# Mechanics

**1.Units, dimensional analysis and trigonometry**

**1.1.Dimensional analysis**

- Dimensional analysis is a way of making sure that an equation is correct, and that you have the right units in your answer.
- The dimensions in an equation are the units that we are working in (cm, s, Kg etc).
  - We will mostly work with *SI* units (kg, m, s), with some use being made of *cgs* (cm, gm, s) and US customary units.
- If you write an answer with no units, or incorrect units, you will drop a point.



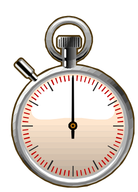
# 2-1Scalars

1.A **scalar quantity** is a quantity that has **magnitude only** and has **no direction** in space

Examples of Scalar Quantities:

- Length
- Area
- Volume
- Time
- Mass



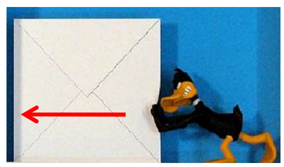


## 2-2Vectors

2- A **vector quantity** is a quantity that has both **magnitude** and a **direction** in space

Examples of Vector Quantities:

- Displacement
- Velocity
- Acceleration
- Force

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## 3-Properties of Vectors

1. **Equality of Two Vectors**
   – Two vectors are **equal** if they have the same magnitude (same units) and same direction
2. **Parallel translation in a diagram**
   – Any vector can be moved parallel to itself without being affected
3. **Negative** vectors are directed in the opposite direction of positive vectors
4. **Resultant Vector**
   – The **resultant** vector is the sum of a given set of vectors $\vec{R} = \vec{A} + \vec{B} + \ldots$

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## 4- Unit Vector:

Is a vector whose magnitude is equal to one only and dimensionless, show the direction of the original vector $\vec{A}$, $|\vec{A}|$ is the magnitude of $\vec{A}$ .

$\vec{A} = \hat{a}A$ or $\vec{A} = \hat{a}\,|\vec{A}|$

So that, the unit vector is equal to: $\hat{a} = \frac{\vec{A}}{A}$ or $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$

The symbols **i, j** and **k** represent unit vectors pointing in the directions x, y and z positives, respectively



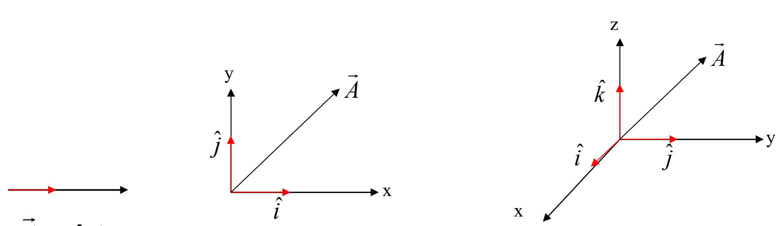
## 4- Unit Vector:

Now $V$ can be written $V = V_x\,\mathbf{i} + V_y\,\mathbf{j}$.

If we need to add the vector **A** = $A_x$ **i**+$A_y$ **j** with the vector **B**= $B_x$i + $B_y$**j** we write

**R** = **A** + **B** = $A_x$ **i** + $A_y$ **j** + $B_x$ **i** + $B_y$ **j** = ($A_x$ + $B_x$)**i** + ($A_y$ + $B_y$)**j**.

1. The components of **R** are **$R_x$=$A_x$+$B_x$** and **$R_y$=$A_y$+$B_y$**




$\vec{A} = \hat{a}A$ $\vec{A} = \hat{i}A_x + \hat{j}A_y$ $\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$

## 5- Working out the components of a vector

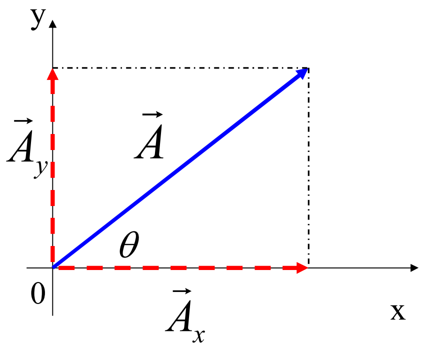
- The projection of vectors along coordinate axes are called component of a vector.

  Projection of $\vec{A}$ on x-axis is $\mathbf{A_x}$

  Projection of $\vec{A}$ on y-axis is $\mathbf{A_y}$

- All vectors can be split into horizontal ($A_x$) and vertical ($A_y$) components.
- **So that This vector can be represented as the sum:**
- Thus, $$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$|A| = \sqrt{|A_x|^2 + |A_y|^2}$$




**Where**

$$A_x = A\cos\theta \qquad A_y = A\sin\theta \qquad \tan\theta = \frac{A_y}{A_x}$$

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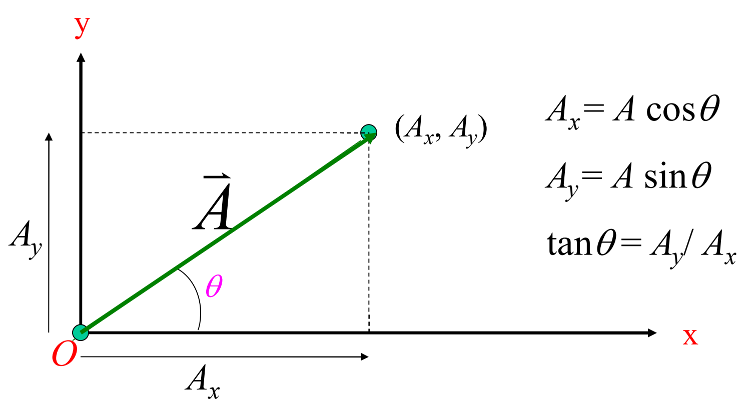


$A_x = A\cos\theta$

$A_y = A\sin\theta$

$\tan\theta = A_y / A_x$

$$A = \sqrt{A_x^2 + A_y^2} = |\vec{A}|.\ \vec{A} \text{ is a unit vector, if } |\vec{A}| = 1.$$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in the $x-$ and $y-$directions, respectively.

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## 6- Properties of Vectors

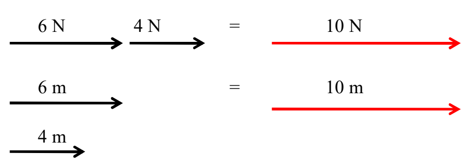
| | |
|---|---|
| $\vec{a}+\vec{b}=\vec{c}$ | **Addition** |
| $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ | **Commutative Property** |
| $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ | **Associative Property** |
| $c(\vec{a}+\vec{b})=c\vec{a}+c\vec{b}$ | **Distributive Property** |
| $(c+d)\vec{a}=c\vec{a}+d\vec{a}$ | **Distributive Property** |



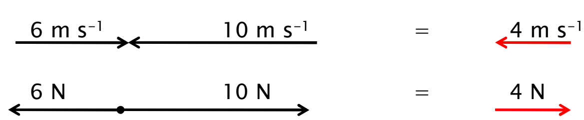
## 7- Resultant of Two Vectors

The resultant is the sum or the combined effect of two vector quantities

**Vectors in the same direction (θ=0):**



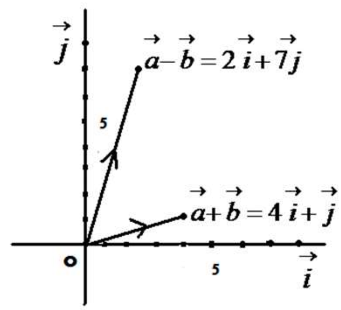

**Vectors in opposite directions (θ=180):** :



## A. Operations with the $\vec{i}$ and $\vec{j}$ Vectors

**(i) Adding and Subtracting**: you add (or subtract) the $\vec{i}$ parts and $\vec{j}$ parts separately.

$$\vec{a} = 3\,\vec{i} + 4\,\vec{j} \quad \text{and} \quad \vec{b} = \vec{i} - 3\,\vec{j}$$






## (ii) Multiplication by a Scalar

$$\vec{a} = 3\,\vec{i} - 2\,\vec{j} \quad \text{and} \quad \vec{b} = -2\,\vec{i} + \vec{j}$$

$$\text{Then } 3\,\vec{a} = 3\left(3\,\vec{i} - 2\,\vec{j}\right) = 9\,\vec{i} - 6\,\vec{j}$$

$$\text{and } -3\,\vec{b} = -3\left(-2\,\vec{i} + \vec{j}\right) = 6\,\vec{i} - 3\,\vec{j}$$

## (iii) The Vector $\vec{ab} = \vec{b} - \vec{a}$

$$\vec{a} = 3\,\vec{i} - 2\,\vec{j} \quad \text{and} \quad \vec{b} = -2\,\vec{i} + \vec{j}$$

## B. The Modulus of a vector in terms of $\vec{i}$ and $\vec{j}$

The modulus of the vector $x\vec{i}+y\vec{j}=\sqrt{x^2+y^2}$

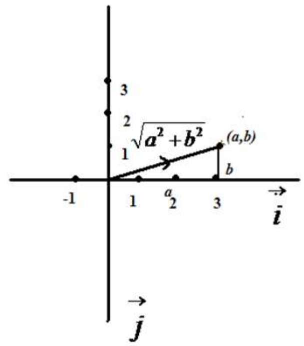
Also written as $\left|x\vec{i}+y\vec{j}\right|=\sqrt{x^2+y^2}$

If $\vec{a}=6\vec{i}+8\vec{j}$ and $\vec{b}=2\vec{i}-3\vec{j}$ find :

(i) $\left|\vec{a}\right|$

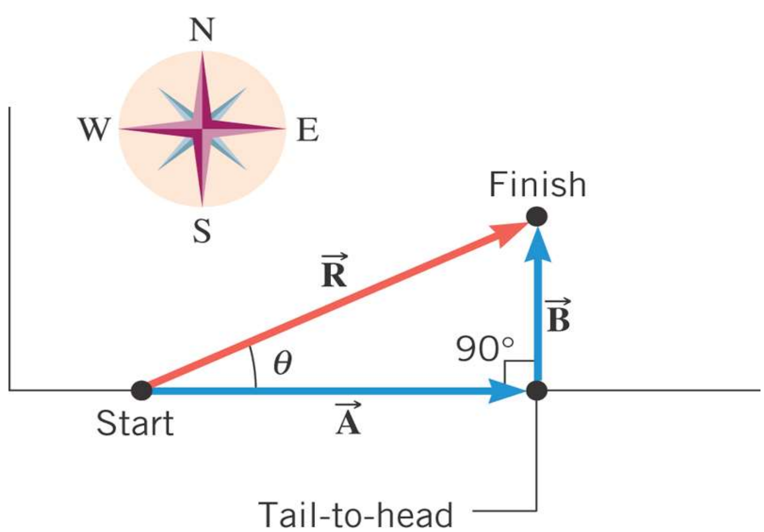
(ii) $\left|\vec{b}\right|$

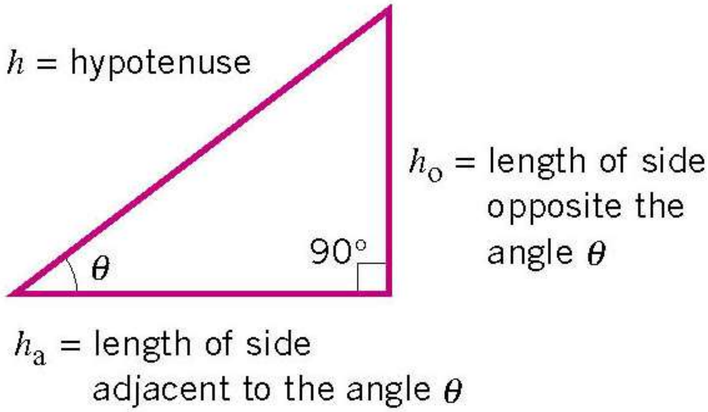
(iii) $\left|\vec{a}+\vec{b}\right|$






## 2. When the Vectors are perpendicular to each other (q=90)






$$\sin\theta = \frac{h_o}{h} \qquad \cos\theta = \frac{h_a}{h} \qquad \tan\theta = \frac{h_o}{h_a}$$

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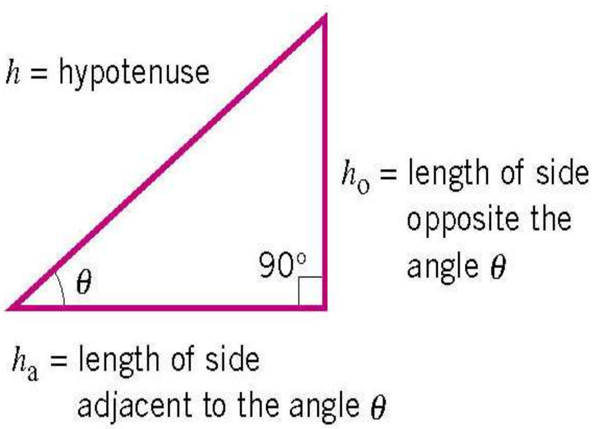


$$\theta = \sin^{-1}\left(\frac{h_o}{h}\right)$$

$$\theta = \cos^{-1}\left(\frac{h_a}{h}\right)$$

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right)$$

**Pythagorean theorem:** $$h^2 = h_o^2 + h_a^2$$

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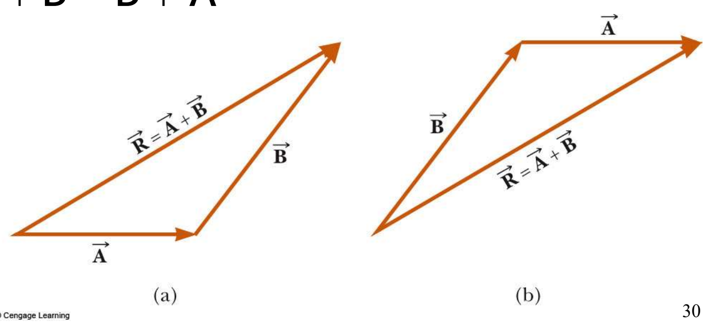
## Notes about Vector Addition

1. **Commutative Law of Addition**
   - The order in which the vectors are added doesn't affect the result
   - $\vec{A} + \vec{B} = \vec{B} + \vec{A}$



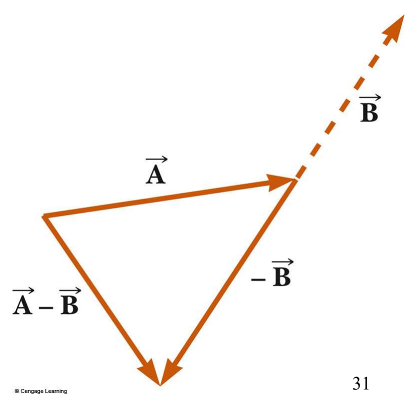




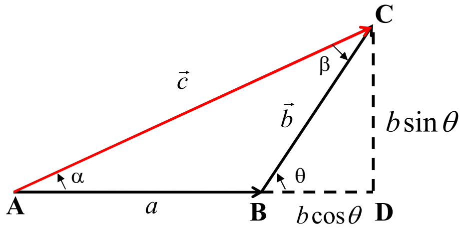

## 2- Vector Subtraction

**Special case of vector addition**

- Add the negative of the vector
- $\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$



# Cosine Law




$$(\overrightarrow{AC})^2 = (\overrightarrow{AD})^2 + (\overrightarrow{DC})^2$$

**but** $(\overrightarrow{AD}) = (\overrightarrow{AB}) + (\overrightarrow{BD})$ **and** $(\overrightarrow{DC}) = b\sin\theta$

**So that** $c^2 = [a + b\cos\theta]^2 + [b\sin\theta]^2$

**or**

$$c^2 = a^2 + b^2 + 2ab\cos\theta$$

**then**

$$c = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

***Cosine Law***

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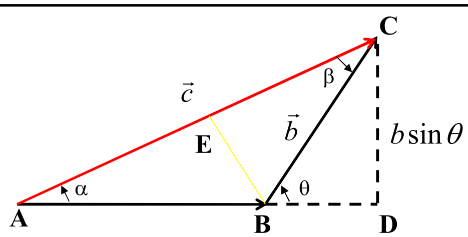
# Sine Law




In ΔACD, $CD = AC\sin\alpha$ & ΔBDC, $CD = BC\sin\theta$

$\therefore$ $c\sin\alpha = b\sin\theta$ **or** $\dfrac{b}{\sin\alpha} = \dfrac{c}{\sin\theta}$

Similarly, $BE = a\sin\alpha = b\sin\beta$ **or** $\dfrac{b}{\sin\alpha} = \dfrac{a}{\sin\beta}$

$$\therefore \quad \frac{c}{\sin\theta} = \frac{b}{\sin\alpha} = \frac{a}{\sin\beta}$$

***Sine Law***

***This called Sine law, To find the direction of or*** $\vec{a} + \vec{b}$

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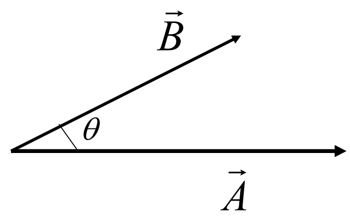
## 3- Scalar Product (Dot Product)

- The dot product of vectors **A** and **B** written **A.B** and read "**A** dot **B**" is defined as the product of the magnitudes of A and B and the cosine of the angle between their tails.

$$\vec{A}.\vec{B} = AB\cos\theta$$




where $0° \le \theta \le 180°$.

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## 8. Laws of Operation

- There are three (3) Laws of Operations:

1. Commutative law:

   A • B = B • A

2. Multiplication by a scalar law:

   a(A•B) = (aA)•B = A•(aB) =(A•B)a

3. Distributive law:

   A • (B + D) = (A • B) + (A • D)

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## 9.Cartesian Vector Formulation

- We can now find the dot product for each of the Cartesian unit vectors, i.e.:

$\mathbf{i}\bullet\mathbf{i} = (1)(1)\cos 0° = 1$ and $\mathbf{i}\bullet\mathbf{j} = (1)(1)\cos 90° = 0.$

In similar way, we can see that:

| | | |
|---|---|---|
| $\mathbf{i}\bullet\mathbf{i} = 1$ | $\mathbf{j}\bullet\mathbf{j} = 1$ | $\mathbf{k}\bullet\mathbf{k} = 1$ |
| $\mathbf{i}\bullet\mathbf{j} = 0$ | $\mathbf{j}\bullet\mathbf{k} = 0$ | $\mathbf{k}\bullet\mathbf{i} = 0$ |

$$\begin{aligned}\mathbf{A}\bullet\mathbf{B} &= (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k})(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\ &= A_xB_x\,(\mathbf{i}\bullet\mathbf{i}) + A_xB_y\,(\mathbf{i}\bullet\mathbf{j}) + A_xB_z\,(\mathbf{i}\bullet\mathbf{k}) \\ &\quad + A_yB_x\,(\mathbf{j}\bullet\mathbf{i}) + A_yB_y\,(\mathbf{j}\bullet\mathbf{j}) + A_yB_z\,(\mathbf{j}\bullet\mathbf{k}) \\ &\quad + A_zB_x\,(\mathbf{k}\bullet\mathbf{i}) + A_zB_z\,(\mathbf{k}\bullet\mathbf{j}) + A_zB_z\,(\mathbf{k}\bullet\mathbf{k})\end{aligned}$$

$\mathbf{A}\bullet\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$

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## 9.Cartesian Vector Formulation (Scalar Product)

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$(4\hat{i} + 3\hat{j} + 2\hat{k}) \bullet (3\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\vec{a}\cdot\vec{b} = 12 + (-6) + 4 = 10$$

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## Some Applications

- **To test whether two vectors are perpendicular.**

*If the dot product of two non zero vectors is zero, the vectors are perpendicular.*

$$(3\hat{i} + 2\hat{j} - 1\hat{k}) \bullet (\hat{i} - 2\hat{j} - 1\hat{k})$$
$$= 3 - 4 + 1 = 0$$

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## Some Applications

- **Finding the angle between two vectors.**

$$\vec{a} \bullet \vec{b} = ab\cos\theta$$

$$\cos\theta = \frac{\vec{a} \bullet \vec{b}}{ab}$$

$$\theta = \cos^{-1}\left(\frac{\vec{a} \bullet \vec{b}}{ab}\right)$$

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## What is the angle between $\vec{a}$ and $\vec{b}$ ?

$$\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k} \qquad \vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$$

$$a = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

$$b = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = 7$$

$$\vec{a} \bullet \vec{b} = 12 - 6 - 2 = 4$$

$$\cos\theta = \frac{4}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right) = 79^\circ$$

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