = Z_{p.u} \times Z_{base} = 0.06 \times 207 = 12.4\ \Omega.$$

Base Impedance of 100KVA unit is:

$$Z_{np1} = \frac{(7200)^2}{100000} = 518\ \Omega$$

Transformer Impedance referred to the Primary side is:

$$Z_{actual} = Z_{p.u} \times Z_{base} = 0.04 \times 518 = 20.7\ \Omega$$

d- Referring to Fig (24 b) we find that 46 AMPS load current divides in the following way:

$$I_1 = 46 \times \frac{20.7}{12.4+20.7} = 28.8 \text{ AMP.}$$

$$I_2 = 46 \times \frac{12.4}{12.4+20.7} = 17.2 \text{ AMP.} \qquad \text{OR} \qquad I_2 = 46 - 28.8 = 17.2 \text{ AMP}$$

4.8 VOLTAGE REGULATION AND EFFICIENCY

Because a real Transformer has a series Impedance within it, the output Voltage of a Transformer varies with the load even if the input Voltage remains constant.

Full- Load Voltage regulation is a quantity that compares the output Voltage of the Transformer at NO- LOAD with Voltage at FULL- LOAD.

It is defined by:

$$VR = \frac{V_{S\ (no-load)} - V_{S\ (full-load)}}{V_{S}(_{full-load})} \times 100\% \qquad \text{......... (1.45)}$$

Since at No- Load:

$$V_S = \frac{V_P}{N}$$

The Voltage regulation can also expressed as:

$$\textbf{VR} = \frac{\left(\frac{V_P}{N}\right) - V_{S\,(full-load)}}{V_S(full-load)} \times 100\% \qquad \textbf{......... (1.46)}$$

4.8.1 THE TRANSFORMER PHASOR DIAGRAM

To determine the Voltage regulation of a Transformer it is necessary to understand the Voltage drops within it.

The Voltage regulation of a Transformer depends on both the magnitude of Impedance and on the phase angle of the current flowing through the Transformer.

In the phasor diagrams, the phasor Voltage $\overrightarrow{V_S}$ is assumed to be at an angle= **ZERO**, and all other Voltage and Currents are compared to that reference.

By applying Kirchhoff's Voltage Low the Primary Voltage can be found as:

$$\frac{\overrightarrow{Vp}}{N} = \overrightarrow{V_S} + R_{eq} \times \overrightarrow{I_S} + j\, X_{eq} \times \overrightarrow{I_S} \qquad \textbf{......... (1.47)}$$

Fig (25), shows a phasor diagram of Transformer operating at a Lagging Power Factor (P.F).

It is easy to see that $\frac{\overrightarrow{Vp}}{N} > \overrightarrow{V_S}$ in lagging load.

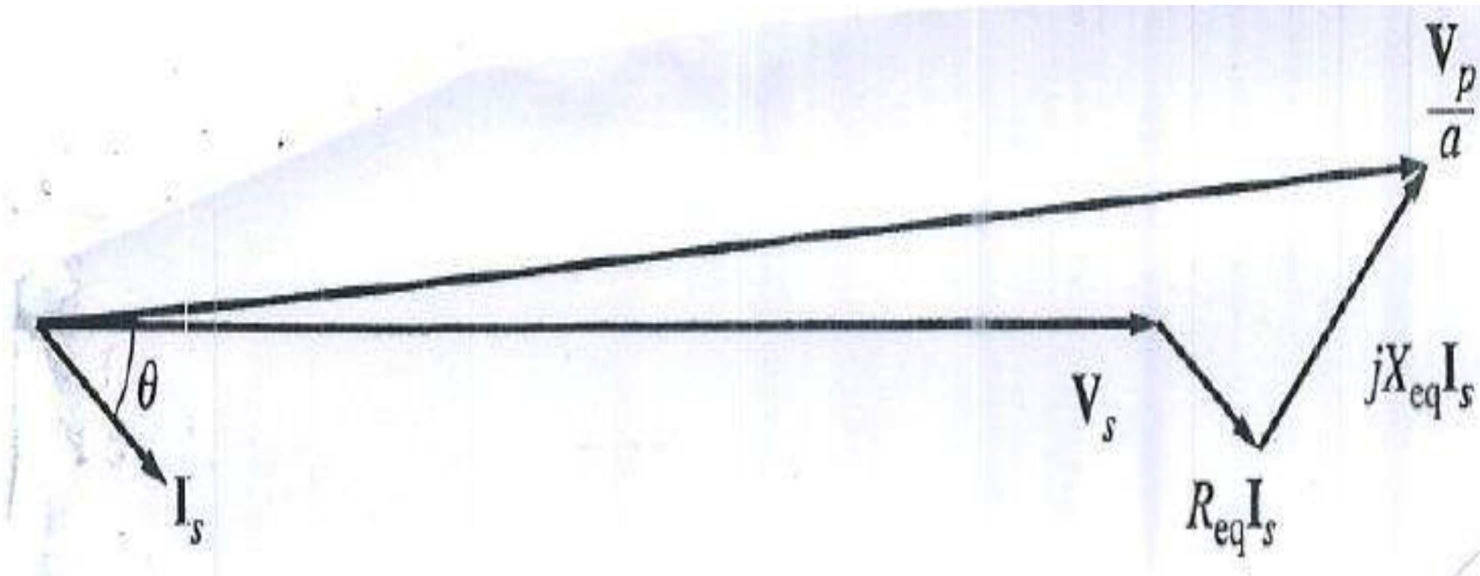


Fig (25) phasor diagram of Transformer operating at a Lagging Power Factor (P.F).

A phasor diagram of unity P.F (P.F= 1) is shown in Fig (26 a).

Here again the Voltage at the Secondary is Lower than the Voltage at Primary. This time the Voltage regulation is a smaller number than it was in Lagging current.

If the Secondary current is Leading, the Secondary Voltage can actually be higher than the referred Primary Voltage, Fig (26 b).

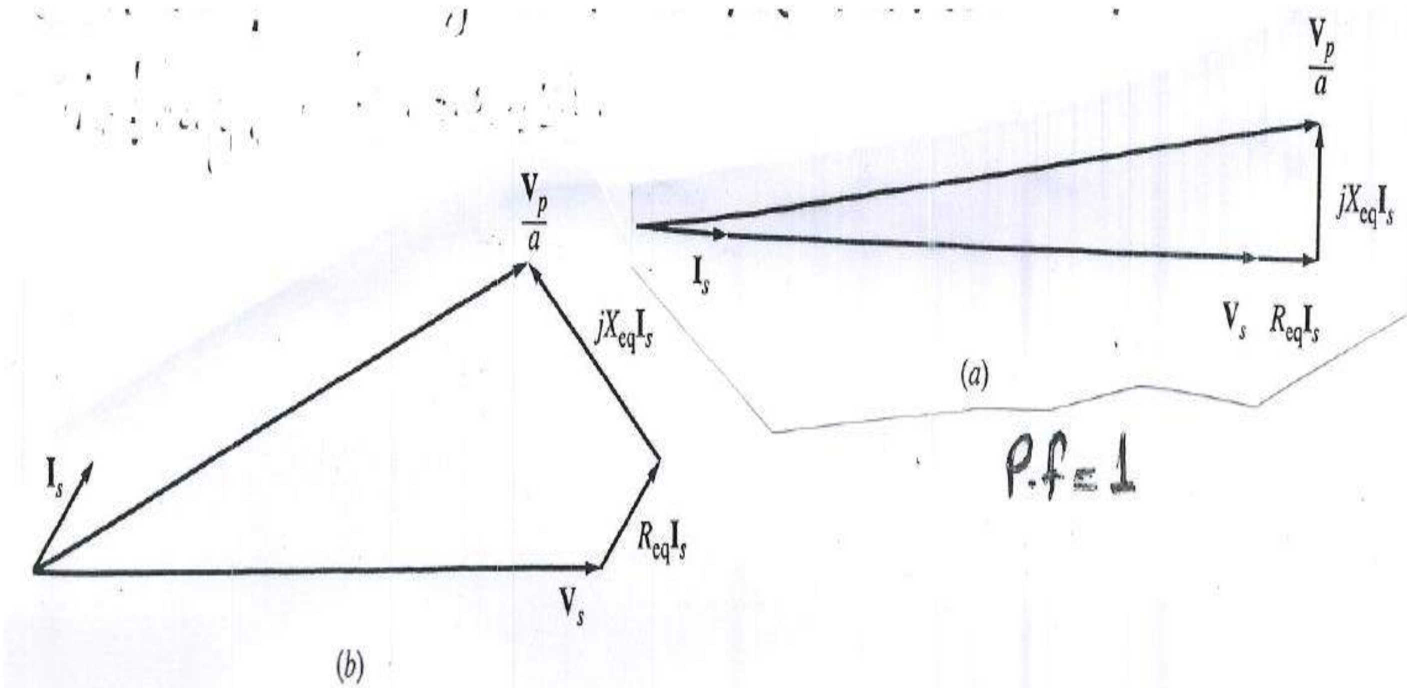


Fig (26) Phasor diagram of Transformer operating (a) unity and (b) Leading P.F.

4.8.2 THE TRANSFORMER EFFICIENSY (η)

The efficiency of a device is defined by:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% \quad \rightarrow \quad \eta = \frac{P_{out}}{P_{out} + P_{Loss}} \times 100\% \quad \text{... (1.48)}$$

These equations apply to motors and generators as wall as to Transformers.

The Transformer equivalent circuit make efficiency calculations easy.

There are **3** types of losses present in Transformer:

1- copper$(I^2 \times \mathbf{R})$ Losses: these losses are accounted for the series resistance.

2- Hysteresis Losses: these losses are accounted for by $\mathbf{R_c}$.

Eddy current Losses: Eddy current loss is power loss in a transformer or motor due to currents induced in the metal parts of the system from the changing of magnetic field. To calculate the η of a Transformer just add the losses to the output Power, which is given by:

$$P_{out} = V_S \times I_S \times COS_{\theta_S}$$

$$\eta = \frac{V_S \times I_S \times COS_{\theta_S}}{P_{CU} + P_{core} + V_S \times I_S \times COS_{\theta_S}} \quad \text{......... (1.49)}$$

Example (11):

A 15 KVA, (2300/230) V Transformer is to be tested to determine its excitation branch components, its series Impedance, and its Voltage regulation, the test data have been taken from the Primary side:

Open circuit test	Vo.c =2300 V	**Io.c = 0.21 AMP**	**Po.c =50 Watt**
Short circuit test	**Vs.c = 47 V**	**Is.c =6.0 AMP**	**Ps.c = 160 Watt**

a) Find the equivalent circuit referred to High- Voltage side.

b) Find the equivalent circuit referred to Low- Voltage side.

c) Calculate the Full- Load Voltage regulation at 0.8 Lagging P.F, and at P.F= 1.0, using the exact equation of V_P

d) What is the efficiency of Transformer at Full- Load and P.F =0.8 Lagging.

Solution:

a) The excitation branch components ca be calculated from open – circuit test, the series element from short – circuit test.

$$\theta_{o.c} = COS^{-1} \frac{P_{o.c}}{V_{o.c} \times I_{o.c}} = COS^{-1} \frac{50\ Watt}{2300V \times 0.21\ AMP} = 84°$$

The excitation Admittance (Y_E) is thus:

$$Y_E = \frac{I_{o.c}}{V_{o.c}} \angle -84° = \frac{0.211}{2300} \angle -84° = 9.13 \times 10^{-5} \mho = (0.0000095 - J0.0000908)\ \mho$$

The elements of excitation are:

$\boldsymbol{R_C} = \frac{1}{0.0000095}$ = 105 KΩ ; $\boldsymbol{X_m} = \frac{1}{0.0000908}$ = 11 KΩ.

From short- circuit test:

$$\boldsymbol{\theta_{s.c}} = \boldsymbol{COS^{-1}} \frac{P_{s.c}}{V_{s.c} \times I_{s.c}} = \boldsymbol{COS^{-1}} \frac{160\ Watt}{47V \times 06\ AMP} = \mathbf{55.4°}$$

The equivalent Impedance is thus:

$$Z_{S.C} = \frac{V_{S.c}}{I_{S.c}} \angle \boldsymbol{\theta_{s.c}} = \frac{47}{6} \angle 55.4° = 7.833\angle 55.4° = 4.45 + j\,6.45$$

The series elements referred to Primary side are:

$\boldsymbol{R_{eq}}$ = **4.45** Ω ; $\boldsymbol{X_{eq}}$ = 6.45 Ω.

This equivalent circuit is shown in Fig (27):

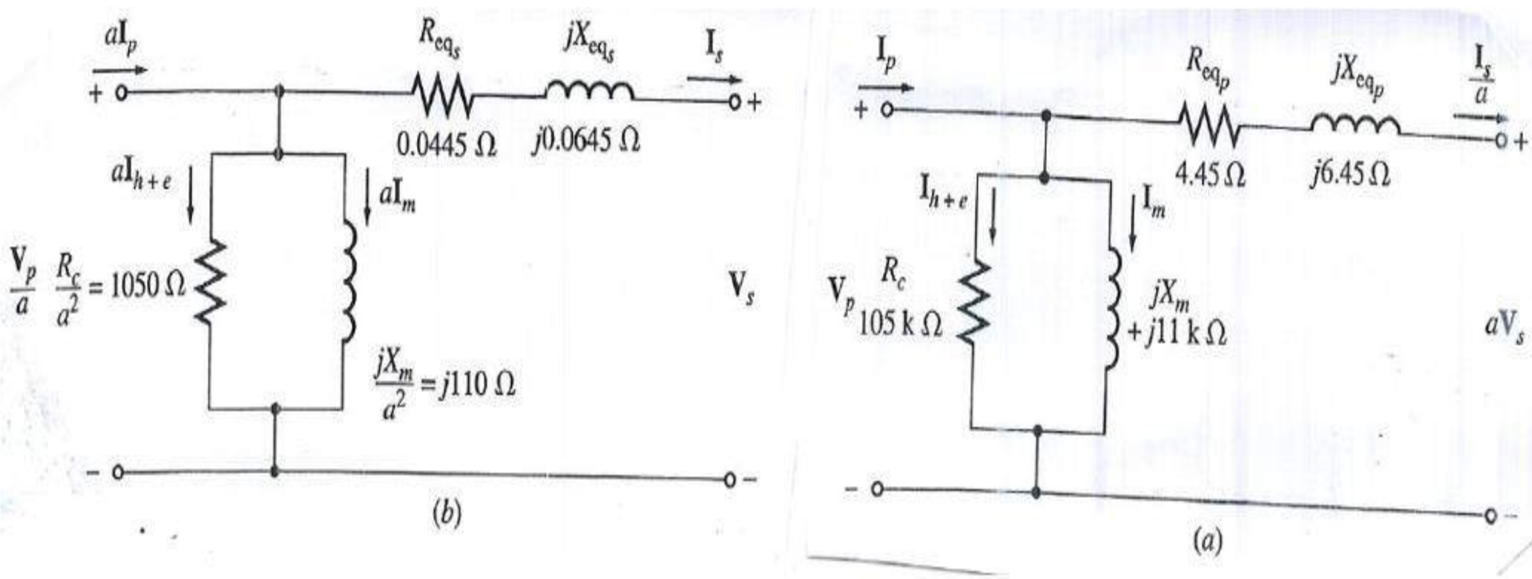


Fig (27) Transformer CCT of example (11) referred to a) primary b) secondary

a) To find the equivalent circuit referred to Low- Voltage side, it is simply necessary to divide the Impedance by N^2 . Since: $N = N_P / N_S = 110$

The resulting are:

$\boldsymbol{R_c}$ = **1050** Ω	$\boldsymbol{X_m}$ = 110 Ω.	$\boldsymbol{R_{eq}}$ = **0.0445** Ω	$\boldsymbol{X_{eq}}$ = 0.0645 Ω.

The equivalent circuit is shown in Fig (27b).

c)The Full- Load on the Secondary side is:

$I_{S(\text{rated})} = \dfrac{S_{rated}}{V_{s(rated)}} = \dfrac{1500}{230}$ **= 65.2 AMP.**

To calculate (**V_P /N)** use equation (1.47):

$$\frac{\overrightarrow{Vp}}{N} = \overrightarrow{V_S} + R_{eq} \times \overrightarrow{I_S} + j\,X_{eq} \times \overrightarrow{I_S}$$

At P.F = 0.8 Lagging $\rightarrow\rightarrow$ $\overrightarrow{I_S}$ **= 65.2** $\angle -36.9°$ AMP

$$\frac{\overrightarrow{Vp}}{N} = 230\angle 0° + (0.0445 \times 65.2\angle -36.9°) + j\,(0.0645 \times 65.2\angle -36.9°)$$

$$= 230\angle 0° + 2.9\angle -36.9° + 4.21\angle 53.1°$$

$= 230 + 2.32 - j1.74 + 2.52 + j3.36 = 234.84 + j1.62 = 234\angle 0.40°$ Volt.

At P.F =1.0 $\rightarrow\rightarrow$ $\overrightarrow{I_S}$ **= 65.2** $\angle 0°$ AMP.

$$\frac{\overrightarrow{Vp}}{N} = 230\angle 0° + (0.0445 \times 65.2\angle 0°) + j\,(0.0645 \times 65.2\angle 0°)$$

$= 230\angle 0° + 2.90\angle 0° + j\,4.21\angle 0° = 232.9 + j4.21 = 232.94\angle 1.04°$ Volt.

$$VR = \frac{\left(\frac{V_P}{N}\right) - V_{S\,(full-load)}}{V_{S(full-load)}} \times 100\% = \frac{234.85 - 230}{230} \times 100\% = 2.1\ \%\ (0.8\ \text{Lag})$$

$$VR = \frac{232.94 - 230}{230} \times 100\% = 1.28\ \% \qquad (P.F = 1)$$

d)To find efficiency η :

$$P_{cu} = I_s^2 \times R_{eq} = 65.2^2 \times 0.0445 = 189\ \text{Watt}$$

$$P_{core} = \left(\frac{V_P}{N}\right)^2 / R_C = (234.85)^2 / 1050 = 52.5\ \text{Watt.}$$

(o/p) Power: $P_{out} = V_s\ I_s$ COS Ө = 230× 65.2 ×COS 36.9= 1200 W

$$\eta = \frac{V_S \times I_S \times COS_{\theta_S}}{P_{CU} + P_{core} + V_S \times I_S \times COS_{\theta_S}} \times 100\% = \frac{1200}{189 + 52.2 + 1200} \times 100\% = 98.03\%$$

4.9 THREE – PHASE TRANSFORMERS

All most all the major Power generation and distribution systems are Three-Phase (3- Ph) A.C system, it is necessary to understand how (3- Ph) Transformer are used in them. Transformers for (3- Ph) circuits can be constructed in one of TWO way.

One approach is simply to take three single phase – Transformers and connect them in three phase, Fig (28).

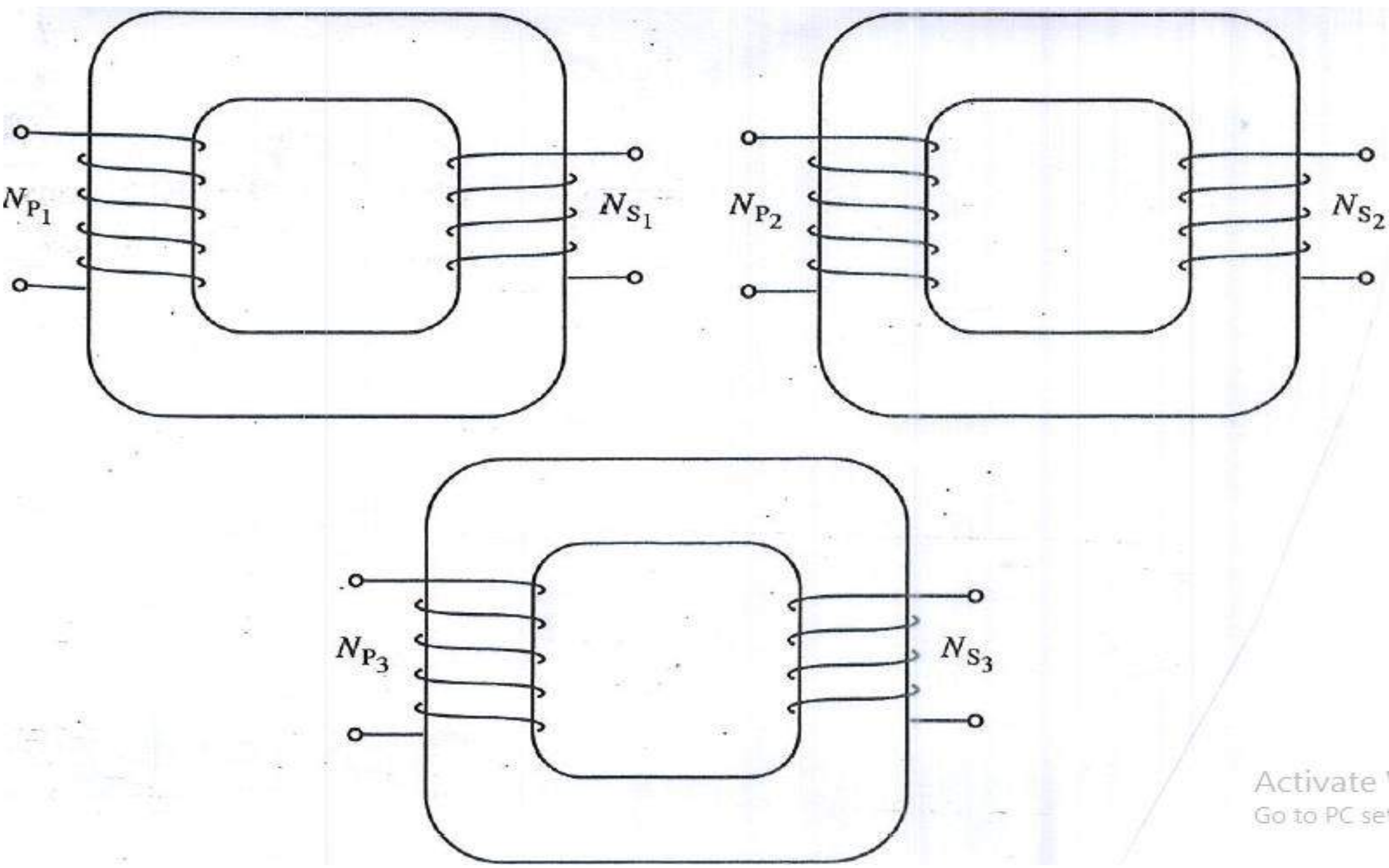


Fig (28) **A 3- Ph Transformer bank composed of independent Transformers**

An alternative approach is to make a three- phase Transformer consisting of three sets of winding warped on the same core, Fig (29).

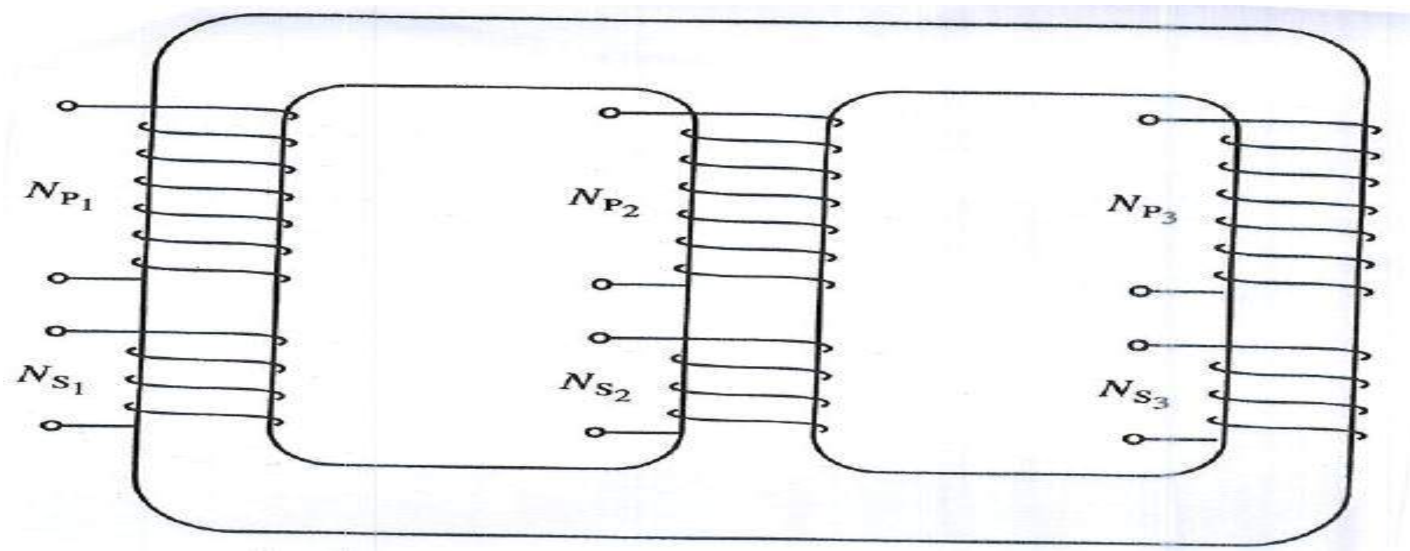
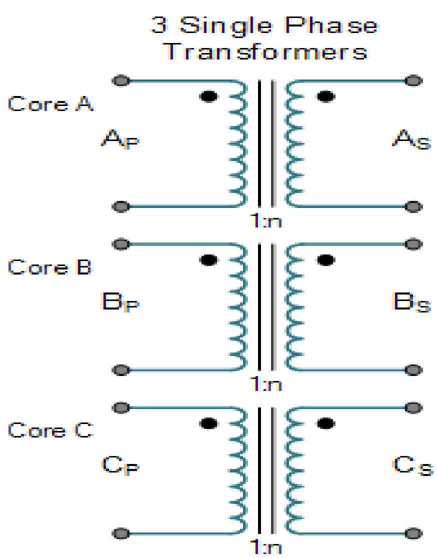


Fig (29) **A 3- Ph Transformer wound on a single three- legged core.**

4.9.1 THREE – PHASE TRANSFORMER CONNECTIONS

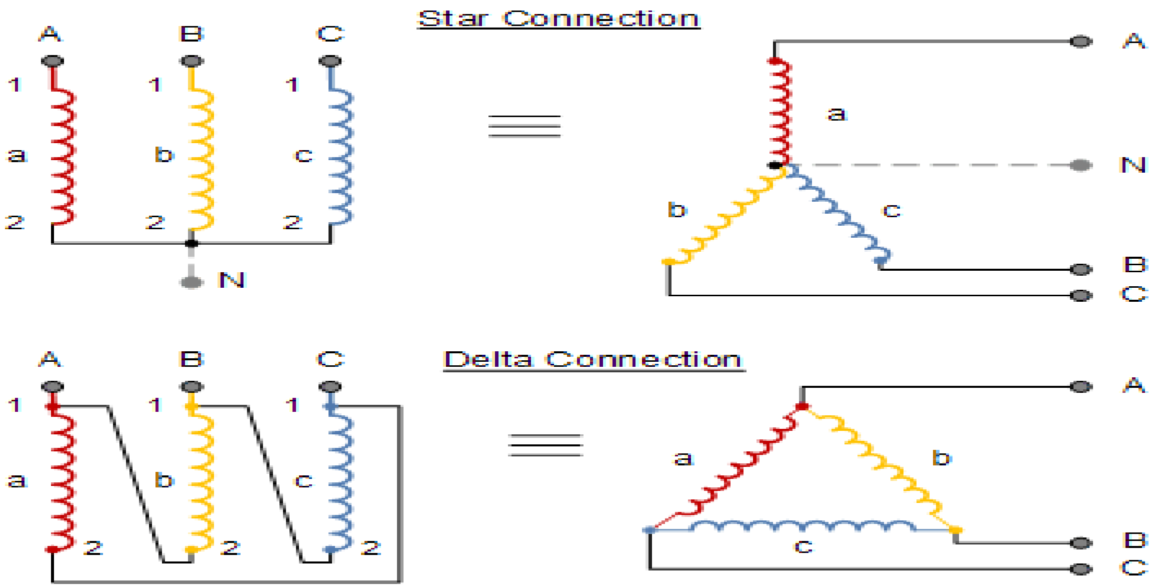
A 3-Ph Transformer consists of three Transformer, either separate or combined on one core.

The Primaries and Secondaries of any 3-Ph Transformer can be connected in either wye (Y) or a delta (**Δ**) .This gives a total of four main possible connections, see Electrical machines and their applications by John Hind Marsh, Fig (32)



Primary Configuration		Secondary Configuration	
Delta (Mesh)	△	Delta (Mesh)	△
Delta (Mesh)	△	Star (Wye)	Y
Star (Wye)	Y	Delta (Mesh)	△
Star (Wye)	Y	Star (Wye)	Y
Interconnected Star		Delta (Mesh)	△
Interconnected Star		Star (Wye)	Y

Transformer Star and Delta Configurations



1- **Y- Y connection** : Fig (30)

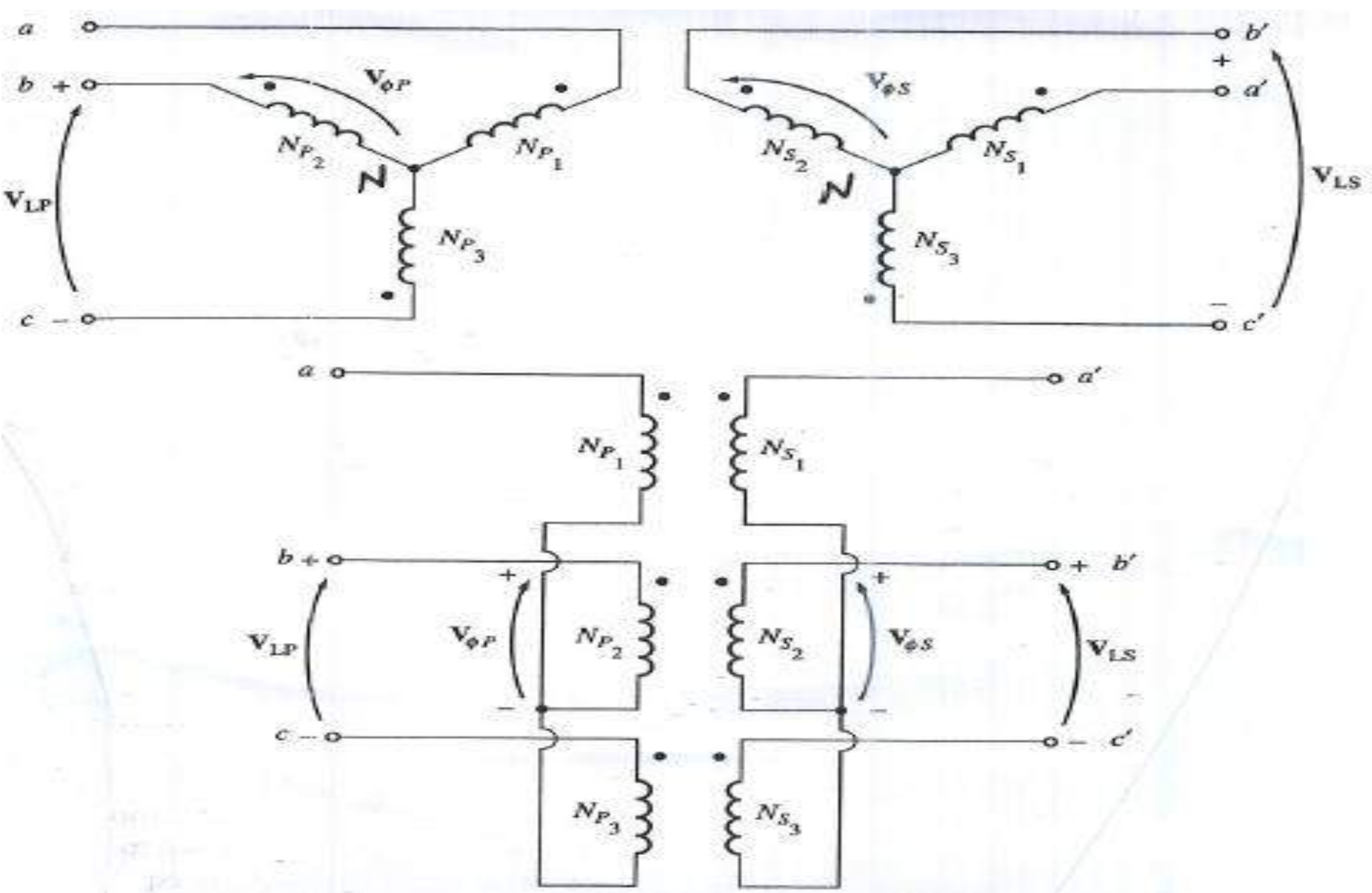


Fig (30) 3- Ph Transformer connections and wiring diagram: **Y- Y**

The Primary Voltage of each phase of the Transformer is given by:

$$V_{phase(\emptyset s)} = \frac{1}{\sqrt{3}} \times V_{Line(LS)} \quad \textbf{and} \quad I_{phase(\emptyset s)} = V_{Line(LS)}$$

Also the Phase Voltage on the Secondary is related to the Line Voltage on Secondary side by:

$$V_{Line(LS)} = \sqrt{3} \times V_{phase(\emptyset s)}$$

$V_{Line(LS)}$ = the Line Voltage on Secondary

$V_{phase(\emptyset s)}$ = the Phase Voltage on the Secondary.

Therefore the Voltage ratio on the Transformer is:

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\emptyset P}}{\sqrt{3}V_{\emptyset S}} = N \qquad \text{(Y- Y)} \qquad \text{......... (1.50)}$$

The (Y- Y) connection has two serious Problems:

1-If loads on the Transformer are unbalanced then the Voltages on the Phases can become severely unbalanced.

2-Third- harmonic Voltages can be large. If a 3- Ph set of Voltages is applied to (Y- Y) Transformer, the Voltages in any Phase will be ($120°$) apart from the Voltages in any other Phase. However, the – harmonic components of each of three Phases will be in Phase with each other. Both the unbalance problem and the third- harmonic problem can be solved using one of two techniques.

1- Solidly ground the neutrals (**N**-point) of the Transformers

2- Add a third (tertiary) winding connected in (**Δ**) to transformer.

2- **Y- Δ** connection : Fig (31)

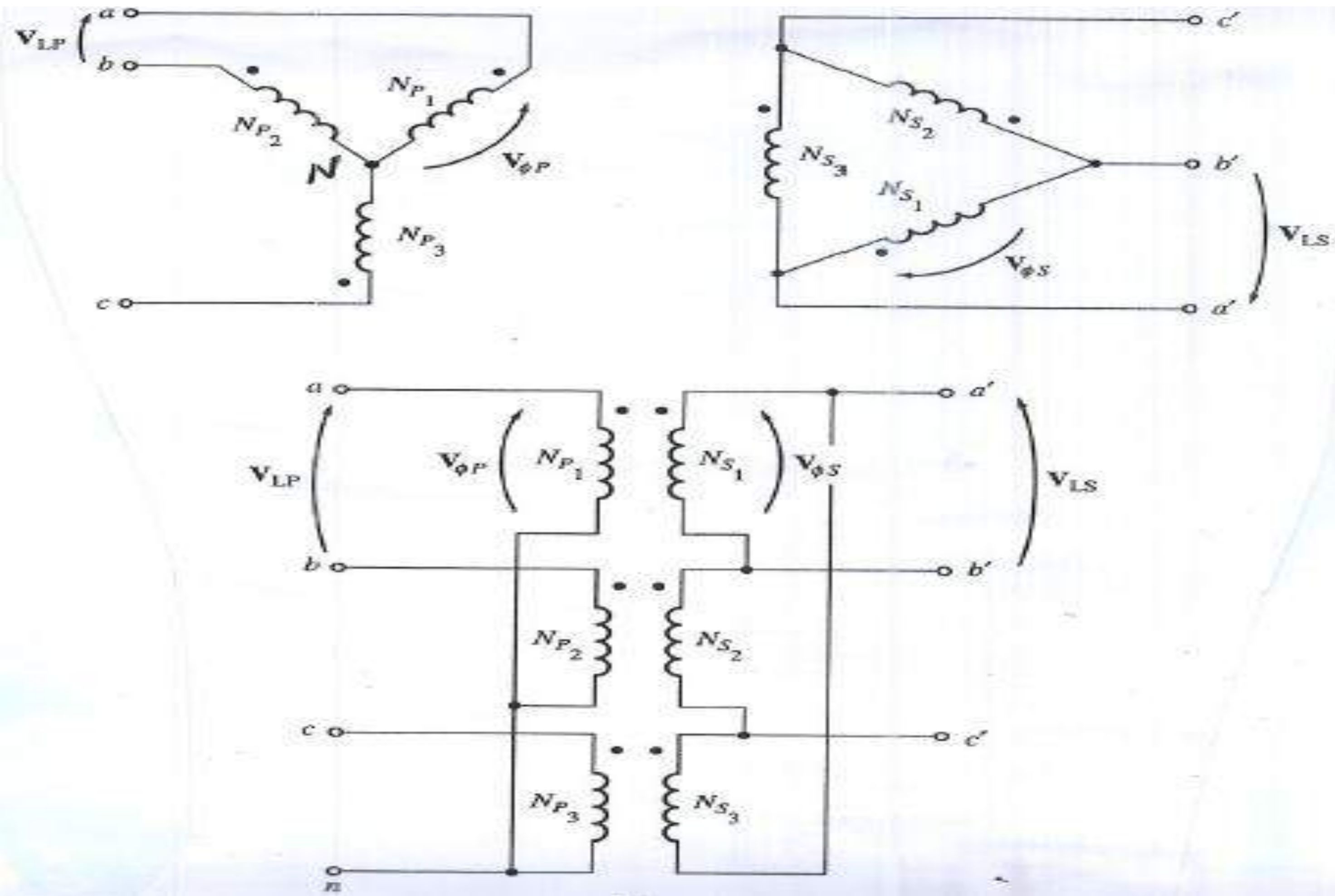


Fig (31) 3- Ph Transformer connections and wiring diagram: **Y- Δ**.

The Primary Line Voltage is related to the Primary Phase Voltage as:

$$V_{LP} = \sqrt{3}V_{\emptyset P} \qquad \text{......... (1.51)}$$

While the Secondary Line Voltage is equal to the Secondary Phase Voltage:

$$V_{LS} = V_{\emptyset S} \qquad \text{......... (1.52)}$$

The Voltage ratio of each phase is:

$$\frac{V_{\phi P}}{V_{\phi S}} = N$$

So the overall relationship between the Line Voltages is:

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}} = \sqrt{3}\,N \qquad (Y\text{-}\ \Delta) \qquad \text{.........}\ (1.53)$$

The (Y- Δ) connection has no problem with Third- Harmonic Components in its Voltages. The (Y- Δ) connection is also more stable with respect to unbalanced loads, since (Δ) partially any imbalance that occurs. The arrangement have ONE problem, the Secondary Voltage is shifted by 30° relative to the Primary Voltage, the Phase occurred can cause problems in paralleling the secondaries of two Transformers.

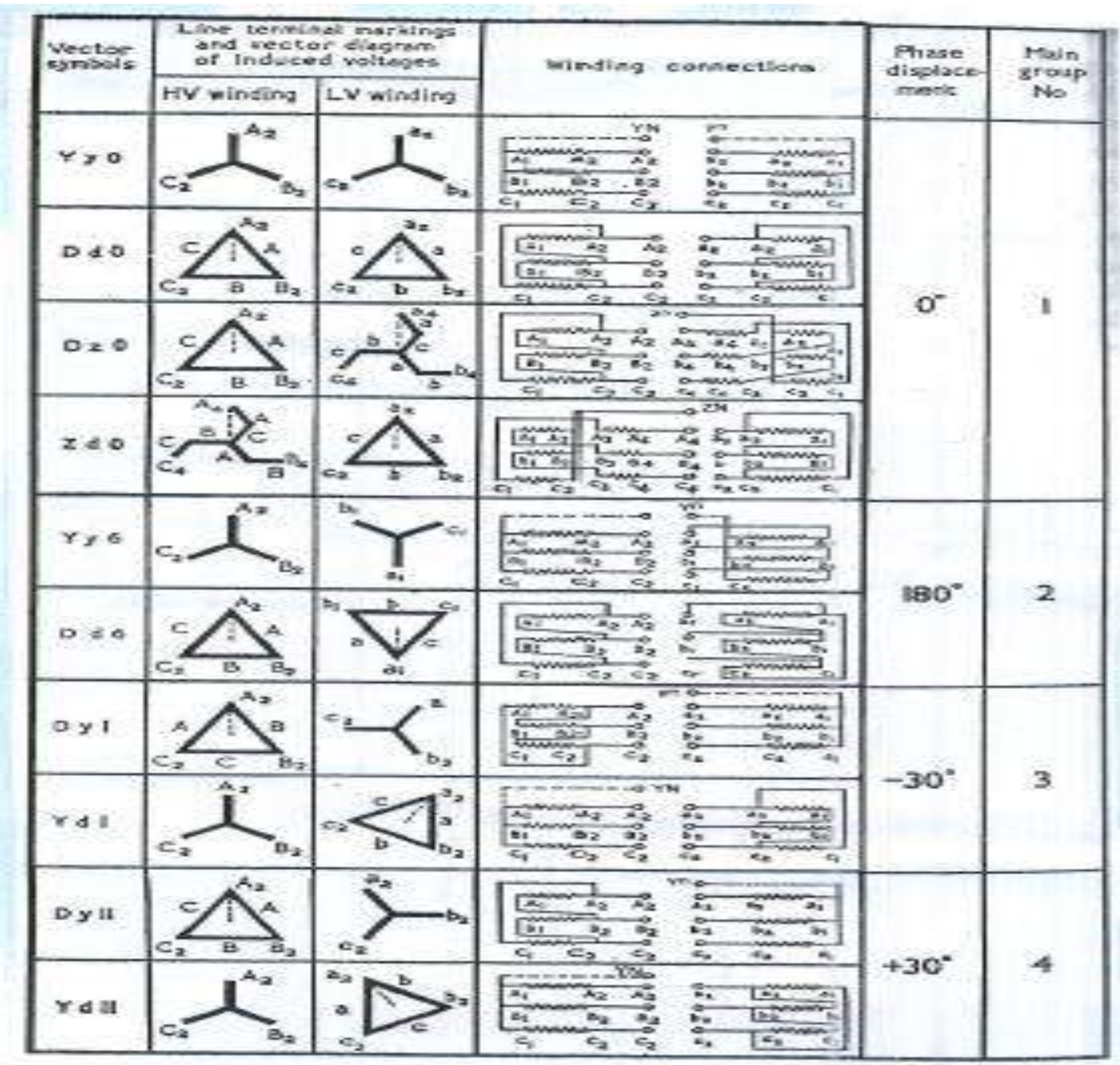


FIG. 4.30 Time-vector diagrams for three-phase transformers.
(By courtesy of the British Standards Institution.)

Fig (32) 3- Ph Transformer connections.