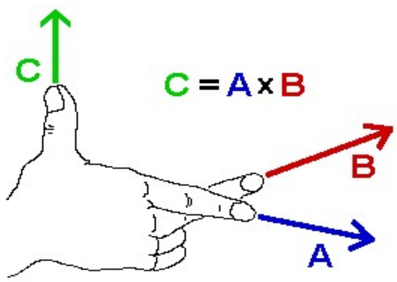
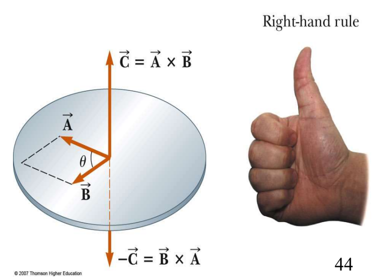
10- Vector Product

1. Vector Product uses the right hand rule
2. Direction of vector product is perpendicular to the two vectors that multiply together
3. Define vector product at the component level first Cross your middle finger under your index finger. Your middle finger represents the second vector. **Your thumb is then the resultant.**





10- Vector Product

1. Given two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$
2. The vector (cross) product of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is defined as a *third vector* **C** is read as "**A** cross **B**"
3. The magnitude of vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$

$$|\vec{C}| = |\vec{A} \times \vec{B}| = AB\sin\theta$$

θ is the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$

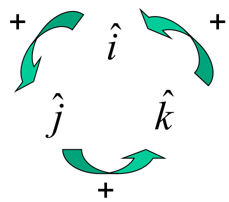
If $\boldsymbol{A = A_x i + A_y j + A_z k}$ ***and*** $\boldsymbol{B = B_x i + B_y j + B_z k}$

$$\vec{A}\times\vec{B} = \ldots \det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix} = (A_yB_z - B_yA_z)\hat{i} - (A_xB_z - B_xA_z)\hat{j} + (A_xB_y - B_xA_y)\hat{k}$$

Magnitude only… $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi$

11- Multiplication of Unit Vectors

$$\hat{i}\times\hat{i}=0 \qquad \hat{j}\times\hat{j}=0 \qquad \hat{k}\times\hat{k}=0$$
$$\hat{i}\times\hat{j}=\hat{k} \qquad \hat{j}\times\hat{k}=\hat{i} \qquad \hat{k}\times\hat{i}=\hat{j}$$
$$\hat{j}\times\hat{i}=-\hat{k} \qquad \hat{k}\times\hat{j}=-\hat{i} \qquad \hat{i}\times\hat{k}=-\hat{j}$$



Can multiply by components, but it usually takes longer and is easier to make mistakes $A_x\hat{i}\times B_y\hat{j}=A_xB_y\hat{k}$

Prove that

AxB= i($A_y.B_z$-$A_z.B_y$) –j($A_x.B_z$-$B_x.A_z$) +k($A_x.B_y$-$B_x.A_y$)

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Example:

Find the cross product of $\vec{A}$ and $\vec{B}$

$$\vec{A}=(12\,m)\hat{i}+(23\,m)\hat{j}+0\hat{k}$$
$$\vec{B}=(-31\,m)\hat{i}+(18\,m)\hat{j}+0\hat{k}$$

Only in the xy-plane

$$\vec{A}\times\vec{B}=$$

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12- Properties of the Vector Product

1. The vector product is *not* commutative. The order in which the vectors are multiplied is important
   – To account for order, remember $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

2- If $\vec{A}$ is parallel to $\vec{B}$ ($\Theta = 0^o$ or 180^o), then $\vec{A} \times \vec{B} = 0$

Therefore $\vec{A} \times \vec{A} = 0$

3. If $\vec{A}$ is perpendicular to $\vec{B}$, then $\left|\vec{A} \times \vec{B}\right| = AB$

4- The vector product obeys the distributive law $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

13-Vector Multiplication Differences

Dot Product

-result is a scalar

-projection of one vector onto the other

$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\phi}$$

Cross Product

-result is a vector

-resultant vector is perpendicular to both vectors

$$\boxed{\left|\vec{A} \times \vec{B}\right| = AB\sin\phi}$$

1- Branches of physics

1. Classical physics

- **Is generally concerned with matter and energy on the normal scale of observation.**
- **Includes the traditional branches and topics that were recognized and well-developed before the beginning of the 20th century — Classical mechanics, acoustics, optics, thermodynamics, and electromagnetism.**

2- Modern physics

- Is concerned with the behavior of matter and energy under extreme conditions or on a very large or very small scale.

- For example, atomic and nuclear physics studies matter on the smallest scale at which chemical elements can be identified.

2-Classical Mechanics:

Is concerned with bodies acted on by forces and bodies in motion and may be divided into:

A-Statics Mechanics :

Study of the forces on a body or bodies not subject to an acceleration).

- The branch of mechanics that deals with forces in the absence of changes in motion.

-Is the branch of mechanics that is concerned with the analysis of loads (force and torque, or "moment") acting on physical systems that do not experience an acceleration ($a=0$), but rather, are in static equilibrium with their environment.

-When in static equilibrium, the acceleration of the system is zero and the system is either at rest, or its center of mass moves at constant velocity.

- The application of Newton's second law to a system gives: $F=ma$

- The assumption of static equilibrium of a = 0 leads to: $F=0$

B- Kinematics Mechanics :

- Study of motion without regard to its causes.
- The branch of mechanics concerned with the motion of objects without reference to the forces which cause the motion.
- Analyzes the positions and motions of objects as a function of time, without regard to the causes of motion. It involves the relationships between the quantities displacement (d), velocity (v), acceleration (a), and time (t). The first three of these quantities are vectors. is often referred to as the "geometry of motion" and is occasionally seen as a branch of mathematics.
- Describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without considering the forces that caused the motion.

C- Dynamics:

-A branch of mechanics that deals with forces and their relation primarily to the motion but sometimes also to the equilibrium of bodies.

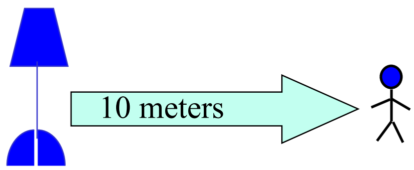
- Study of motion and the forces that affect it.
- Is concerned with the relationship between motion of bodies and its causes.
- Namely the forces acting on the bodies and the properties of the bodies, particularly mass and moment of inertia.

Dynamics is a branch of mechanics that deals with the “motion” of bodies under the action of forces. Dynamics has two different parts:

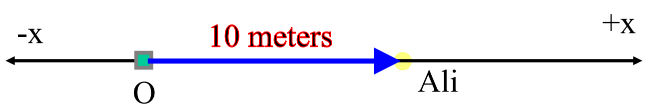
2-Motion in One-Dimension (Kinematics):

2-1 Position

1. Position is usually measured and referenced to an origin:



- At time= 0 seconds Ali is 10 meters to the right of the lamp
- Origin → lamp
- Positive direction → to the right of the lamp
- Position vector **:**



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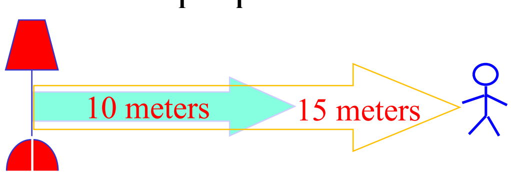
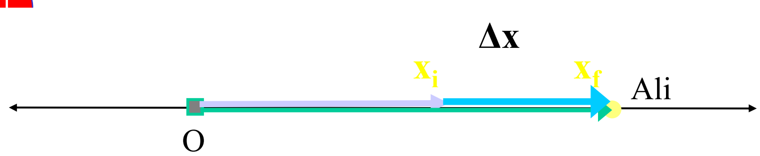
2-2 Displacement

1. One second later Ali is 15 meters to the right of the lamp
2. Displacement is just change in position
   - $\Delta x = x_f - x_i$





$x_f = x_i + \Delta x$

$\Delta x = x_f - x_i$ = 5 meters to the right !

$\Delta t = t_f - t_i$ = 1 second

Relating Δx to Δt yields velocity

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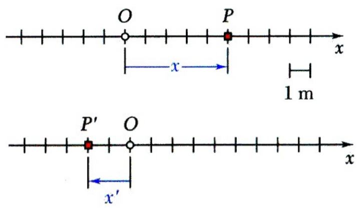
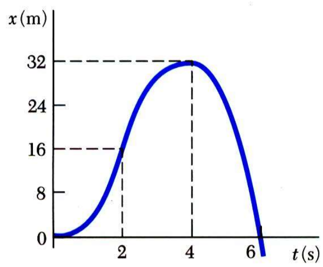
3- Rectilinear Motion: **Position**



- **Particle moving along a straight line is said to be in *rectilinear motion*.**
- *Position coordinate* of a particle is defined by positive or negative distance of particle from a fixed origin on the line.



- The *motion* of a particle is known if the position coordinate for particle is known for every value of time t. Motion of the particle may be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$

or in the form of a graph x vs. t.

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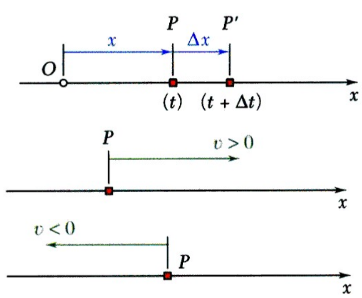
3- Rectilinear Motion: **Velocity**

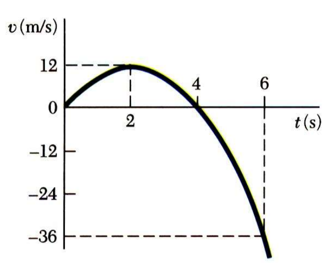


- Consider particle which occupies position P at time t and P' at $t+\Delta t$,

$$\textit{Average velocity} = \frac{\Delta x}{\Delta t}$$

$$\textit{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as *particle speed*.



- From the definition of a derivative,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

e.g., $x = 6t^2 - t^3$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

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4- Average Velocity

Changes in position vs Changes in time

- Average velocity = net distance covered per total time, includes BOTH magnitude and direction

$$\bar{v}(\text{average velocity}) = \frac{\Delta x(\text{net displacement})}{\Delta t(\text{total time})}$$

$$\bar{v}(\text{average velocity}) = \frac{\Delta x(5\text{ m to the right})}{\Delta t(1\text{ sec})}$$

- Ali's average velocity was 5 m/s to the right

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5- Average Speed

- Speed, s, is usually just the magnitude of velocity.
  - The "how fast" without the direction.
- However:
  Average speed references the total distance travelled

$$\bar{s}(\text{average speed}) = \frac{\text{distance taken along path}}{\Delta t(\text{total time})}$$

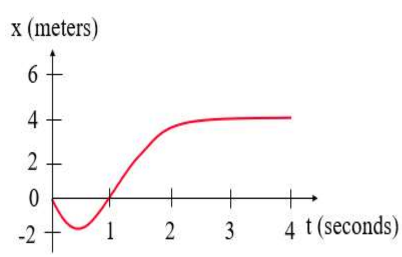
- Ali's average speed was 5 m/s

Exercise 1 Average Velocity

What is the magnitude of the **average** velocity over the first 4 seconds ?

(A) -1 m/s (C) 1 m/s

(B) 4 m/s (D) not enough information to decide.



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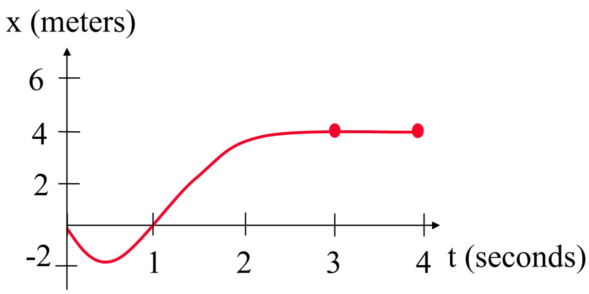
Average Velocity Exercise 2

What is the average velocity in the last second (t = 3 to 4) ?



A. 2 m/s
B. 4 m/s
C. 1 m/s
D. 0 m/s

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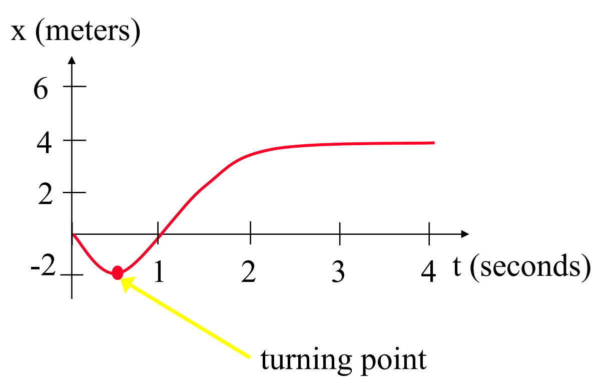
Average Speed Exercise 3

What is the average speed over the first 4 seconds ?
0 m to -2 m to 0 m to 4 m → 8 meters total

A. 2 m/s
B. 4 m/s
C. 1 m/s
D. 0 m/s



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