Collage of Basic Education
General Science
FOURTH STAGE

Chapter Three Superconductivity

Some properties of superconductor (Lodon theory , Entropy , BCS THEORY and Cooper pairs, two fluid model energy gap and …… etc

Dr Abbas H Rostam

London Theory

Knowing the experimental properties of superconductors, London introduced a phenomenological theory that can be described as follows:

1. The superconducting material contains two fluids below T_C, $\frac{n_S(T)}{n}$ is the fraction of the electron fluid that is in the *super fluid state*

$\frac{n_N(T)}{n} = \left[1 - \frac{n_S(T)}{n}\right]$ is the fraction in the *normal state*. The total density of electrons in the superconducting material is $n = n_N + n_S$.

2. Both the normal fluid and super fluid respond to external fields, but the superfluid is dissipation less while the normal fluid is not. We can write the electrical conductivities for the normal and super fluids as follows:

$$\sigma_N = \frac{n_N e^2 \tau_N}{m}, \qquad \sigma_S = \frac{n_S e^2 \tau_S}{m} \qquad \text{but } \tau_S \to \infty \text{ giving } \sigma_S \to \infty.$$

3.

$$n_S(T) \to n \text{ as } T \to 0,$$
$$n_S(T) \to 0 \text{ as } T \to T_c.$$

$$n_S = n\left[1 - \left(\frac{T}{T_C}\right)^4\right]$$

4. To explain the Meissner effect, London proposed the *London equation*

$$\nabla \times \mathbf{j} + \frac{n_S e^2}{mc}\mathbf{B} = 0.$$

London Equations or Theory

The London theory is based on the rather old two-fluid model of superconductors. According to this theory, the superconductor can be thought of being composed of both normal electrons and superfluid electrons or superconducting electrons. The concentrations and speed of normal electrons and superfluid electrons are different in a superconductor. Let $n_n(N_n)$ = concentration of normal electrons in the superconductor, $n_{S(N_S)} =$ concentration of superfluid electrons or superconducting electrons in the superconductor, If $n\,(N_0) =$ is the concentration of all types of electrons in the superconductor then we have.

$$N_0 = N_n + N_s$$

For $T < T_C$, the total density of electrons $n = n_s + n_n$.
n_s = density of superconducting electrons;
n_n = density of normal electrons.
For $T \to 0$, $n_s \to n$; and for $T \to T_C$, $n_s \to 0$.
The normal electrons conduct with finite resistance, while the superconducting electrons have dissipationless flow.

Now the penetration depth λ can be defined, using this equation, as magnetic induction can penetrate only into a very thin layer of the superconductor and vanishes in the interior. The penetration the distance at which magnetic field *B* is reduced to 1/*e* of its initial value at the surface. This equation shows that depth λ is also found to depend strongly on temperature of the superconductor and to become much larger as $T \to T_C$. This observation can be fitted extremely well by a simple expression of the form .

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}}$$

$$\lambda^2(T) = \lambda^2(0)\,\frac{T_c^4}{T_c^4 - T^4}$$

In this equation

$$\lambda_{(0)} = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

$\lambda_{(0)}$ = pénétration depth at température 0 *T K*.

Entropy

We know that the entropy is a measure of the disorder of a system. In all superconductors, the entropy decreases significantly on cooling below the critical temperature Tc.

- **Therefore, the observed decrease in entropy between the normal state and superconducting state shows that the superconducting state is more ordered than the normal state**

. • For aluminum, the change in entropy was observed to be small of the order of 10−14 k per atom, where k is the Boltzmann constant.

- **The variation of entropy of aluminum in the normal and superconducting states with temperature is shown in Fig (2).**

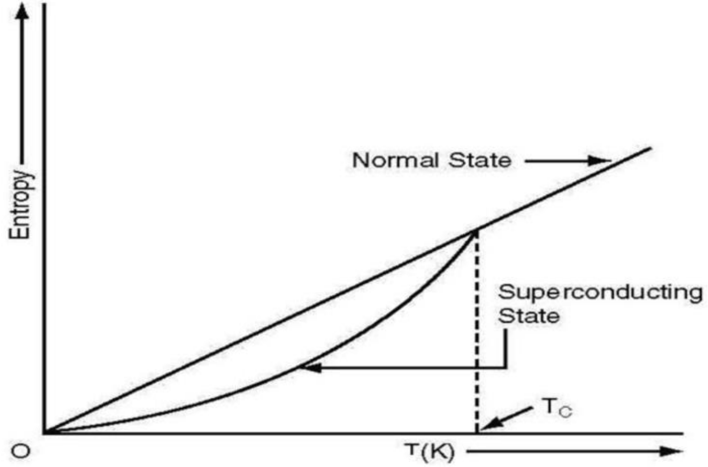


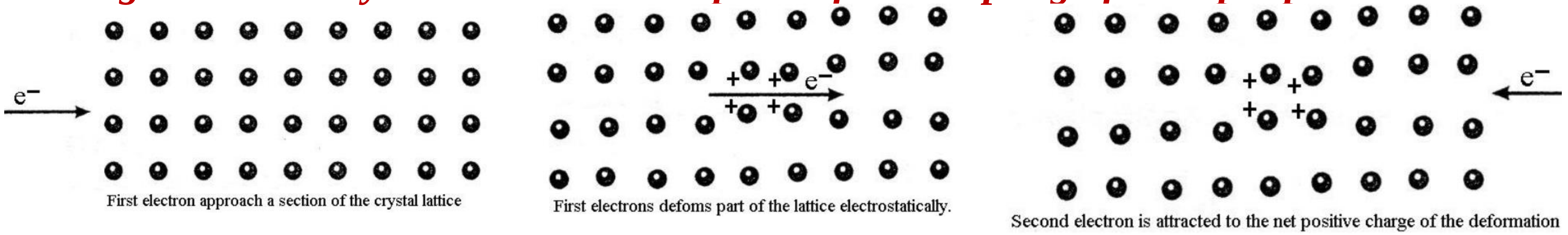
Figure (2) : Entropy S of aluminum in the normal and superconducting states as a function of the temperature. The Entropy is lower in the superconducting state because the electrons are more ordered here than the normal state. At any temperature below the critical temperature T_C the specimen can be put in the normal state by application of a magnetic field stronger than the critical field. The small entropy change must mean only a small fraction of the order of 10^{-4} of the conductor electrons participate in the transition to the ordered superconducting state.

BCS theory and Cooper pairs

According to classical physics, part of the resistance of a metal is due to collisions between free electrons and the crystal lattice's vibrations, known as phonons. In addition, part of the resistance is due to scattering of electrons from impurities or defects in the conductor.

The superconducting state of a metal may be considered to be resulting from a cooperative behavior of conduction electrons. Such a cooperation or coherence of electrons takes place when a number of electrons occupy the same quantum state. Thus, however, appears to be impossible for both statistical and dynamic reasons. First Statistically, electrons are fermions and hence occupy the quantum states singly. This impossible for electrons in cooper pairs because two electrons in the same levels or in single quantum state.

Figure: This may the classical description of the coupling of a cooper pair



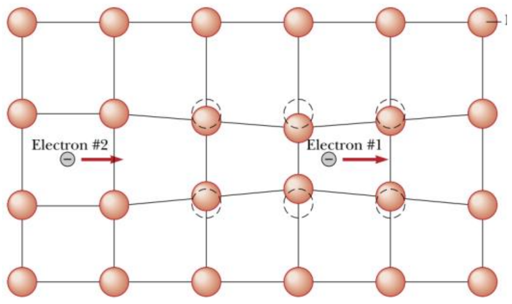


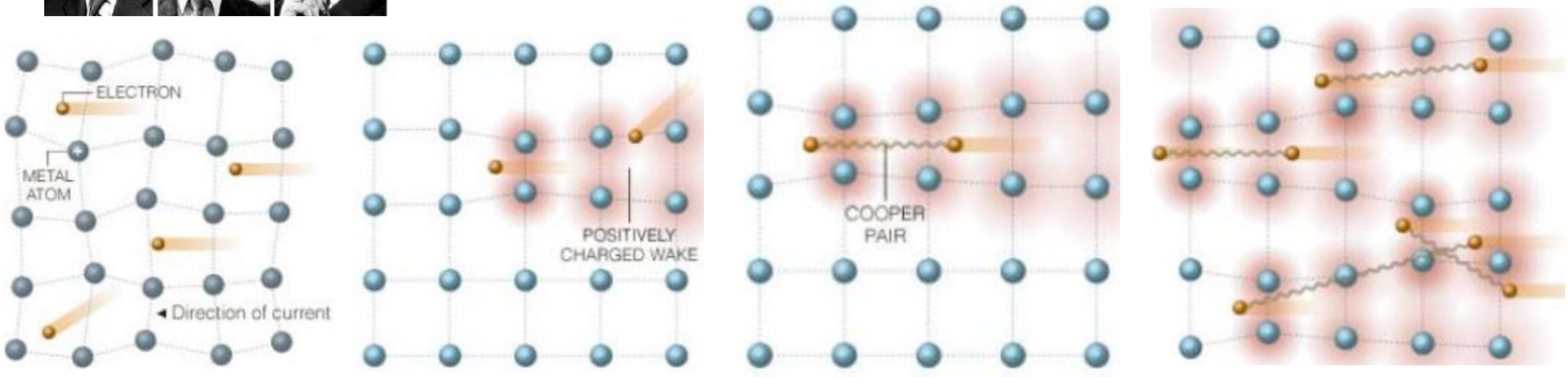
Figure : The basis for the attractive interaction between two electrons via the lattice deformation. Electron 1 attracts the positive ions, which move inward from their equilibrium position. This distorted region of the lattice has a net positive charge,

Hence the second electron (the Cooper pair partner) comes along and is attracted by the displaced ions. If the second electron pass through this distorted region, it experiences a force which is one of attraction and is of the type of polarization force.

Microscopic theory of superconductivity

Low-Temperature Superconductivity

Bardeen-Cooper-Schrieffer (BCS) theory (1957).
Nobel prize in 1972



Electrical Resistance

Electrons carring an electrical current through metal wire typically encounter resistance, which is caused by collision and scattering as the particles move through the vibrating lattice of metal atoms.

Critical Temperature

As the metal is cooled to low temperature, the lattice vibration slows. A moving electron attracts nearby metal atoms, which create a positively charged wake behind the electron. This wake can attract another nearby electron.

Cooper Pairs

The two electrons form a weak bond, called Cooper pair, which encounters less resistance than two electrons moving separately. When more Cooper pairs form, they behave in the same way.

Superconductivity

If a pair is scattered by an impurity, it will quickly get back in with other pairs. This allows the electrons to flow undisturbed through the lattice of metal atoms. With no resistanace, the current may for years.

Two fluid model

1. Assume there are two kind of carriers – normal and superconducting. In here, let the normal carriers form component 1, and the superconducting carriers form component 2.

2. Superconducting carriers are in a condensed state, they are at the lowest energy state and they carry no entropy.

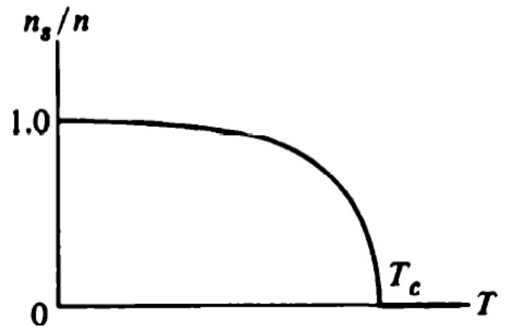
3. As a result, there is no scattering for the superconducting carriers and there is no resistance for them – they cause the phenomenon of superconductivity.

4. When $T>T_c$, all carriers are normal. When $T=0$, all carriers will be superconducting. When $0<T<T_c$, $\omega = n_s/N$ will be superconducting and $(1-\omega)$ will be normal. ω can be considered as an order parameter. We want now to determine the value of ω for the equilibrium between the two components.

In 1934 in order to explain thermodynamic properties of superconductors in fuller detail, Goter and Casimir introduced the two –fluid model of superconductivity, Many properties of superconductors can be described in terms of a two-fluid. According to this model, the conduction electrons in superconducting substance fall into two classes: superelectrons and normal electrons. The normal electrons behave as charged particle flowing in a viscous medium. The two fluids interpenetrate but do not interact

When a superconductor is cooled below T_C, normal electrons begin to transform to the super electron state. The densities of the normal and the super electrons n and n_S respectively, and sum to the total density n of the conduction electrons,

$$n_n(T) + n_S(T) = n_{Total}$$



The fraction of superelectrons $\frac{n_S}{n}$ versus temperature

But the superelectron have novel properties which endow the superconductor with its distinctive features these electrons experience no scattering have zero entropy (perfect order, and along coherence length

The number of superelectrons depends on temperature. To obtain agreement with experiment, Gorter and Casimir found that the concentration of these electrons is given by the formula

The ratio of normal electrons to electron pairs depend upon the temperature, with the concentration of paired electrons decreasing as the temperature increases and going to zero at the transition temperature.

Which is plotted in fig ( ) Thus at $\boldsymbol{T = 0\,K,}$ **all the electrons are superelectron, but as** $\boldsymbol{T}$ **increase, the superelectrons decrease in number, and eventually they all become normal electrons at** $\boldsymbol{T = T_C}$**.**

$$n_S = n\left[1 - \left(\frac{T}{T_C}\right)^4\right]$$

where at $\boldsymbol{T = 0}$ **we have** $\boldsymbol{n_n(0) = 0}$ **and** $\boldsymbol{n_n s = n}$ **.**

If we assume that from equation below is valid for any temperature below T_C,

•

Then $\lambda_{(0)} = \sqrt{\dfrac{m}{\mu_0 n(T) e^2}}$ **and we can write** $\lambda_{(T)} = \sqrt{\dfrac{m}{\mu_0 n_S(T) e^2}}$

From equation penetrate depth and equation superelectron and normal electron we obtain

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}} \qquad n_S = n\left[1-\left(\frac{T}{T_C}\right)^4\right] \qquad \frac{n_S}{n} = \left(\frac{\lambda_{(0)}}{\lambda_{(T)}}\right)^2$$

For $T < T_C$, the total density of electrons $n = n_s + n_n$.
n_s = density of superconducting electrons;
n_n = density of normal electrons.
For $T \to 0$, $n_s \to n$; and for $T \to T_C$, $n_s \to 0$.
The normal electrons conduct with finite resistance, while the superconducting electrons have dissipationless flow.

What is the concentration of super electrons at $T = 0\,K$, $T = \frac{1}{4}T_c$, $T = \frac{1}{2}T_c$, and $T = 1.1T_c$ in a superconductor with a penetration depth of 150 nm? What is the concentration of normal conduction electrons at these temperatures?

Solution

$$n_S = n\left[1 - \left(\frac{T}{T_C}\right)^4\right]$$

$$1 - \quad n_S = n\left[1 - \left(\frac{0}{T_C}\right)^4\right] = n[1 - (0)^4] = n[1 - 0]$$

$n_S = n$ All electron is super electron

$$2 - \quad n_S = n\left[1 - \left(\frac{\frac{T_C}{4}}{T_C}\right)^4\right] = n\left[1 - \left(\frac{T_C}{4} \times \frac{1}{T_C}\right)^4\right]$$

$$n_S = n\left[1 - \left(\frac{T_C}{4} \times \frac{1}{T_C}\right)^4\right] = n\left[1 - \left(\frac{1}{4}\right)^4\right]$$

$$n_S = n\left[1 - \frac{1}{64}\right] = n\left[\frac{64 - 1}{64}\right] = n\left[\frac{63}{64}\right]$$

$$n_S = \frac{63}{64}n$$

$$3 - \quad n_S = n\left[1 - \left(\frac{\frac{T_C}{2}}{T_C}\right)^4\right] = n\left[1 - \left(\frac{T_C}{2} \times \frac{1}{T_C}\right)^4\right]$$

$$n_S = n\left[1 - \frac{1}{16}\right] = n\left[\frac{16 - 1}{16}\right]$$

$$n_S = n\left[\frac{15}{16}\right] = \qquad n_S = \frac{15}{16}n$$

The concentration of the normal electron on these temperature

$$1 - \quad at \quad T = 0 \qquad n_n = n\left(\frac{T}{T_C}\right)^4$$

$$n_n = n\left(\frac{0}{T_C}\right)^4 \qquad n_n = n(0)^4 = 0$$

$$3 \quad - \quad at \quad T = \frac{T_C}{2} \qquad n_n = n\left(\frac{T}{T_C}\right)^4$$

$$n_n = n\left(\frac{\frac{T_C}{2}}{T_C}\right)^4 = n\left(\frac{T_C}{2} \times \frac{1}{T_C}\right)^4 = n\left(\frac{1}{2}\right)^4$$

$$n_n = \frac{1}{16}n$$

Q :**Calculate the number of super electron and normal electrons for Ta superconductor at temperature 3 K , if the critical temperate is 4.46 K and critical penetrate depth is 35 nm .**

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}} \qquad \lambda = \frac{35\,nm}{\sqrt{1-\left(\frac{3}{4.46}\right)^4}} \qquad \lambda = \frac{35\,nm}{\sqrt{1-(0.67)^4}} = \frac{35\,nm}{\sqrt{1-0.2015}} = \frac{35\,nm}{\sqrt{0.7985}} = \frac{35\,nm}{0.8935} = 39.17\,nm$$

Number os super electron is

$$n_S = n\left[1-\left(\frac{T}{T_C}\right)^4\right] \qquad \frac{n_S}{n} = \left(\frac{\lambda_{(0)}}{\lambda_{(T)}}\right)^2 \qquad \frac{n_S}{n} = \left(\frac{\mathbf{35\,nm}}{\mathbf{39.176}}\right)^2$$

$$\frac{n_S}{n} = (\mathbf{0.8935})^2 \qquad \frac{n_S}{n} = 0.7983 \qquad n_S = 0.7983\,N$$

2 − $n_n(T) + n_S(T) = n_{Total}$

$$n_n(T) = n_{Total} - n_S(T) \qquad n_n(T) = n - 0.7983n = 0.2017\,n$$

$$n_n(T) = 0.2017\,n$$

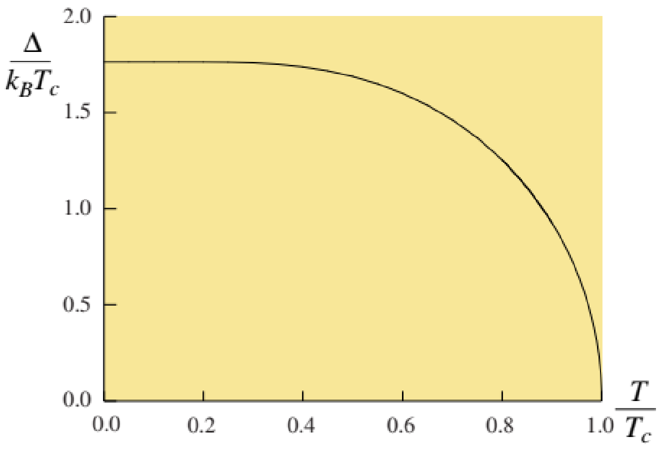
The energy gap and electron pairing

In conductors there is almost no energy gap and the valence band is overlap with the conduction band And it was Belief that there is no gap energy in superconducting materials as in conductors. But it turns out the opposite He found that there is an energy gap arising through the interaction between electrons to form what are called Cooper pairs.

The temperature dependence of the penetration depth suggests a density n_s of superconducting electrons that increased from zero at T_C to the full electron density at $T = 0$. The behavior is consistent with the existence of an energy gap Δ separating the states of the superconducting electrons from those of the 'normal' electrons. Both experiment and theory indicate that Δ is temperature-dependent, vanishing at T_C this figure, and attaining its maximum value Δ(0) at $T = 0$. At low temperatures $(T \ll T_C)$ one would expect that the number of excited (normal) electrons would fall off as $EXP^{\frac{-\Delta(0)}{K_B T}}$. energy gap $E_g = 2\Delta =$ $E_{gap}(T=0) = 3.53\, K_B T_C$

Direct evidence for an energy gap is provided by measurements of the absorption of electromagnetic waves. At low temperature ($T < T$c) the absorption is vanishingly small at low frequencies but increases sharply when the photon energy is sufficient to excite electrons across the energy gap. The frequency for the onset of absorption is given by

$$ h\nu = 2\Delta(0). $$



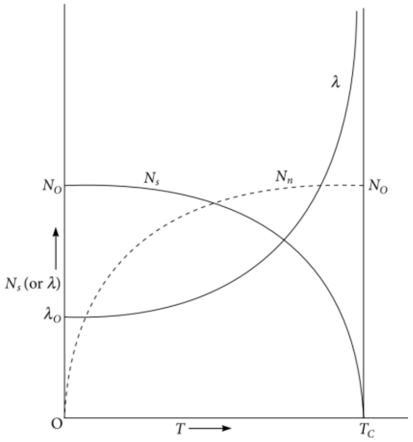


Figure ( ): Temperature dependence of the BCS gap function

The factor 2 arises because absorption of a photon creates two excited electrons. A natural explanation for this is provided by the BCS theory of superconductivity, according to which the superconducting electrons are bound together in pairs, known as Cooper pairs. Thus 2Δ is the binding energy of a Cooper pair so that $h\nu = 2\Delta(0)$. describes the breaking of a pair by absorption of a photon. The attractive interaction that binds the pairs is due to the lattice vibrations .

The binding energy of a Cooper pair is largest when all the pairs are in the same state.

At $T = 0$ all the electrons are paired but at $T > 0$ some pairs are broken by thermal excitation.

The "distance" between the two electrons of a Cooper pair is called the *coherence length*, In the framework of the BCS theory, the coherence length and the energy gap relate to each other at *T* = 0 as

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta(0)}$$

$$E_g = 3.52 K_B T_C \sqrt{1 - \frac{T}{T_C}}$$

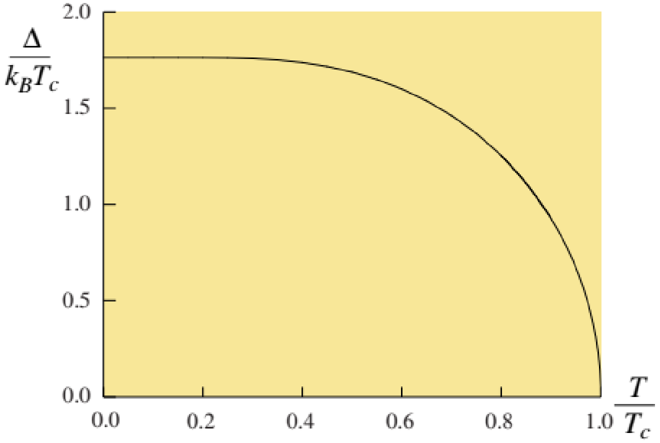
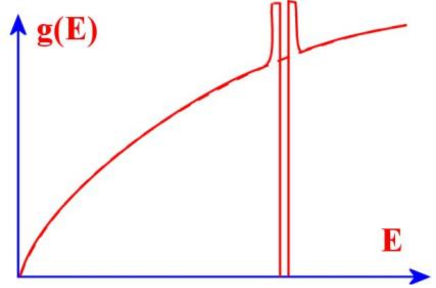


Figure ( ): Temperature dependence of the BCS gap function

The effect of the interaction is to ensure that within Δ of the Fermi surface there are no occupied states. The density of states immediately above and below the gap is increased correspondingly.

$$E_g \approx 3.52\, k_B T_c \left[1 - \frac{T}{T_c}\right]^{1/2}$$



The gap is $E_g = 2\Delta$ wide. The Fermi energy is in the middle of the gap.
An energy 2Δ will break up a pair and create two 'normal' electrons.
The pairs have many of the properties of bosons.

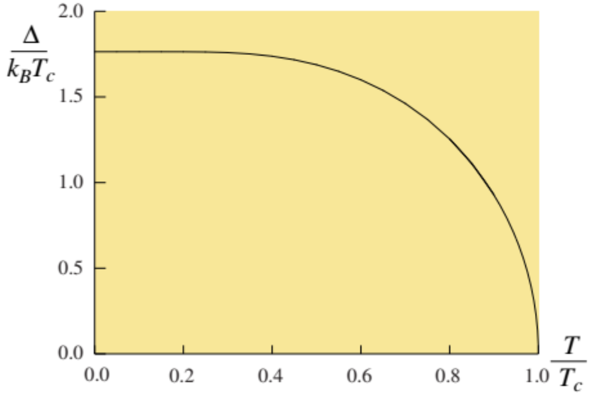
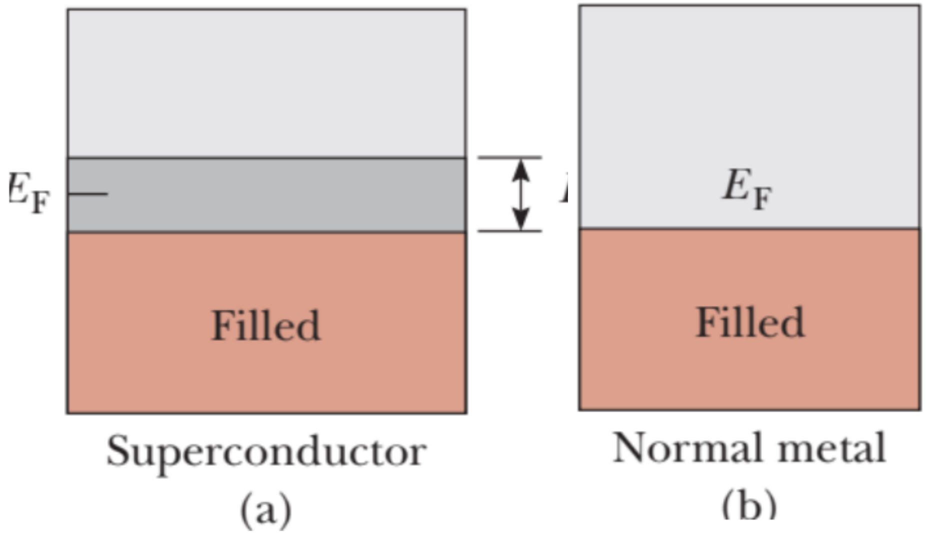


Figure ( ): Temperature dependence of the BCS gap function

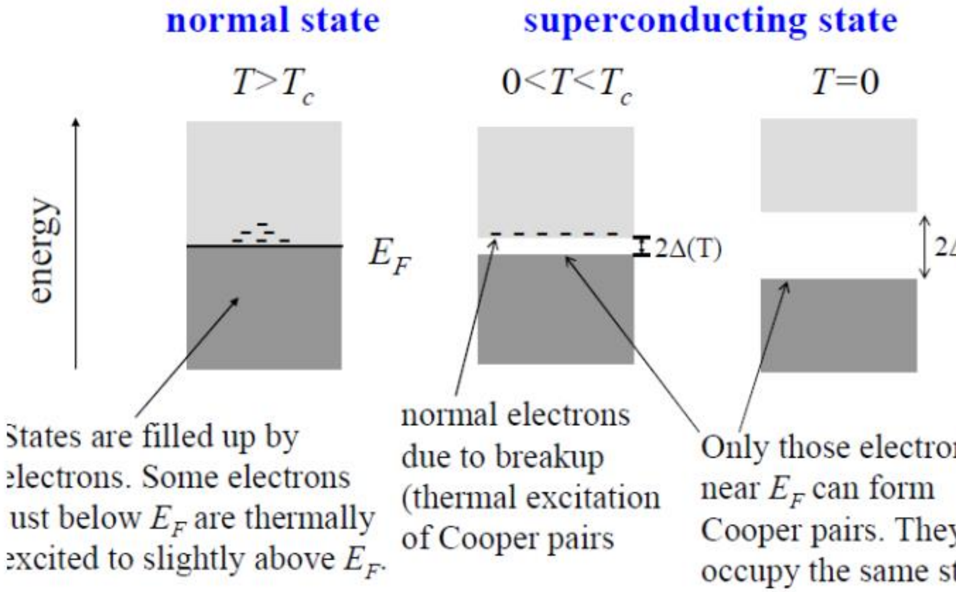
$$E_{gap}(T = 0) = 3.53\, K_B T_C$$

Energy gap a Cooper pair has a lower energy than 2 individual electron BCS gives

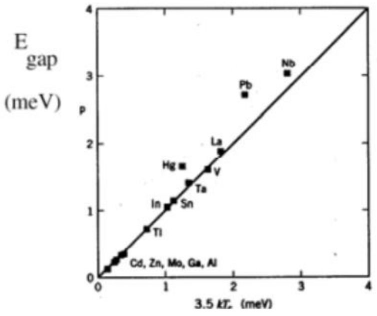


The binding energy for superconductor introduces an energy gap $2\Delta(T)$ near the Fermi energy E_F. ~ eV

$2\Delta(0) \sim 10^{-4}\, E_F$



BCS calculated gap



$E_g(0)/k_B T_c = 3.52$ **Weak electron-phonon coupling**

$E_g(0)/k_B T_c > 3.52$ **Strong electron-phonon coupling**

Normal and superconducting electrons

In London theory electrons in a superconductor are considered as a mixture of superconducting electrons and normal electro

$$n_S = n\left[1 - \left(\frac{T}{T_C}\right)^4\right]$$

The number density of superconducting and normal electrons is respectively denoted by n_s and n. At T = 0, n is equal to the total density of conduction electrons and as the temperature is increased to T_C the number density of the superconducting electrons decreases while that of the normal electrons increases. At a temperature $(T) = (T_C)$, the number density of the superconducting electrons is equal to zero. The two–fluid model of a superconductor in the flow of superconducting electrons does not encounter any resistance. This means that the superconducting electrons do not generate any constant electric field.

Table (2-1) comprising between superconducting and normal state

Superconducting electrons	Normal electrons
condensed in ordered state below T_c	condensed in disordered state above T_c
These electrons experience are not scattering by collision	These electrons experience are scattering by collision
have zero entropy (perfect order)	have not zero entropy (disorder)
have perfect electrical conductivity	have not perfect electrical conductivity
Long coherence length about 10^4 Å	short coherence length

Main Differences Between Conductors & Superconductors

Characteristics	Conductor	Superconductor
Definition	An object or material that allows the flow of charge when applied with a voltage	It is a conductor that has zero electrical resistance & expels magnetic fields
Electrical Resistivity	Normal conductors have some resistance that depends on its length, width, and material.	Superconductors have no electrical resistance below critical temperature.
Energy consumption	Due to resistance, the conductor consumes energy in the form of heat.	There is no energy loss or consumption in superconductors
Current storage	The current in a conductor die out when the power source is removed	The current in superconductor keeps flowing even after the source is removed
Magnetic field	The magnetic field lines of force pass within the conductor	The superconductor expels the magnetic field from the inside.
Quantum locking or levitation	A simple conductor does not have such a feat.	The quantum locking is a state in which the superconductor stays locked (hovering) in a magnetic field.
Energy Storage	A conductor cannot store energy due to its resistance. It dissipates energy.	A superconductor does no dissipate energy so it can store energy.

Power Transmission	You need multiple conductors with huge gauges to transfer power between stations.	A single superconductor can transmit power between stations without any loss or insulation burnout.
Operating temperature	Conductors can operate in normal range of temperatures.	These conductors achieve superconductivity when it is supercooled down below 5 K.
	Conductors have specific current	The superconductors can tolerate an infinite

Limitation	ratings & they cannot withstand heavy current.	amount of current but its temperature needs to be below the critical temperature.
Applications	The conductors are made into wires for conduction of electrical current	The superconductors have no efficient application to date due to its very low temperature but it will revolutionize our life once its temperature is maintained efficiently.