1. **Binomial theorem**
2. Find the first four terms, in ascending powers of x, in the binomial expansion of$(1+3x)^8$.
3. Determine the coefficient of x^6in the binomial expansion of $(1+3x)^8$,
4. Find, without using a calculator, the coefficient of x^3in the expansion of $(2+3x)^6$.
5. The third term in the expansion of $(x+p)^8$ is $252x^6$ .find the possible values of p
6.

(a) Show that $(2n-1)^3+(2n+1)^3=16n^3+12n$ for $n \in \mathbb{Z}$.

(b) Hence, or otherwise, prove that the sum of the cubes of any two consecutive odd integers is divisible by four.

7.

Consider the expansion of $\left(\frac{x^2}{2}+\frac{a}{x}\right)^6$. The constant term is 960.

Find the possible values of a.

8.

Consider the expansion of $x\left(2x^2+\frac{a}{x}\right)^7$. The constant term is 20 412. Find a.

9.

Consider the expansion of $\left(3x+\frac{p}{x}\right)^8$, where $p>0$. The coefficient of the term in x^4 is equal to the coefficient of the term in x^6. Find p.

10.

Consider the expansion of $\left(2x^6+\frac{x^2}{q}\right)^{10}$, $q \neq 0$. The coefficient of the term in x^{40} is twelve times the coefficient of the term in x^{36}. Find q.

11.

Given that $(5+nx)^2\left(1+\frac{3}{5}x\right)^n=25+100x+\cdots$, find the value of n.

https://www.revisionvillage.com/ib-math-analysis-and-approaches-sl/questionbank/number-and-algebra/the-binomial-theorem/

12. Prove by mathematical induction that $n! + (n-1)! + (n-2)! = n^2(n-2)!$ $for\ all\ n \geq 2$.

13.

Prove $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$ for positive values of using mathematical induction.

14.

15.

Concept Check Questions

1. Prove by Mathematical Induction that $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2}{4}(n+1)^2$ for all $n \geq 1$

2. Prove by Mathematical Induction that $2^{n+2} + 3^{3n}$ is divisible by 5 for all $n \geq 1$

3. Use Mathematical Induction to show that $cos(x + n\pi) = (-1)^n cosx$ for all positive integers $n \geq 1$.

16. Prove by Mathematical Induction that 13+23+33+...+n3=n24(n+1)2 for all n≥1

17. Prove by Mathematical Induction that 2n+2+33n is divisible by 5 for all n≥1

18. Use Mathematical Induction to show that cos(x+nπ)=(−1)ncosx for all positive integers n≥1.

https://www.matrix.edu.au/beginners-guide-year-12-maths-ext-1/mathematical-induction/

19. Test

Q1: Using synthetic division and the upper and lower bound tests, find all real zeros of the function $f(x) = 4x^4 + x^3 - 27x^2 + 18x$.

A $3, 0, -\frac{3}{4}, -2$

B $-3, 0, \frac{3}{4}, 2$

C $-3, 0, -\frac{4}{3}, -2$

D $3, \frac{3}{4}, 2$

E $-3, 0, \frac{4}{3}, 2$

20.

Q2: Noura is trying to find zeros in the function $f(x) = 6x^4 + 19x^3 - 37x^2 - 62x + 24$. She has used synthetic division to find $f(a)$ for $a = -5, -2, 1$, and 3.

−5	6	19	−37	−62	24
		−30	55	−90	760
	6	−11	18	−152	784

1	6	19	−37	−62	24
		6	25	−12	−74
	6	25	−12	−74	−50

−2	6	19	−37	−62	24
		−12	−14	102	−80
	6	7	−51	40	−56

3	6	19	−37	−62	24
		18	111	222	480
	6	37	74	160	504

21.

Use her results to state an interval in which all real zeros of f lie.

A $[-2, 1]$

B $[1, 3]$

C $[-2, 3]$

D $[-5, 3]$

E $[-5, 1]$

22.

Q3: Consider the function $f(x) = x^4 - 4x^3 - 7x^2 + 74x - 104$.

Fawaz is using synthetic division to help him find real zeros of f.

−5	1	−4	−7	74	−104
		−5	45	−190	580
	1	−9	38	−116	476

2	1	−4	−7	74	−104
		2	−4	−22	−104
	1	−2	−11	−52	0

23.

▸ What can he conclude about −5?

A That it is an upper bound on the interval in which all real zeros lie

B That it is a lower bound on the interval in which all real zeros lie

C That it is neither upper bound nor lower bound on the interval in which all real zeros lie

D That it is a real zero of f

▸ What can he conclude about 2?

A That it is a real zero of f

B That it is a lower bound on the interval in which all real zeros lie

C That it is the only real zero of f

24.

▸ Find all the real zeros of f.

A 2, 13

B −2, 2

C 1, 4

D −13, 1

E −4, 2

25.

Q4: Use synthetic division to determine whether $x = 6$ is an upper bound, a lower bound, or neither an upper nor a lower bound on the interval in which all real zeros of the polynomial function $f(x) = x^3 + 3x^2 - 34x - 42$ lie.

A It is a lower bound.

B It is neither an upper nor a lower bound.

C It is an upper bound.

26.

Q5: Which the following is the interval in which all real zeros of the polynomial function $f(x) = x^3 + 9x^2 + 23x + 15$ lie?

A $[-10, -2]$

B $[-9, -1]$

C $[-4, 1]$

D $[-11, -3]$

E $[-2, 2]$

27.

Q6: If $x = 1$ is an upper bound for the real zeros of $f(x)$, which of the following can be $f(x)$?

A $f(x) = x^3 - 4x^2 + x + 6$

B $f(x) = 6x^3 + 19x^2 + 2x - 3$

C $f(x) = 2x^3 - 13x^2 + 17x + 12$

D $f(x) = x^3 - 5x^2 + 2x + 8$

E $f(x) = 3x^3 - 2x^2 - 7x - 2$

28.

Q7: If the numbers in the bottom row of the synthetic division of $f(x)$ by $x - 3$ are alternately positive and negative (zero entries count as positive or negative), where $f(x)$ is a polynomial with real coefficients and a positive leading coefficient, which of the following must be true?

A $x = 3$ is an upper bound for the real zeros of f.

B $x = 3$ is a lower bound for the real zeros of f.

C $x = 3$ is a real zero of f.

D None of the above

29.

Q8: If each number in the bottom row of the synthetic division of $f(x)$ by $x+1$ is either positive or zero, where $f(x)$ is a polynomial with real coefficients and a positive leading coefficient, which of the following must be true?

A $x=-1$ is a lower bound for the real zeros of f.

B $x=-1$ is an upper bound for the real zeros of f.

C $x=-1$ is a real zero of f.

D None of the above

30.

Q9: If the numbers in the bottom row of the synthetic division of $f(x)$ by $x+3$ are alternately positive and negative (zero entries count as positive or negative), where $f(x)$ is a polynomial with real coefficients and a positive leading coefficient, which of the following must be true?

A $x=-3$ is neither an upper bound nor a lower bound for the real zeros of f.

B $x=-3$ is an upper bound for the real zeros of f.

C $x=-3$ is a lower bound for the real zeros of f.

D $x=-3$ is a real zero of f.

E None of the above

31.

Q10: If each number in the bottom row of the synthetic division of $f(x)$ by $x-1$ is either positive or zero, where $f(x)$ is a polynomial with real coefficients and a positive leading coefficient, which of the following must be true?

A $x=1$ is a real zero of f.

B $x=1$ is a lower bound for the real zeros of f.

C $x=1$ is an upper bound for the real zeros of f.

D $x=1$ is neither an upper bound nor a lower bound for the real zeros of f.

E None of the above

32.

Prove all the followings from 33-16

33. *The commutative laws:*

34. $z_1 + z_2 = z_2 + z_1$

35. $z_1 z_2 = z_2 z_1$ *hw*

36. *The associative laws:*

37. $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$,

38. $(z_1 z_2) z_3 = z_1(z_2 z_3)$

39. *The distributive law:* $z(z_1 + z_2) = zz_1 + zz_2$

40 identity

The additive identity 0 = (0,0) and the multiplicative identity 1 = (1,0)

That is,$z + 0 = z$ *and* $z \cdot 1 = z$

40. *for every complex number z . Furthermore, 0 and 1 are the only complex numbers with such properties*

41. *Inverse*

For each complex number $z = (x, y)$ *a unique additive inverse* $-z = (-x, -y)$

- *satisfying the equation* $z + (-z) = 0$.

42. *For* ***any nonzero*** *complex number* $z = (x, y)$, *there is a number* z^{-1} *such that*$zz^{-1} = 1$ *where* $z^{-1} = \left(\frac{x}{x^2+y^2}, \frac{-y}{x^2+y^2}\right)$*to here*

Find the followings from 17-23

43. Cube root of $1 - i$
44. Forth roots of $-i$
45. Square roots of $\frac{-1-\sqrt{3}i}{-1+i}$
46. $(-8i)^{\frac{1}{3}}$
47. $z = \sqrt{\sqrt{6} - \sqrt{2}i}$
48. $z = 1^{1/3}$
49. $z^2 + 2z - 1 = 0$

50. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$
51. Solve the equation $9x^3 - 36x^2 + 40x - 16 = 0$if the roots form $\alpha - \beta, \alpha, \alpha + \beta$
52. Solve the cubic equation $3x^2 - 26x^2 + 52x - 24$ if the roots of the form $\alpha\beta^{-1}, \alpha, \alpha\beta$
53. Find all roots of $2x^3 + 4x^2 + 6x + 4 = 0$ if the sum of second and third roots is equal to -1.
54. Find the sum of squares of roots of the equation $x^3 - 2x^2 - 5x + 6 = 0$.

55. Q: Let $B = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ with $|B| = m \; where \; m \epsilon \mathbb{R}$ then calculate the following determinants with explain

[4.5 Marks]

56. $|C|$ where $C = \begin{bmatrix} 2a_{11} & 6a_{12} & 2a_{13} \\ a_{21} & 3a_{22} & a_{23} \\ a_{31} & 3a_{32} & a_{33} \end{bmatrix}$

57. $|D|$ where $D = \begin{bmatrix} a_{11} & a_{12} & 2a_{11} + a_{13} \\ a_{21} & a_{22} & 2a_{21} + a_{23} \\ a_{31} & a_{32} & 2a_{31} + a_{33} \end{bmatrix}$

58. $|E|$ where $E = \begin{bmatrix} a_{13} & a_{23} & a_{33} \\ a_{12} & a_{22} & a_{32} \\ a_{11} & a_{21} & a_{31} \end{bmatrix}$

59. **Q: If** α_1 and α_2 are the roots of $a_0x^2 + a_1x + a_2 = 0$ then $\alpha_1 + \alpha_2 = \frac{-a_1}{a_0}$ and $\alpha_1\alpha_2 = \frac{a_2}{a_0}$

60. Show that If α_1, α_2 and α_3 are the roots of $a_0x^3 + a_1x^2 + a_2x + a_3 = 0$ then
$\alpha_1 + \alpha_2 + \alpha_3 = \frac{-a_1}{a_0}$

and

$$\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1 = \frac{a_2}{a_0}$$

$$\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}$$

61. Solve the equation $3x^2 - 16x^2 + 23\,x - 6 = 0$

If the product of two roots is 1

https://www.nagwa.com/en/worksheets/529109865759/

greatest common divisior

upper bound lower bound

Find $q(x)$and $r(x)$,for $f(x)$(62-64)

62. $f(x) = x^3 - 12x^2 - 4x$, $g(x) = x - 3$
63. $f(x) = x^5 - 4x^3 + 2x^2 - 58 - 8$, $g(x) = x - 3$.
64. $f(x) = x^4 + 7x^2 + 12x$,$g(x) = x^2 - 3x + 1$.

- Fill the following blanks

65. If $|A_{3\times3}| = 5$ then $\left|\frac{2}{5}A\right| =_{(a)} -----$.
66. In cardans method the transformation (b) $-------$ reduce equation $x^3 + ax^2 + bx + c = 0$ to the norm form $y^3 + py + q = 0$.
67. $(kA)^{-1} =_{(c)} -----$for scalar k and matrix A.
68. A matrix is said to be in Reduced row echelon form it satisfies the following four conditions:
    - (d) $------$, (e) $------$, (f) $-----$and (g) $-------$.
69. For matrices A and B if $AB = 0_h$ $------$ $A = 0$ or $B = 0$.
70. If α_1, α_2 and α_3 are the roots of $a_0x^3 + a_1x^2 + a_2x + a_3 = 0$ then relation between root a are

(i) $------$, (j) $---------$, and (k) $----------$.

71. The steps of Ferrari's method to solve quartic equation $x^4 + 2ax^3 + bx^2 + cx + d = 0$ are

    (l) $------$, (m) $-------$ and (n) $------$.

72. Let A and B be two invertable matrices of order n show that $(AB)^{-1} = B^{-1}A^{-1}$.

73. Find inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

74. Use Gauss-Jordan elimination method to solve the following linear system.

$$x - y + 3z = 1$$
$$2x + y + 2z = 6$$
$$-2x - 2y + z = 3$$

75.

Example 5. *Write the following linear system in the augmented form and then solve it by using Gauss elimination method.*

$$\begin{aligned} x_1 + 2x_2 - x_3 + 4x_4 &= 12 \\ 2x_1 + x_2 + x_3 + x_4 &= 10 \\ -3x_1 - x_2 + 4x_3 + x_4 &= 2 \\ x_1 + x_2 - x_3 + 3x_4 &= 6 \end{aligned}$$

76.

Example 6. *Solve the following linear system using Gauss elimination method by using forward substitution technique*

$$\begin{aligned} x_1 + 2x_2 + x_3 + 4x_4 &= 13 \\ 2x_1 + 0x_2 + 4x_3 + 3x_4 &= 28 \\ 4x_1 + 2x_2 + 2x_3 + x_4 &= 20 \\ -3x_1 + x_2 + 3x_3 + 2x_4 &= 6 \end{aligned}$$

77.

Example 7. *Solve the following linear system using Gauss-Jordan elimination method*

$$\begin{aligned} 3x_1 + 4x_2 + 3x_3 &= 10 \\ x_1 + 5x_2 - x_3 &= 7 \\ 6x_1 + 3x_2 + 7x_3 &= 15 \end{aligned}$$

78.

Example 8. *Solve the following linear system using Gauss-Jordan elimination method*

$$\begin{aligned} -2x_1 + x_2 + 5x_3 &= 15 \\ 4x_1 - 8x_2 + x_3 &= -21 \\ 4x_1 - x_2 + x_3 &= 7 \end{aligned}$$

Solving_Systems_of_Linear_Equations.pdf

Properties of Determinants - Differentiation and Integration of Determinants (byjus.com)

79.

Question 1: Using properties of determinants, prove that

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)\left(ab+bc+ca-a^2-b^2-c^2\right)$$

80.

Question 3: Show that

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \theta & \phi & \psi \\ \lambda & \mu & v \end{vmatrix} = \begin{vmatrix} \beta & \mu & \phi \\ \alpha & \lambda & \theta \\ \gamma & v & \psi \end{vmatrix}$$

81.

Question 4: If a, b, c are all different and if

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0,$$

prove that abc = −1.

82.

Question 5: Prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

83.

Question 6: Prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Properties of Determinants - Differentiation and Integration of Determinants (byjus.com)

84.

In each of the problems 1 through 4, the augmented matrix shown has been obtained by a sequence of row operations. In each case, decide which of the following statements is true about the associated system of equations.

(A) The system has a unique solution.
(B) The system has no solution.
(C) The system has an infinite number of solutions with **one** arbitrary parameter.
(D) The system has an infinite number of solutions with **two** arbitrary parameters.
(E) None of the above.

1. $\left[\begin{array}{ccc|c} 1 & 0 & 5 & 10 \\ 0 & 3 & -3 & 6 \\ 0 & 1 & -1 & 2 \end{array}\right]$

2. $\left[\begin{array}{ccc|c} 1 & 0 & 6 & 4 \\ 0 & 3 & -3 & 6 \\ 0 & 1 & -1 & -2 \end{array}\right]$

3. $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & -1 & -4 & 0 \\ 0 & 2 & -2 & -8 & 0 \end{array}\right]$

4. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -3 & 1 \\ 0 & 0 & 1 & 1 & -2 \end{array}\right]$

85. Q

9. Which of the following systems of equations has a unique solution?

(A) $\begin{cases} 2x-3y=2 \\ -4x+6y=4 \end{cases}$ (B) $\begin{cases} 2x-3y=2 \\ 4x-6y=4 \end{cases}$ (C) $\begin{cases} 2x-3y=2 \\ 4x+6y=-4 \end{cases}$

(D) $\begin{cases} 2x-3y=2 \\ -4x+6y=-4 \end{cases}$ (E) none of the others

86.

87. Q

2. Find all solutions of the following system of equations. **Show all work.**

$$\begin{cases} 2x - 5y = 0 \\ x - 3y - z = -1 \\ -x + 2y - z = -1 \end{cases}$$

3. Find the inverse of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & 4 \end{bmatrix}$. **Show all work.**

88.

4. Select the correct statement about the three lines given by the equations:

$$x + y = 2, \quad x = -2, \quad x + 2y = 7.$$

(A) Two of these lines are parallel.
(B) Two of these lines have negative slopes, and the slope is undefined for the other line.
(C) Two of these lines have negative slopes, and the slope is zero for the other line.
(D) Two of these lines have positive slopes, and the slope is undefined for the other line.
(E) These lines all go through a single point
(F) none of the above.

89.

For each of the augmented matrices in the next three problems, determine which of the following statements is true about the associated system of linear equations:
(A) The system has no solution.
(B) The system has exactly one solution.
(C) The system has exactly two solutions.
(D) The system has infinitely many solutions in which one variable can be selected arbitrarily.
(E) The system has infinitely many solutions in which two variables can be selected arbitrarily.
(F) none of the above.

6. $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 7 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

90.

9. Determine which of the following matrices are in reduced form.

$$A = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 1 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \qquad B = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$C = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \qquad D = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

(A) C only (B) B and C only (C) A, B and C
(D) B, C and D only (E) A and C only (F) none of these

91.

14. Find all solutions to the following system of equations:

$$\begin{aligned} x - y + 3z &= 5 \\ 2x + y + 3w &= 7 \\ -x + y - 3z + 4w &= 3 \end{aligned}$$

(A) $x = 2$, $y = -3 + 3z$, z arbitrary, $w = 2$
(B) $x = 2 + z$, $y = -3 + 3z$, z arbitrary, $w = 2$
(C) $x = 3 - z$, $y = -3 + 3z$, z arbitrary, $w = 2$
(D) $x = 2 - z$, $y = -3 + 2z$, z arbitrary, $w = 2$
(E) $x = 2 - z - 2w$, $y = -3 + 2z + w$, z arbitrary, w arbitrary
(F) none of these

92.

15. A certain 3×3 matrix A has as its inverse the matrix

$$A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & -1 & 2 \end{bmatrix}.$$

Determine the value of y where

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) none of these

93.

3. Solve the following system of linear equations by using an augmented matrix and putting it into row-reduced form.

$$\begin{aligned} 2y + 6z &= 4 \\ x + 4z &= 2 \\ 3x + 2y + 10z &= 2 \end{aligned}$$

94.