Question Banks

Question 1. *How to interpret the differential equation*

$$\frac{dy}{dt} = 0.028y - 10.$$

Question 2. *Give an example of a differential equation for which a degree is not defined.*

Question 3. Question: *Is it possible for a differential equation to have more than one dependent variable?*

Question 4. *Show that every function of the form* $y = \frac{1}{x}e^{cx}$*, where* c *is a constant is a solution of the differential equation* $xy' + y - y\ln(xy) = 0$ *for all* $x \neq 0$*.*

Question 5. *Does a given differential equation have always a solution over an interval?*

Question 6. *Eliminate the constant* a *from the equation* $(x-a)^2 + y^2 = a^2$

Question 7. *Eliminate* α *and* β *from the relation* $x = \beta\cos(\omega t + \alpha)$*, in which* ω *is a parameter (not to be eliminate).*

Question 8. *Solve the following differential equations:*

1) $ydx - xdy = xydx.$

2) $(x + y)(dx - dy) = dx + dy.$

3) $x^2(1 - y)dx + y^2(1 + x)dy = 0.$

4) $3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0.$

Question 9. *Suppose that* $\frac{dy}{dx} = g(\frac{y}{x})$*, derive a formula for solving this type of differential equation.*

Question 10. *Solve the following differential equations:*

1) $xydx + (x^2 + y^2)dy = 0.$

2) $(x^2 + xy + y^2)dx - xydy = 0.$

3) $y' = \frac{x+y}{x-y}.$

4) $\frac{dy}{dx} = \frac{xe^{y/x}+y}{x}.$

5) $(2x \sinh(\frac{y}{x}) + 3y \cosh(\frac{y}{x}))dx - 3x \cosh(\frac{y}{x})dy = 0.$

Question 11. *Solve the following differential equations:*

1) $(y - 2)dx - (x - y - 1)dy = 0.$

2) $(x - 4y - 9)dx + (4x + y - 2)dy = 0.$

3) $(x + y - 1)dx + (2x + 2y + 1)dy = 0.$

Question 12. *Solve the following differential equations:*

1) $(\cos x \cos y - \cot x)dx - \sin x \sin y dy = 0.$

2) $2xydx + (x^2+1)dy = 0$.

3) $\frac{dy}{dx} = -\frac{3x^2+4xy}{2x^2+2y}$.

4) $y' = (xy^2-1)/(1-x^2y)$.

Question 13. *Solve the following differential equations (Find the general solution of the following):*

1) $y\frac{dx}{dy} + 2x = y^3$.

2) $x\frac{dy}{dx} + y = x$.

3) $y' + \tan(x)\, y = \cos^2(x)$, *over the interval* $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

4) $3xy' - y = \ln(x) + 1$, $x > 0$ *satisfying* $y(1) = -2$.

Question 14. *Solve the following differential equations:*

1) $y(6y^2 - x - 1)dx + 2xdy = 0$.

2) $\frac{dy}{dx} + y = (xy)^2$.

3) $xy - \frac{dy}{dx} = y^3 e^{-x^3}$.

Question 15. *Solve the following differential equations:*

1) $\frac{dy}{dx} = -\frac{x^2+2xy+y^2}{1+(x+y)^2}$.

2) $\frac{dy}{dx} - (3x-2y)^3 = 0$.

Question 16. *Formulate the following and solve them:*

1) The slope at any point (x, y) of a curve is $\frac{y}{x}$ and it passes through the point $(2, 3)$. Find the equation of the curve.

2) During a chemical reaction, substance A is converted into substance B at a rate that is proportional to the square of the amount of A. When 60 grams of A are present, and after 1 hour only 10 grams of A remain unconverted. How much of A is present after 2 hours?

3) Suppose that a petri dish initially contains 3000 bacteria and that 12 minutes later there are 3500 bacteria.
a) Find a formula for the bacteria population t hours (not minutes) after the initial measurement.

b) Predict the bacteria population in 4 hours.

4) Let $N(t)$ be the number of people at time t. Assume that the land is intrinsically capable of supporting L people and that the rate of increase is proportional to both N and $L - N$.

Question 17. *Solve the following differential equations. 1)* $(x^3 + x^2 + x + 1)p^2 - (3x^2 + 2x + 1)yp + 2xy^2 = 0.$
2) $y + x\frac{dy}{dx} - x^4(\frac{dy}{dx})^2 = 0.$

3) $y = y^2(y')^3 + 2y'x.$

Question 18. *Can you give an example that two function are linearly independent even that their Wronskian is zero?*

Question 19. *Solve the following differential equation:*

1) $y'' + y = 0.$

2) $y'' - 4y' + 4y = 0.$

3) $y'' - 7y' = 0.$

4) $y'' - 2\sqrt{2}y' + 2y = 0.$

5) $4y'' + 4y' + y = 0.$

Question 20. *Solve the following differential equations:*

1) $y^{(6)} - y^{(5)} + 2y^{(4)} - 2y''' + y'' - y' = 0.$

2) $(D^3 + 1)y = 0.$

3) $(D^3 + 2D^2 - 5D - 6)y = 0.$

4) $(D^4 + 4D)y = 0.$

5) $(D^5 - 5D^4 + 12D^3 - 16D^2 + 12D - 4)y = 0.$

6) $y^{(5)} - y^{(4)} + 4y' - 4y = 0.$

Question 21.

$$(D^3 + aD^2 + bD + c)y = 0,$$

where a, b and c are constants, has a solution

$$y = C_1 e^{-x} + e^{-2x}(C_2 \sin 4x + C_3 \cos 4x).$$

Determine the values of a, b and c.

Question 22. *Solve the following differential equations:*

1) $(2D^2 + 2D + 3)y = x^2 + 2x - 1.$

2) $(D^3 - 2D + 4)y = x^4 + 3x^2 - 5x + 2.$

Question 23. *Solve* $(D^2 - 2D + 2)y = e^x \sin x.$

Question 24. *Solve the following nonhonogeneous equations:*

1) $y'' - 3y' + 2y = \frac{e^{3x}}{e^x + 1}.$

2) $y'' + 2y' + y = e^{-x} \ln x.$

Question 25. *Solve the following differential equations:*

1) $(x^2 D^2 - xD + 4)y = \cos \ln x + x \sin \ln x.$

2) $[(3x + 2)^2 D^2 + 3(3x + 2)D - 36]y = 3x^2 + 4x + 1.$

Question 26. *Solve the following differential equations:*

1) $y'' - 2xy' + (x^2 + 2)y = e^{\frac{1}{2}(x^2 + 2x)}.$

2) $(1 + x)^2 y'' + (x + 1)(x - 2)y' + (2 - x)y = 0.$

Question 27. *Use Laplace transforms method to solve the following.*

1. *Solve*

$$y'' + y = 4te^t,$$

subject to the initial condition $y(0) = -2$ *and* $y'(0) = 0$.

2. *Solve the initial value problem*

$$x'' - x' - 6x = 0; \quad x(0) = 2, \quad x'(0) = -1.$$

3. *Solve the initial value problem*

$$y'' + 4y = \sin 3t; \quad y(0) = y'(0) = 0.$$