Question Banks

Question 1. *Consider the following system of linear differential equations*

$$\begin{pmatrix} 6 & 5 \\ 2 & -3 \end{pmatrix}.$$

Find the special fundamental matrix $\psi(t)$ which satisfies $\psi(0) = I$.

Question 2. *Show that for any matrix B, we have $Be^B = e^B B$.*

Question 3. *Check whether the following functions satisfy the Lipschitz condition on the respective intervals. If so, find a suitable Lipschitz constant.*

1. $f(t, x) = 2tx^{-4}, \quad (t, x) \in [0, \infty] \times [1, \infty]$.
2. $f(t, y) = \cos(t) + y^3, \quad t \in [0, 1] \times [1, \infty],\ |y| \leq \infty$

Question 4. *Show that every function of the form $y = \frac{1}{x}e^{cx}$, where c is a constant is a solution of the differential equation $xy' + y - y\ln(xy) = 0$ for all $x \neq 0$.*

Question 5. *Find the general solution of the system*

$$Y' = \begin{pmatrix} 2 & -3 & 3 \\ 0 & 5 & -3 \\ 0 & 6 & -4 \end{pmatrix} Y.$$

Question 6. *Find a fundamental matrix for the system*

$$x' = 4x + 2y,$$

$$y' = 3x - y.$$

Then use it to find the solution that satisfies the initial condition $x(0) = 1$ *and* $y(0) = -1$.

Question 7. *Consider the* 3×3 *matrix*

$$Y' = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}.$$

Find a fundamental matrix.

Question 8. *Solve the following differential equations:*

1) $ydx - xdy = xydx$.

2) $(x + y)(dx - dy) = dx + dy$.

3) $x^2(1 - y)dx + y^2(1 + x)dy = 0$.

4) $3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$.

Question 9. *Find the general solution of the system*

$$Y' = \begin{pmatrix} -6 & -7 & -13 \\ 5 & 6 & 9 \\ 2 & 2 & 5 \end{pmatrix} Y.$$

Question 10. *Solve the initial value problem*

$$\begin{aligned} x' &= 2x + 5y + e^t, \qquad x(0) = 1 \\ y' &= x - 2y - 1, \qquad y(0) = -1. \end{aligned}$$

Question 11. *Solve the linear system*

$$\begin{aligned} \dot{x} &= x + 4y, \\ \dot{y} &= -x - 3y. \end{aligned}$$

Using exponential matrix method.

Question 12. *Solve the following differential equation using method of successive approximation*

$$\frac{dy}{dx} = 4xy, \quad y(0) = 3$$

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Question 13. *Discuss the existence and unique solution for the initial value problem*

$$y' = \frac{2y}{x}, \quad y(x_0) = y_0.$$

Question 14. *Find the eigenvalues and the eigenfunctions of*

$$\frac{d^y}{dx^2} + \lambda y = 0, \quad 0 < x < 1, \quad y(0) = 0, \; y(1) = 0$$

.

Question 15. *Express the boundary value problem* $y'' + \lambda y = 0$, $0 < x < \pi$ *which satisfy the boundary conditions to* $y(0) = 0$, $y'(\pi) = 0$ *into a Strum-Liouville problem.*