Salahaddin University
Collage of Basic Education
Department of General Science
Fourth Stage
Second Course

Superconductor

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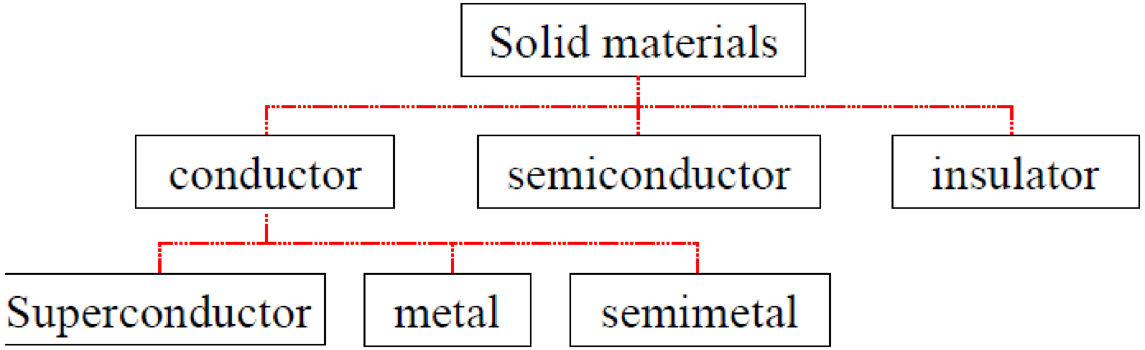
Chapter one Superconductor

Solids have different conductivity:



Solid materials

conductor

semiconductor

insulator

Superconductor

metal

semimetal

Superconductivity

A perfect superconductor is a material that exhibits two characteristic properties, namely zero electrical resistance and perfect diamagnetism, when it is cooled below a particular temperature Tc, called the critical temperature.

At higher temperatures it is a normal metal, and ordinarily is not a very good conductor. In the normal state some super-conducting metals are weakly diamagnetic and some are paramagnetic. Below Tc they exhibit perfect electrical conductivity and also perfect or quite pronounced diamagnetism

A brief history of low temperature

- 1911 Onnes measured the resistance of metal at such a low T. To remove residual resistance, he chose mercury. Near 4 K, the resistance drops to 0.

In 1908, H. Kamerlingh Onnes initiated the field of low-temperature by liquifying helium in his laboratory at the University of Leiden. Three years later (1911) Low temperature superconductivity was discovered by Kamerlingh Onnes below 4.15 K of the dc resistance of mercury dropped to zero. He was awarded in 1913 the Nobel Prize in Physics, partly for this discovery.

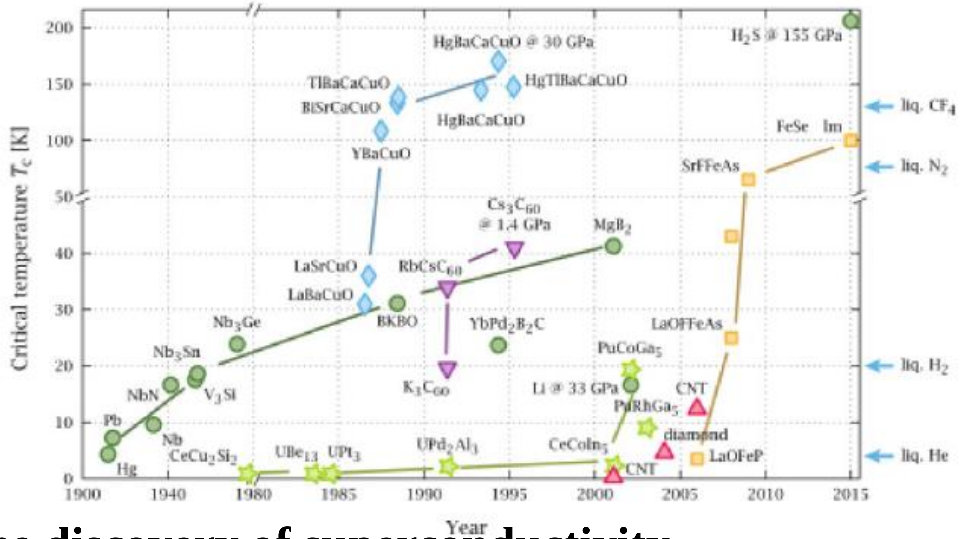
He was one of the famous scientists of the world. In 1933, Meissner and Ochsenfeld observed yet another interesting phenomenon called Meissner effect.

The isotope effect provided support for the electron–phonon interaction mechanism of superconductivity.

- 1956 Cooper pair: attractive interaction between electrons (with the help of crystal vibrations) near the FS forms a bound state.
- 1957 Bardeen, Cooper, Schrieffer: BCS theory

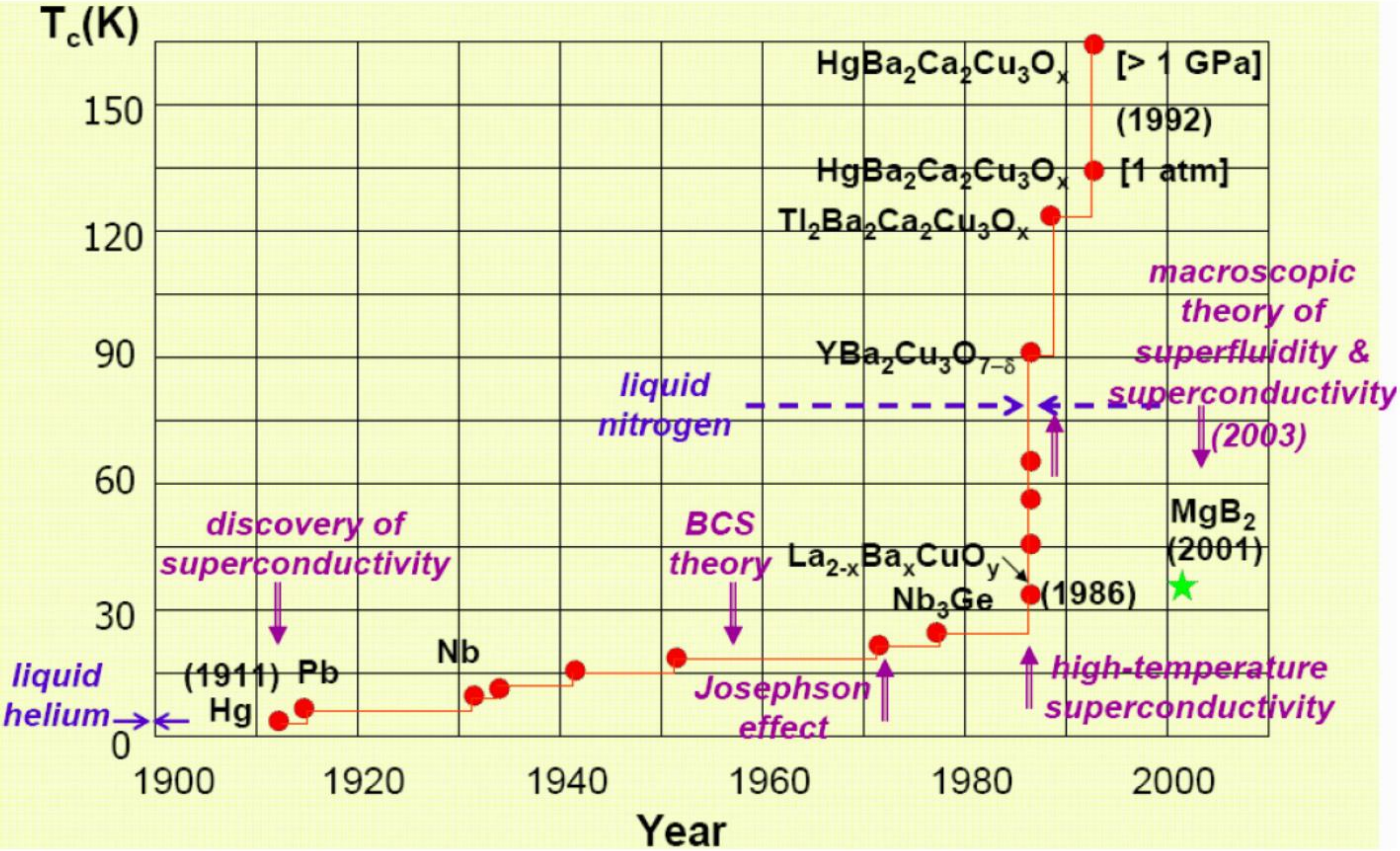
Microscopic wave function for the condensation of Cooper pairs.

Ref: 1972 Nobel lectures by Bardeen, Cooper, and Schrieffer



Evolution of critical temperatures since the discovery of superconductivity

Historical Overview



Significant parameters of superconducting state

There are three significant critical parameters that keep up the superconducting state

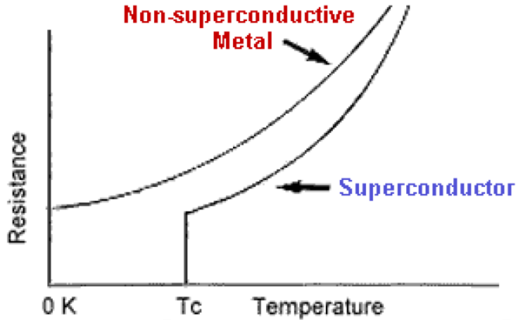
1- Critical temperature (T_C) **2- Critical magnetic field (H_C)** **3- Critical current density (J_C)**

1- Critical temperature (T_C)

Critical temperature is the temperature at which phase transition is observed from normal to superconducting state. The value of electrical resistivity also vanishes at this point.

The critical temperature is the temperature that marks the difference between superconducting and non-superconducting properties within a superconducting material. Above this temperature, the superconductor will behave normally. In the case of metals, the resistance will decrease with a drop in temperature, similar to non-superconducting metals. When the critical temperature is reached, the resistance suddenly drops to zero, and the material behaves as a superconductor. This temperature is not constant for all superconductors, but varies depending on the material, with some superconductors have a lower critical temperature than others.

Those with a critical temperature above 30K are called high temperature superconductors, such as Y- Ba-Cu-O with a critical temperature of 90 K. High temperature superconductors are especially useful due to it being easier to achieve superconductance in these materials because they do not need to be cooled to such low temperatures.



Comparisons between superconductor and non superconductor

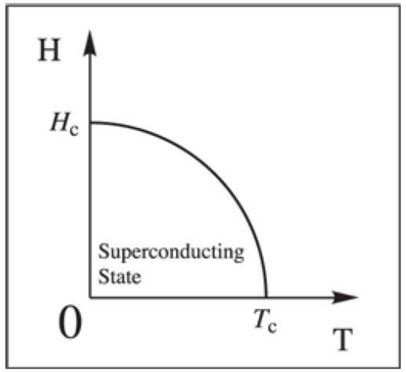
2- Critical magnetic field (H_C)

This field is basically responsible for the destruction in the superconducting properties of a superconductors. Consider a superconductor placed in magnetic field. The superconductivity is destroyed with the increase in the magnetic field till the critical value of magnetic field is achieved. As the value of applied field exceeds the critical value, the material comes back to the normal superconducting state. Fig shows the critical magnetic field being a temperature dependent quantity.

Mathematically it is given as $$H_C\,(T) = H_C(0)\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

Where $H_C(0)$ is the maximum value of applied field at T=0

$H_c(T_c) = 0 \quad (\text{at } T = T_c)$



Critical magnetic field versus temperature

3- Critical current density (Jc)

In order to minimize the resistance and energy losses and maximize the amount of current, thin wires of superconductivity are used. Whenever a superconductor is placed in a colder place it carries large amount of currents i.e. critical current density is temperature subordinate

Magnetic Parameters

The terms which can be used to describe the concepts of magnetism are called magnetic parameters. The important magnetic parameters that are used to characterize the magnetic behaviour of materials are enumerated here.

When a solid is placed in a magnetic field, it gets magnetized. The magnetic moment per unit volume developed inside a solid is called magnetization (M)

$$\chi = \frac{M}{H}$$

Intensity of magnetization M: Intensity of magnetization or (magnetization,) a vector quantity, is defined as the net magnetic dipole moment per unit volume.

χ is magnetic susceptibility which is a measure of the quality of the magnetic material and is defined as the magnetization produced per unit applied magnetic field

Where H is strength of the applied magnetic field , also referred to as the magnetic field intensity. M and H are measured in amperes per meter and χ is a dimensionless quantity. The magnitude and sign of susceptibility vary with the type of magnetism.

Magnetic field intensity H The magnetic field intensity or magnetic field strength is defined as the ratio of magnetic induction in free space to the magnetic permeability of the space.

The magnetic induction or magnetic flux density B produced inside the medium as a consequence of the applied magnetic field H is given by

Magnetic induction

B: The magnetic induction or the magnetic flux density in magnitude is defined as the magnetic flux *j* per unit area, i.e.,

$$\mathbf{B} = \mu_o (\mathbf{H} + \mathbf{M})$$

Where μ_0 is the permeability of the free space or vacuum and is equal to $\left(\mu_0 = 4\pi \times 10^{-7} \ \frac{Henry}{meter}\right)$. The quantity B is measured in Weber per square metre $\left(\frac{Weber}{m^2}\right)$ .or Tesla (T)

$$\mathbf{B} = \mu_o (1 + \chi) \mathbf{H}$$

$$\mathbf{B} = \mu \mathbf{H}$$

Where μ *is called the absolute permeabity of the medium* . For the isotopic medium ,it represents a scalar quantity having dimensionless same as that of μ_o. It is more convenient to introduce a dimensionless parameter μ_r which is called the relative permeability of the medium and is given by

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H}$$

$$\mu_r = 1 + \chi$$

$$\mu = \mu_0 \mu_r$$

For free space in the absence of any material medium $M = 0$, $\chi = 0$ $\mu = \mu_o$ and μ_1 =0 , and from the above relation, we obtain $B = \mu_0$ B= H $B = \mu_0 H$

Superconductor

The superconductivity was first discovered by H. K. Onnes in 1911 when he used liquid helium to cool down a sample of mercury (Kamerlingh-Onnes, H. 1911). He observed a sudden drop in resistance to an immeasurably small value when the temperature of the sample reached a value below 4.2 K. Unfortunately, there was no plausible explanation for the event at the time of the experiment because the quantum theory of metals was not yet developed to its full potential

Superconductivity is a phenomenon in which the electromagnetic properties of certain materials change considerably when they are cooled down under a critical temperature that is characteristic of a material. The well-known properties include perfect conductivity and perfect diamagnetism figure (1-1) demonstrates the former. Thermodynamically the transition of a material into the superconducting state is a second-order phase transition, e.g. there is no change of the crystallographic structure of the material.

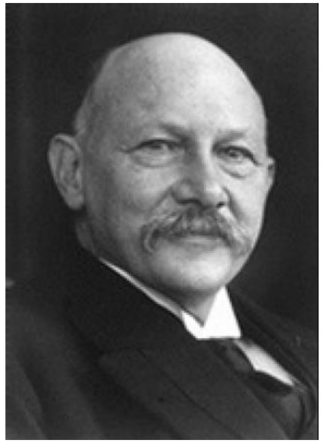
Superconductors have four important characteristics, namely, zero resistivity, Miessner effect, Josephson effect and quantization of magnetic field. Zero resistivity means, a material in its superconducting stage offers no resistance to the flow of direct electric current or in other words superconductor is a perfect electrical conductor. Meissner effect: a superconductor will expel magnetic flux from its interior by an internally induced magnetic field. Thus a superconductor in a weak magnetic field will act as a perfect diamagnetic. Josephson effect is the remarkable electrical property associated with the tunneling of superconducting electron pairs from a superconductor through a thin layer of an insulator in to another superconductor

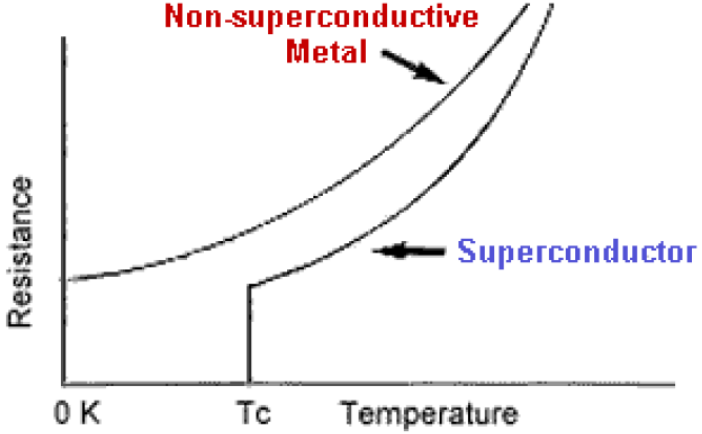
first two properties are related to electric power application and the last two properties are related to microelectronic applications

Onnes called it supra-conductivity and later *superconductivity*.

Nobel Prize (1913)

Heike Kamerlingh Onnes
(1853 - 1926)





Superconductors

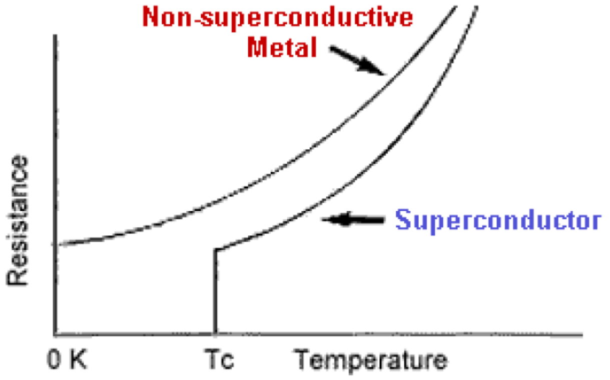
Fundamentally, superconductor is an element or a metallic alloy that possesses two distinguishing properties.

- **Zero dc-resistivity**
- **Perfect diamagnetism**

Zero dc resistivity

A superconductor, when cooled below a certain temperature, loses all its electrical resistance dramatically as shown in fig. 1.1. Eventually the metallic alloy allows all the current to flow through it without any effective electrical resistance. This current flowing through the metal is known as "Supercurrent"[

The certain temperature below which this transition of state occurs is designated as Tc,which is Critical Temperature. Superconductivity is a quantum phenomenon occurring at a macroscopic level. All the elements that possess this property, have their own value of Critical Temperature. Fig 1.1 (a) shows the graph between resistance and temperature for a superconductor.



Perfect diamagnetism

Diamagnetism is the alignment of magnetic moments in such a way that they oppose the externally applied field". A perfect conductor is perfect diamagnetic in nature. In addition to zero dc resistivity, superconductors exhibit another property of perfect diamagnetism in which they cancel or oppose all the magnetic flux inside the material when placed in an externally applied magnetic field.

The magnetism exhibited by these materials is perfect diamagnetism. For example; copper, air, hydrogen, gold, nitrogen. Figure 1.2 shows the perfect diamagnetic behavior of a superconductor.

As magnetic field is applied, current is induced in atomic orbitals causing diamagnetism. This induced current produces magnetization within the diamagnetic material which opposes the applied field, so cancels out it. As the applied field is removed, magnetization also disappears.

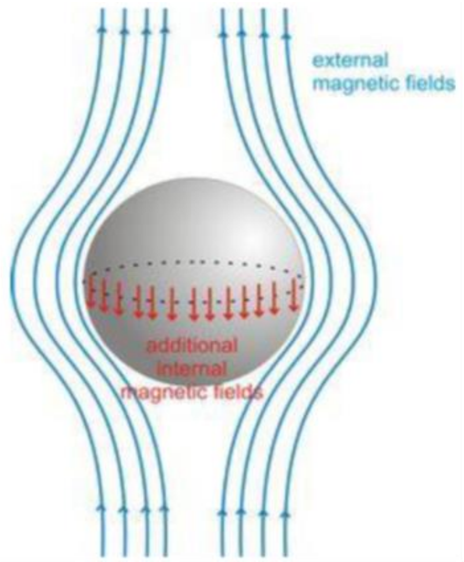
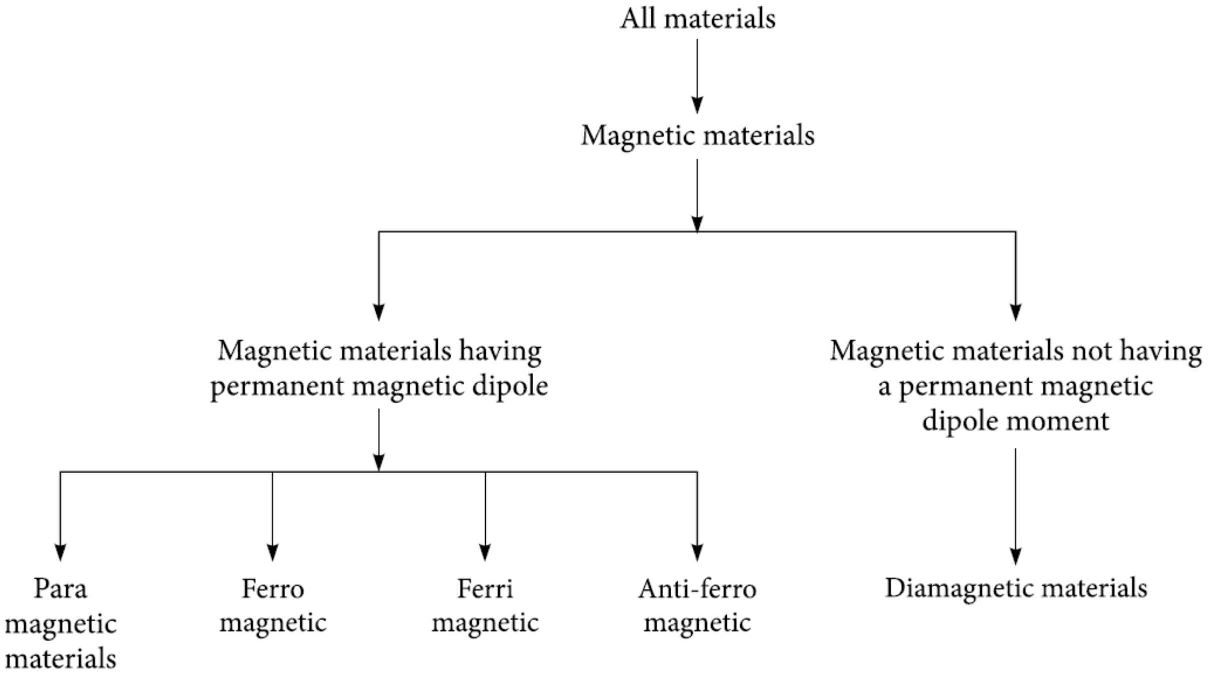


Figure : Perfect diamagnetic behavior of superconductor

Classification of Materials from the Magnetic Point of View



All materials

Magnetic materials

Magnetic materials having permanent magnetic dipole

Magnetic materials not having a permanent magnetic dipole moment

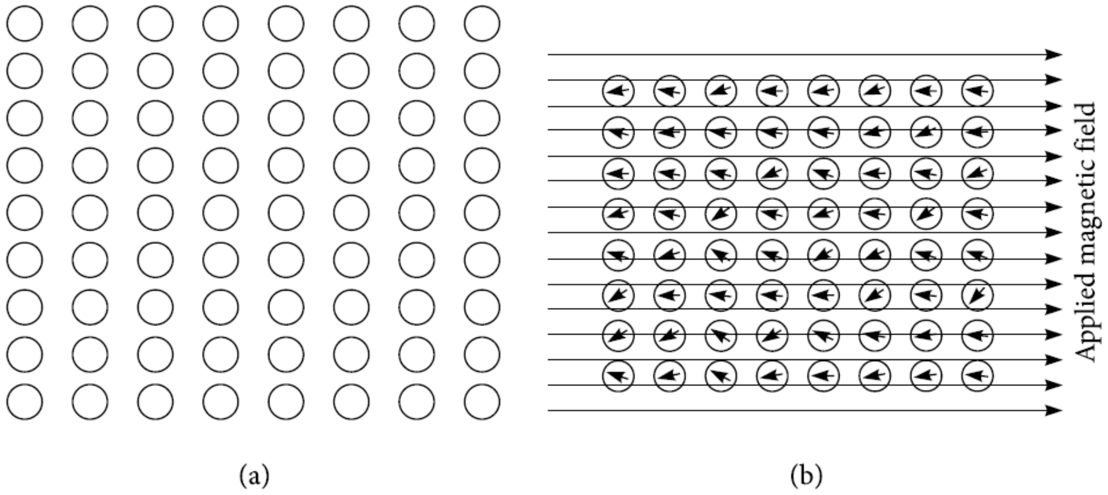
Para magnetic materials

Ferro magnetic

Ferri magnetic

Anti-ferro magnetic

Diamagnetic materials



(a) In the absence of an external magnetic field, no atomic dipole moments exist in diamagnetic materials. (b) The atomic dipole configuration of diamagnetic materials under the action of the applied external field. Here atomic dipole moments are induced in the atoms of the diamagnetic materials and the magnetic dipole moment directions are nearly opposite to that of the applied magnetic field

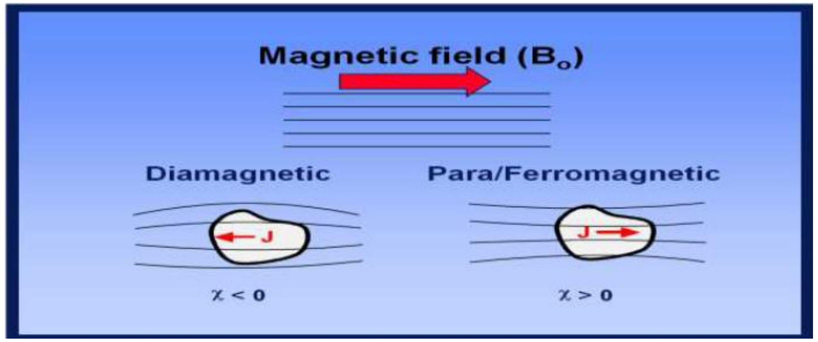
In diamagnetic material $B = \mu_0 (H + M)$

Since in superconductors B=0 (the field is completely screened out from the interior of the material), hence

$0 = \mu_0 (H + M)$ $M = -H$ $\chi = \frac{M}{H} = -1$

Hence magnetic susceptibility of a superconductor can be given as

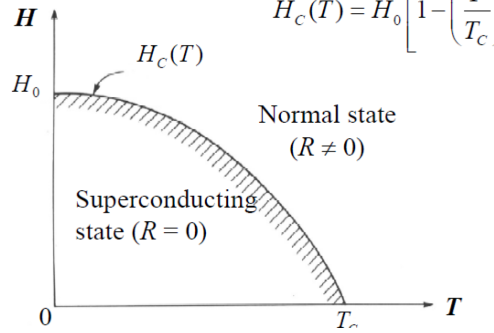
Perfect diamagnetism is basically due to the result of a continuous current flowing in the opposite direction to that of the externally applied magnetic field i.e. Meissner effect. Fig 1.2 (c) shows the behaviour of susceptibility because of externally applied magnetic field. The value of susceptibility varies from some positive number to -1. χ =1 is the perfect value of diamagnetism



Susceptibility in the presence of magnetic field

The superconducting state can be maintained only when $I < I_C$ and $H < H_C$.

H_c depends on temperature: $H_C(T) = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$



Current-carrying wire generates magnetic field:

The magnetic field on the wire surface is $B = \frac{\mu_0 I}{2\pi r}$.

Assume there is no external magnetic field ($H_a = 0$).
In order to maintain superconducting state, the current must not exceed a critical value I_C so that the generated field on wire surface $B \le \mu_0 H_C$.

Critical current: $$\frac{\mu_0 I_C}{2\pi r} \approx \mu_0 H_C \Rightarrow I_C \approx 2\pi r H_C$$

For applied magnetic field $H_a = 0$, $I_C \propto H_C$.

$$I_C = I_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

Conduction Electron Transport

The number density n (electrons/ cm^3) of conduction electrons in a metallic element such as copper or aluminum of density ρ (g/ cm^3) , atomic mass number A (g/mole), and valence Z is given by

$$n = N_A Z \rho_m / A,$$

where N_A is Avogadro's number. A potential difference applied along a conducting wire produces an electric field E and hence the force $F = -eE = m\left(\frac{dv}{dt}\right)$, which accelerates the electrons. They undergo successive periods of acceleration interrupted by collisions, and during the average time r between collisions they attain a component of velocity along the field direction

$$v_{av} = -(eE/m)\tau,$$

which is called the drift velocity, The negative sign means that the electrons move in a direction opposite to that of the electric field. The current density J,

$$J = nev_{av},$$

can be written, wit current density in the form of

$$J = (ne^2\tau/m)E = \sigma_o E,$$

and we have for the dc electrical conductivity $\sigma_o$and its reciprocal the resistivity ρ

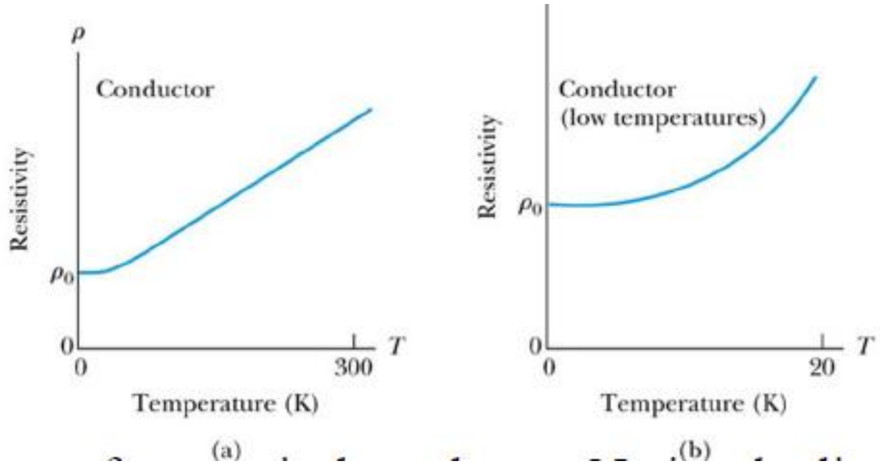
$$\sigma_o = ne^2\tau/m = 1/\rho.$$

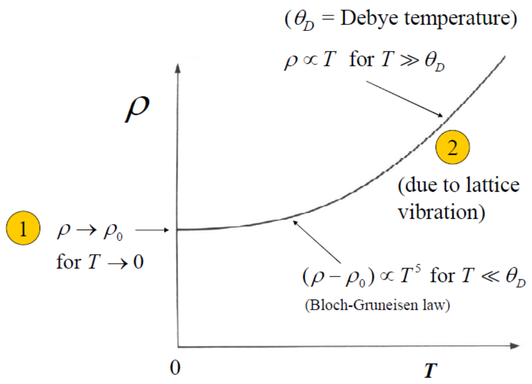
The drift velocity v_{av} Vav is much less than the Fermi velocity v_F at which the conduction electrons actually move on the Fermi surface. The mean free path I, or average distance traveled between collisions, is given by

$$l = v_F\tau.$$

Typically, v_F 106 m/s for good conductors (i.e., 1/300 the speed Of llight) and it is perhaps one-tenth of this value for A-15 compounds and high-temperature superconductors in their normal states.

Resistivity of a metal as a function of temperature:





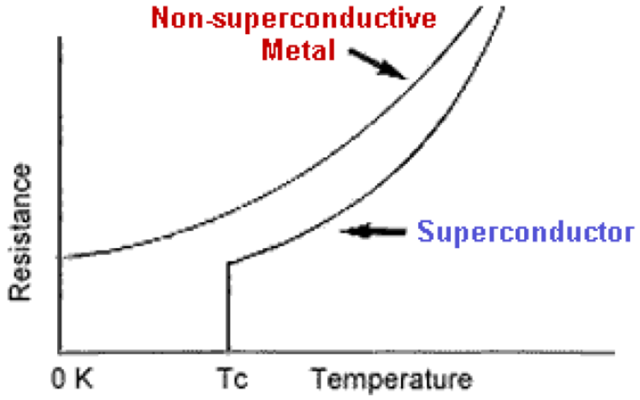


Figure : (a) Resistivity versus temperature for a typical conductor. Notice the linear rise in resistivity with increasing temperature at all but very low temperatures. (b) Resistivity versus temperature for a typical conductor at very low temperatures. Notice that the curve flattens and approaches a nonzero resistance as T → 0.

Origin of resistance:

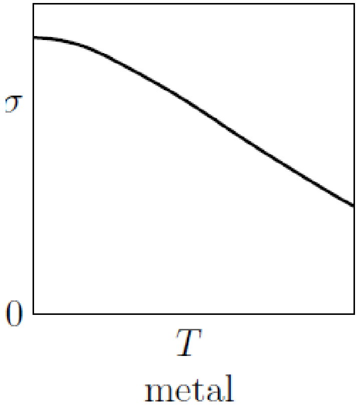
① ρ_0 (residual resistivity) due to defects, impurities, etc.

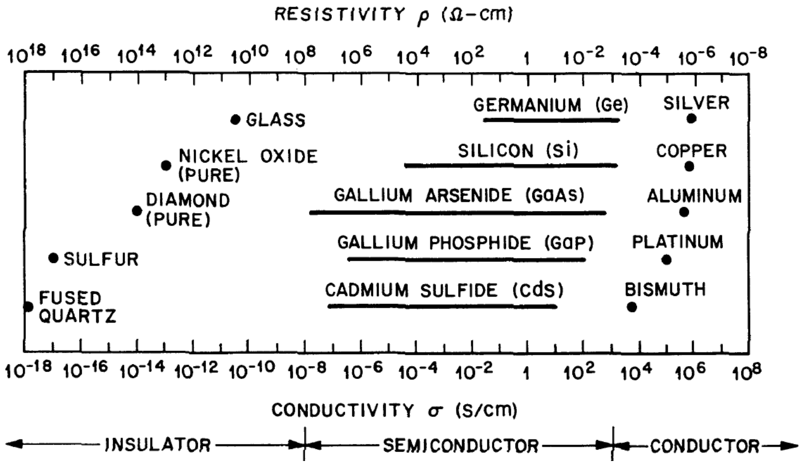
② temperature dependence of ρ depends on electron-ion scattering & lattice vibrations.

$$\rho \propto T \quad \text{for } T \gg \theta_D$$

$$(\rho-\rho_0) \propto T^5 \quad \text{for } T \ll \theta_D$$

If there are no defects and impurities then $\rho = \rho_0 = 0$ at $T = 0$.





Typical range of conductivities for insulators, semiconductors, and conductors

Some problems

Q1- A superconducting tin has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 Tesla at 0 K. Find the critical field at 2 K.

Solution

$$H_C\,(T) = H_C(0)\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

Critical temperature o= 3.7 K

Critical field = 0.0306 Tesla

temperature o= 3.7 K

Find the critical field at 2 K.

$$\boldsymbol{H_C\,(T) = 0.0306\ \text{Tesla}\left[1-\left(\frac{2\ K}{3.7}\right)^2\right]}$$

$$\boldsymbol{H_C\,(T) = 0.0306\ \text{Tesla}\left[1-\frac{4}{13.69}\right]}$$

$$\boldsymbol{H_C\,(T) = 0.0306\ \text{Tesla}[1-0.29218] = 0.0306\times 0.7078 = 0.02166\,Tesla}$$

$$\boldsymbol{H_C\,(T) = 0.02166\,Tesla}$$

Q2- Derive the Temperature as a function of a critical magnetic field.

$$H_C\,(T) = H_C(0)\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$\frac{H_C\,(T)}{H_C(0)} = \left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$\left(\frac{T}{T_C}\right)^2 = 1-\frac{H_C\,(T)}{H_C(0)}$$

$$\frac{T}{T_C} = \sqrt{1-\frac{H_C\,(T)}{H_C(0)}}$$

$$\boldsymbol{T = T_C}\sqrt{1-\frac{H_C\,(T)}{H_C(0)}}$$

Q3: The superconducting transition temperature of lead of 7.26 K. The initial field at 0 K is is 64 × 10^4 Amp per m. Calculate the critical field at 5 K.

Solution **Critical temperature $\boldsymbol{7.26\ k}$**

initial field at 0 K is 64 × 10^4Amp m.

temperature o= 3.7 K

Calculate the critical field at 5 K.

$$H_C\,(T) = H_C(0)\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$H_C\,(T) = \mathbf{64\times 10^4}\times\left[1-\left(\frac{\boldsymbol{5\ K}}{\boldsymbol{7.26.\,KL}}\right)^{\mathbf{2}}\right]$$

$$H_C\,(T) = \mathbf{i64\times 10^4}\times\left[1-\frac{25}{52.7}\right]$$

$$H_C\,(T) = \mathbf{64\times 10^4}\times[1-\boldsymbol{0.474}] = \mathbf{64\times 10^4}\times\boldsymbol{0.526} = \mathbf{336640}\ \text{Tesla}$$

Some problem

Q1- A superconducting tin has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 Tesla at 0 K. Find the critical field at 2 K.

$$H_C\,(T) = H_C(0)\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

Critical temperature o= 3.7 K

Critical field = 0.0306 Tesla

temperature o= 3.7 K

Find the critical field at 2 K.

$$\boldsymbol{H_C\,(T) = 0.0306\ \text{Tesla}\left[1-\frac{4}{13.69}\right]}$$

Solution

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$$\frac{H_C\,(T)}{H_C(0)} = \left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$\left(\frac{T}{T_C}\right)^2 = 1-\frac{H_C\,(T)}{H_C(0)}$$

$$\frac{T}{T_C} = \sqrt{1-\frac{H_C\,(T)}{H_C(0)}}$$

$$\boldsymbol{T = T_C}\sqrt{1-\frac{H_C\,(T)}{H_C(0)}}$$