Chapter Two Superconductor

Some properties of superconductor (Meissner effect, Fundamentals Parameters of The Superconductivity, types of superconductor……… etc

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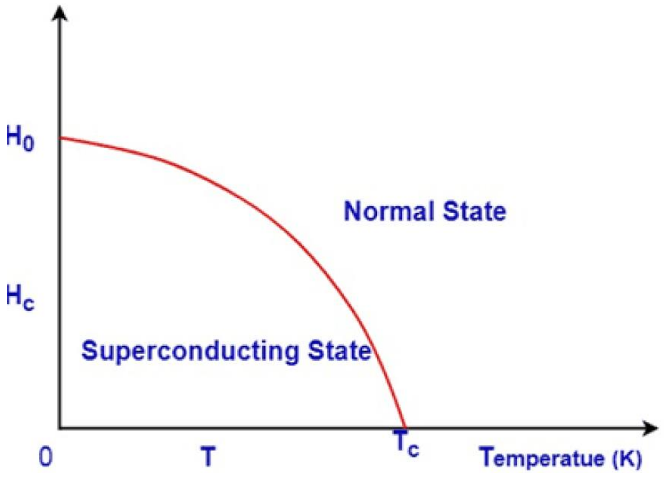
Meissner effect

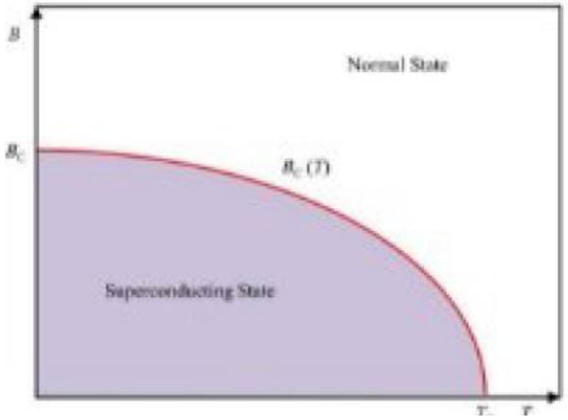
When Superconductors, are cooled below the critical temperature, they expel magnetic field and do not allow the magnetic field to penetrate inside them. This phenomenon is called Meissner effect, was discovered by German physicists "Walther Meissner" and "Robert Ochsenfeld" in 1933 , as shown in figure(2-1)

They observed that when the sample got cooled below the transition (critical) temperature in the presence of an external magnetic field, the value of the magnetic field outside the sample increases.

$$H_C(T) = H_C(0)\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$B_C(T) = B_C(0)\left[1-\left(\frac{T}{T_C}\right)^2\right]$$





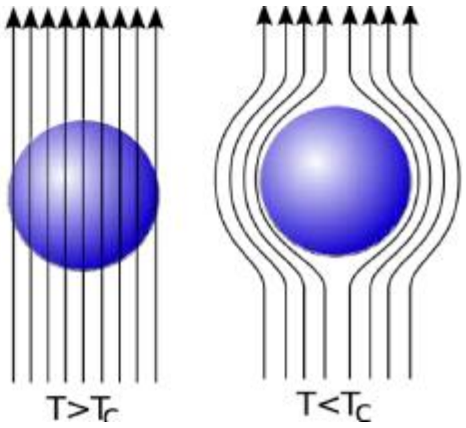


Figure (2-1)A : Variation in critical magnetic field with temperature

Figure : (2-1) B: Meissner effect

What is $\vec{B}$ inside SC?

For perfect conductor, $\sigma = \infty$.
Since $\vec{J} = \sigma \vec{E} < \infty$, $\vec{E} = 0$.
(Note: in electrostatics, $\vec{E} = 0$)

Faraday's law: $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

$\therefore \frac{\partial B}{\partial t} = 0.$

Where $H_C(0)$ is the maximum value of applied field at $T = 0$

Fundamentals Parameters of the Superconductivity

(a) Penetration Depth (λ)

While studying Meissner effect, that the superconductor expels a (weak) magnetic field B from its interior, i.e. $\bar{B} = 0$. (the Meissner effect) is by establishing a persistent supercurrent on its surface which exactly cancels the applied field inside the superconductor.

The finer experiments reveal that the applied externamagnetic field does not suddenly drop to zero, but actually penetrates into the superconductor within a very thin layer of the surface), the magnitude of the penetration depth depends on the material and temperature, and decreases exponentially towards the core of a superconductor, as shown in Figure (2-2). A

The magnetic field thus decays exponentially with distance into the superconductor with a characteristic length scale λ, known as the penetration depth, as shown in Fig (2-2) B, this penetrate decays exponentially according to the relation

$$H_{(X)} = H_{(0)} exp(-\frac{x}{\lambda})$$

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}}$$

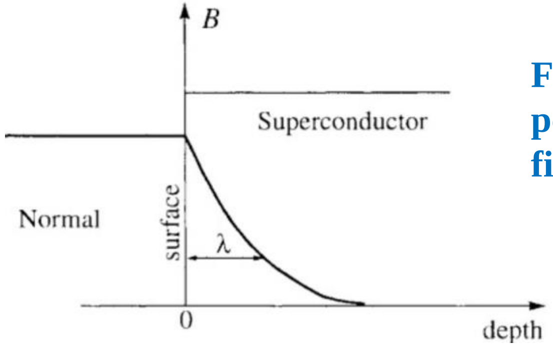


Figure (2-2) A: The penetration of the magnetic field into the superconducting

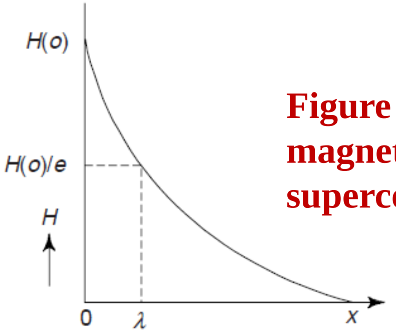


Figure (2-2) B : Decay of the magnetic field in the interior of a superconductor

where H(0) is the value of the magnetic field at the surface and λ is a penetration depth; x is the distance for H to fall from H(0) to H(0)/e.

$$\lambda_{(0)} = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

Where λ_0 is the penetration depth at zero temperature

This equation m represents the mass, e serves as the charge and n_s is the number density of the superconducting electrons. External magnetic field is suppressed over this value of λ.

(b) The Coherence Length (ξ)

The Coherence Length (ξ) It is a measure of the distance over which the gap parameter (Δ),

It was also referred as the distance between two electrons of the cooper pair within the highly coherent superconducting state.

coherence length ξ_0 is defined as :

$$\xi_0 = \frac{\hbar \upsilon_F}{\pi \Delta}$$

where v_F is the Fermi velocity (on the Fermi surface).

and Δ is the energy gap. Using order of magnitude values for v_F and Δ.

In the framework of the BCS theory, the coherence length ξ and the energy gap relate to each other at $T = 0$

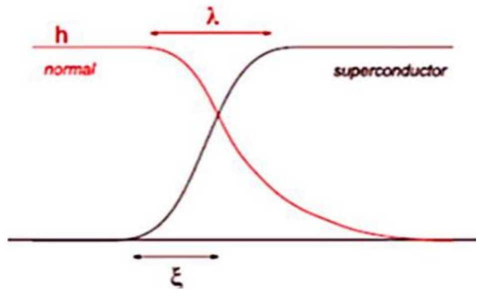
The quantity ξ_0 is the intrinsic coherence length which is temperature-independent The coherence length is large in metal superconductors. In spite of the fact that two electrons in a Cooper pair are far apart from each other, the other Cooper pairs are only a few nanometers away.

(c) The Cooper Pairs

When A superconductor material is cooled below the critical temperature , transforms the free electron gas from the normal state to a quantum fluid of coherent electron pairs . These charge carriers are called the Cooper pairs

(d) Ginzburg-Landau Parameter (K

Is the ratio of two characteristic lengths, λ and ξ is called the Ginzburg-Landau ratio. The GL parameter defines The ratio of the characteristic lengths



$$\kappa(T) = \frac{\lambda(T)}{\xi(T)}.$$

Close to κ is independent of temperature and if allows one to distinguish between type 1 and type 11 superconductors.

Figure (2-3) :The Coherence Length (ξ) and Penetration Depth (λ) at superconducting / normal surface boundary.

If $\kappa < 0.7$ *material in type* 1 *superconductor and if* $\kappa > 0$, thematerial is type 11 superconductor. The exact critical value of K which separates type 1 from type 11 between is $\frac{1}{\sqrt{2}}$ (~ 0.7) case the magnetic flux does penetrate the sample in the form of the cylindrical of tubes called vortices

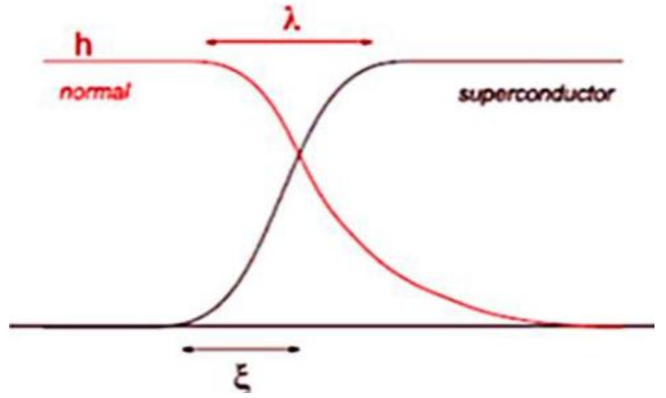


Figure (2-3) :The Coherence Length (ξ) and Penetration Depth (λ) at superconducting / normal surface boundary.

Isotope effect (1950)

In 1950 during experiment with different isotopes of mercury in order to understand the effect of the mass on superconductivity a very astonishing effect appears: different mass of the isotope have different T_C as in figure (2-4). This discovery gives the key to understand the basic mechanism for superconductivity (formation of Cooper pairs by electron -phonon- electron interaction).

Theory of superconductivity which is the BCS theory in 1957. T_C varied with the mass of the atom for different isotopes from figure (2-4) a relation can be found T_C and atomic mass as in the following figure.

$$T_C = \frac{1}{\sqrt{M}}$$

$$\frac{T_c}{T'_c} = \frac{(M')^{1/2}}{(M)^{1/2}}$$

$$\frac{T_{C1}}{T_{C2}} = \sqrt{\frac{M_2}{M_1}}$$

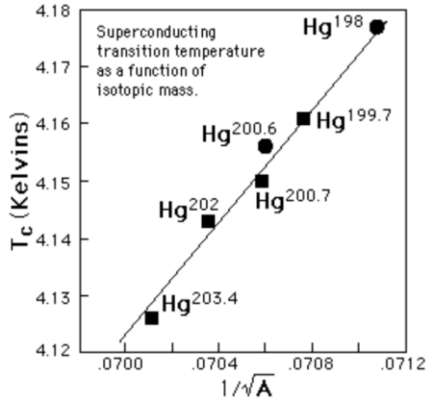


Figure (2-4) :Superconducting transition temperature as a function of isotopic mass.

Josephson effect

1- when two superconductors are joined by a thin insulating layer, it is easier for electron pairs to pass from one superconductor to another without resistance. This effect called the Josephson effect

This effect has applications for superfast electrical switches that can be used to make small , high speed computer.

Exampe : The critical temperature for mercury with isotopic mass 199.5 is 4.18 K. Calculate its critical temperature when its isotopic mass changes to 203.4

Solution

The data given are $M1$ = 199.5, $M2$ = 203.4 and T_{C1} = 4.18 K. The critical temperature in terms of its isotopic mass is given by

$$\frac{T_{C1}}{T_{C2}} = \sqrt{\frac{M_2}{M_1}} \qquad \frac{T_{C2}}{T_{C1}} = \sqrt{\frac{M_1}{M_2}} \qquad T_{C2} = T_{C1} \times \sqrt{\frac{M_1}{M_2}} \qquad T_{C2} = 4.18\,K \times \sqrt{\frac{199.5}{203.4}}$$

$$T_{C2} = 4.18\,K \times \sqrt{0.9808 = 4.18 \times 0.99} = 4.14\,K \qquad \boldsymbol{T_{C2} = 4.14\,K}$$

Heat capacity of superconductor

The heat capacity C of a material is the amount of heat ΔQ needed to raise the temperature by ΔT = 1 K per mole, namely ΔQ = CΔT. That heat capacity of a metal is a combination of lattice and electronic heat capacities.

The electronic specific heat (C_e) of the electrons is defined as the ratio of that portion of the heat used by the electrons to the rise in temperature of the system. For normal metals the heat capacity exhibits a temperature dependence $C = \gamma T + \beta T^3$

The first term ($Ce = \gamma T$) arises from the electronic contribution, while the second term ($CL = \beta T^3$) arises from the lattice.

The specific heat of the electrons in a superconductor varies with the absolute temperature (T) in the normal and in the superconducting state,

At low temperatures the specific heat is dominated by the electronic part, $Ce = \gamma T$.

The electronic specific heat in the superconducting state (designated C_{es}) is smaller than in the normal state (designated C_{en}) at low enough temperatures, but C_{es} becomes larger than C_{en} as the transition temperature T_c is approached. Although the curve has a cusp shape near T_c for the high-T_c superconductors. at which point it drops abruptly to C_{en} for the classic superconductors, Precise measurements have indicated that, at temperatures considerably below the transition temperature, the logarithm of the electronic specific heat is inversely proportional to the temperature.

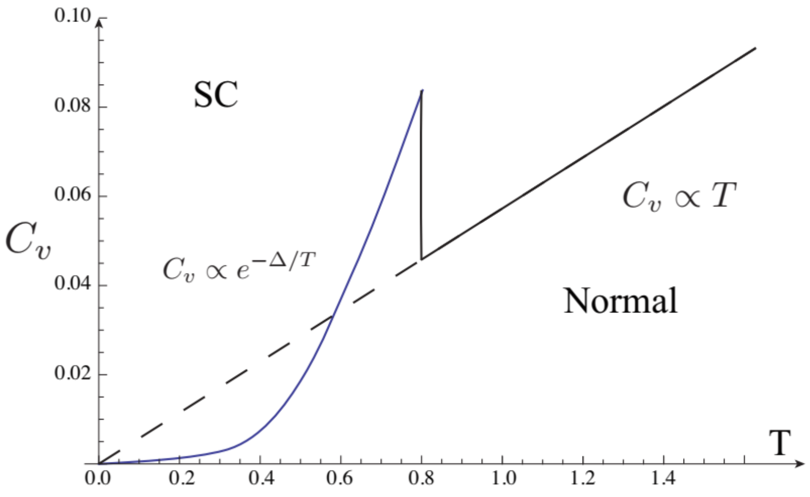


Fig. (2-5): Variation of heat capacity with temperature of a specimen in superconducting and normal states (Heat capacity jump at critical temperature and exponential decay for $T < Tc$)

CRITICAL FIELD AND CURRENT

application of a sufficiently strong magnetic field to a superconductor causes its resistance to return to the normal state value, and each superconductor has a critical magnetic field $B_{C,}$, below which is superconductor, above which it returns to normal. There is also a critical transport current density J_C that will induce this critical field at the surface and drive the superconductor normal. As in this equation

$$B_C(T) = \mu_0(T)\boldsymbol{\lambda}(T)J_C(T) \qquad (2-1)$$

where all three quantities are temperature dependent. Either an applied field or an applied current can destroy the superconductivity if either exceeds its respective, critical value. At absolute zero, we have

$$B_C(0) = \mu_0(0)\boldsymbol{\lambda}J_C(0) \qquad (2-2)$$

and this is often written

$$B_C = \mu_\lambda\boldsymbol{\lambda}J_C \qquad (2-3)$$

A particular superconducting wire of radius R has a maximum current, called the critical current I_C,

$$\boldsymbol{I_C = 2\pi R\lambda J_C} \qquad (2-4)$$

Subsisted the value of current density. (2-3 in 2-4), the value of the critical current may be written as

$$J_C = \frac{B_C}{\mu_0\lambda}$$

$$\boldsymbol{I_C} = \boldsymbol{2\pi R}\lambda \times \frac{B_C}{\mu_0\lambda} \qquad \boldsymbol{I_C} = \boldsymbol{2\pi R} \times \frac{B_C}{\mu_0} \qquad \boldsymbol{I_C} = \frac{\boldsymbol{2\pi R}B_C}{\mu_0}$$

The average current carried by a superconducting wire is not very high when most of the wire carries zero current.

To achieve high average super current densities, the wire must have a diameter less than the penetration depth,

TEMPERATURE DEPENDENCES

In the normal region above the transition temperature there is no critical field $B_C = 0$ and there is total magnetic field penetration $(\lambda = 0)$. As a superconductor is cooled down through the transition temperature Tc, the critical field gradually increases to its maximum value $B_C(0)$ at absolute zero $(T = 0)$

$$B_C(T) = B_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

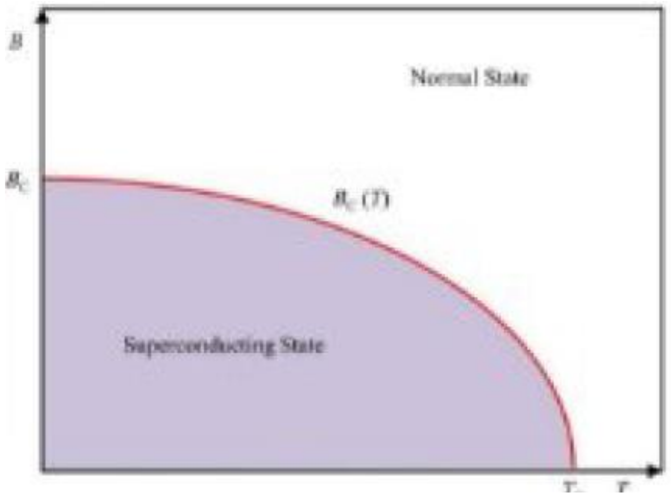


Figure (2-6) :Temperature dependence B_C relation depicts boundary that separate between superconductor and normal state

while the penetration depth decreases from infinity to its minimum at absolute zero as shown fig (2-7), . The explicit temperature dependences of $B_C(T)$ and T are given by the Ginzburg–Landau theory where $\lambda_0 = \lambda_L$ as given by this equation

$$\lambda_{(0)} = \sqrt{\frac{m}{\mu_0 n e^2}}$$

which assumes that all of the conduction electrons are superelectrons at T = 0. The critical current density may be written as the ratio

$$J_C(T) = \frac{B_C(T)}{\mu_0 \lambda(T)}$$

$$B_C(T) = B_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

In order to obtain the temperature dependence of $J_C(T), B_C(T)$ *and* λ . These temperature dependences have the form

$$B_C(T) = B_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}}$$

$$J_C(T) =\approx J_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

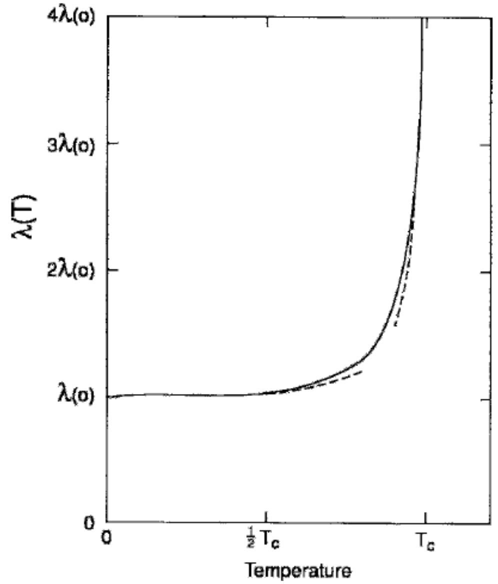


Figure (2-7): **Temperature dependence of the penetration depth T corresponding. The asymptotic behaviors near T = 0 and T = Tc are indicated by dashed lines.**

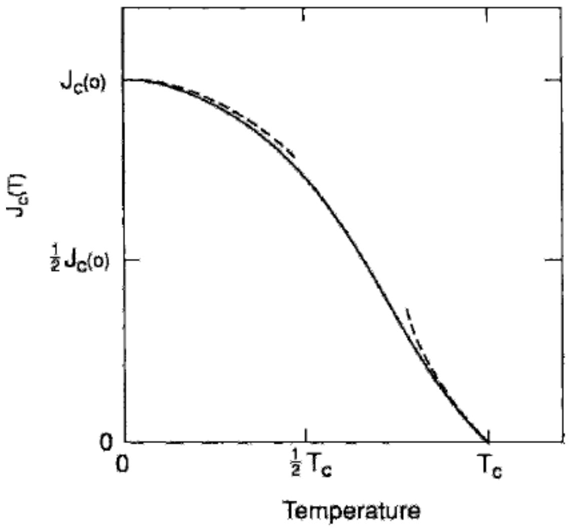


Figure (2-8): **Temperature dependence of the critical current density** $J_C(T)$ **.**

Types of superconductors

Depending upon the response of superconductors in magnetic field, Nicholas Gerbis classified them into two main types.

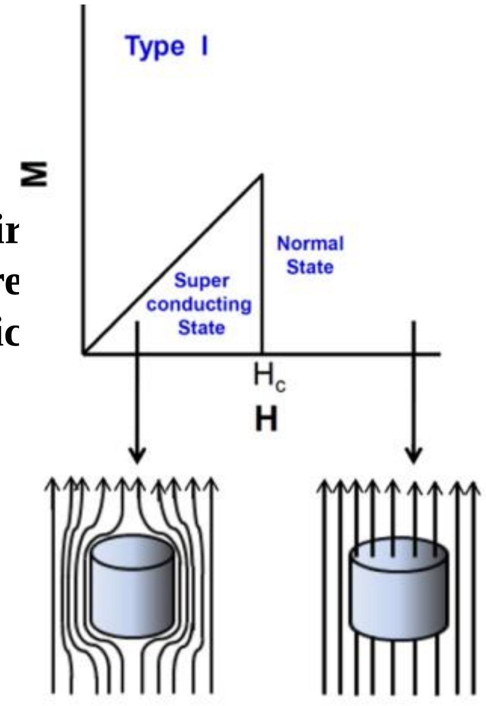
(i) Type-I superconductors

(ii) Type-II superconductors

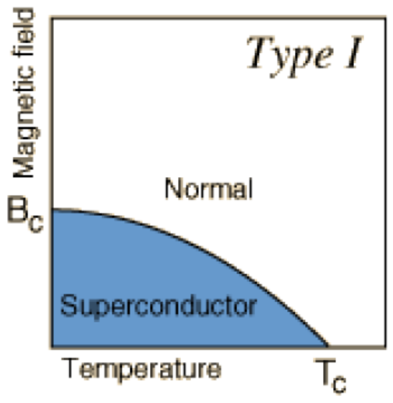
Type-I superconductors

Type I superconductor is (soft) according to their magnetization behavior. Type I superconductors were discovered first and mainly observed in pure metallic elements.

Type I superconductors shows a complete Meissner effect up to certain critical field, H_C, at which complete penetration occurs as the superconductor becomes normal.



(a)



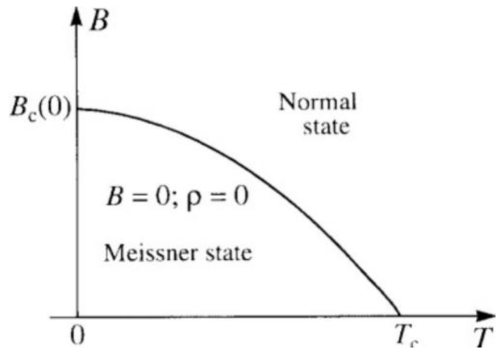
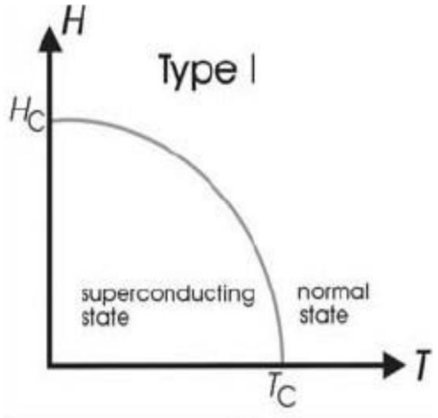


Fig.: (2-9) (a) Magnetization vs External Magnetic Field for type-I superconductors
(b) H-T phase diagram of type-I superconductors

These superconductors exhibit the property of zero electrical resistivity below critical temperature Tc. They also possess the property of expulsion of magnetic field from the interior of the superconductor (Meissner Effect).
Moreover, there is a critical magnetic field above which superconductivity ceases. At this value of critical applied field, they undergo sudden transition from diamagnetic to paramagnetic state. Figure: (2-9) a shows the magnetization versus externally applied magnetic field and the H-T phase diagram for Type-I superconductors figure (2-9) (b) .



(b)

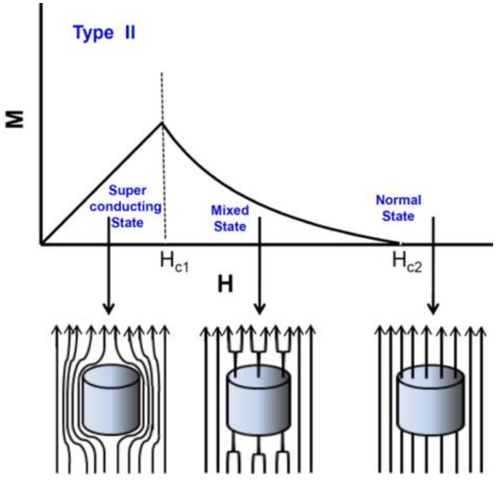
Type-II superconductors

Superconductor type II (hard) according to their magnetization behavior. compounds and alloys are in general type II superconductors.

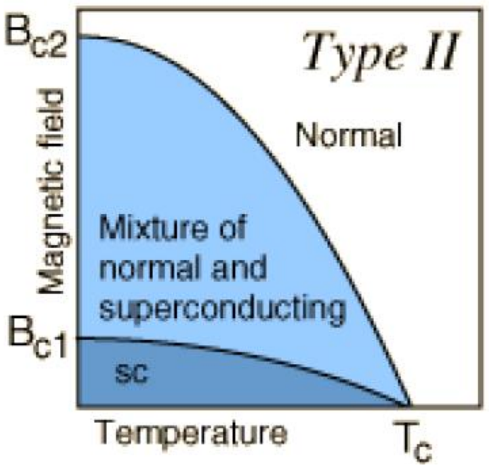
In the starting of 1930, many alloys exhibiting superconducting properties were found. They were classified as type-II superconductors. It was observed that these superconductors <u>have higher values of critical fields and resistivity</u> which allows them to withstand higher current densities. High field superconducting magnets are constructed using type-II superconductors for example niobium-titanium (NbTi).

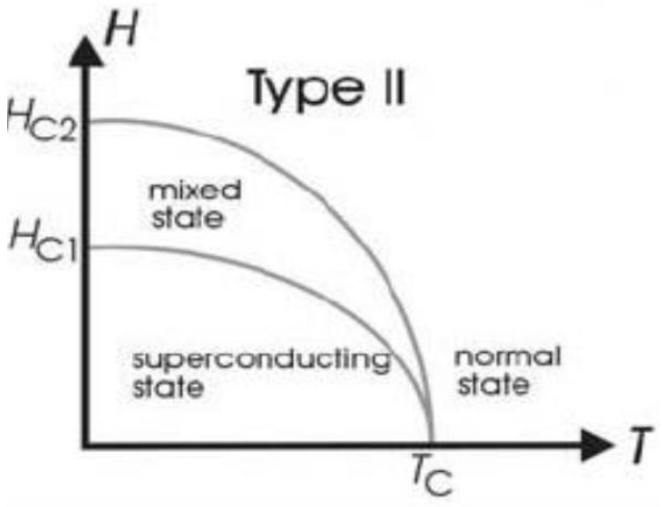
Type-II superconductors are those superconductors that mostly endure in mixed state i.e. normal and superconducting state. This mixed state is called "vortex state". In the region $H_{C1} < H < H_{C2}$ superconductivity is partially destroyed. In this region the superconductor is in a mixed state. Whereas, in the region $H < H_{C1}$ superconductor completely obeys the Meissner effect i.e. material is perfect diamagnetic and does not allows the magnetic flux to penetrate.

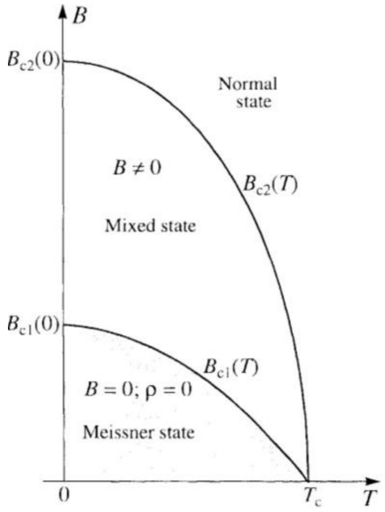
Figure 1.5(b) shows magnetization versus externally applied magnetic field and the H-T phase diagram for Type-II superconductors.



(b)







(a) Magnetization vs External Field of Type-II sup

(b)

(b) H-T phase diagram of Type-II superconductor

Comparison table of Type-I and Type-II superconductors

Type-I Superconductors	Type-II Superconductors
➢ Critical field is represented by "H_c"	➢ Critical field is represented by "H_{c1}, H_{c2}, H_{c3}"
➢ Low values of critical field.	➢ High values of critical field.
➢ Current flows through the surface only.	➢ Current flows through the whole material.
➢ Coherence length >> Penetration depth. ($\xi >> \lambda_L$)	➢ Coherence length <<< Penetration depth ($\xi << \lambda_L$)

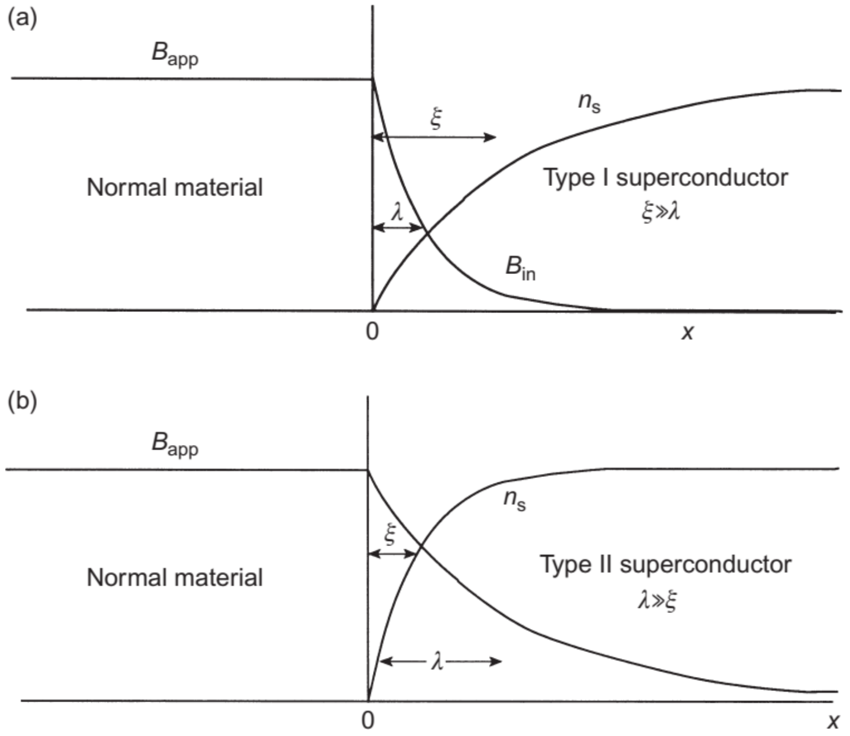


Figure 9.6 Increase in the number of superconducting electrons n_s and decay of the magnetic field B_{in} with distance x from the surface of the superconductor. The coherence length ξ and penetration depth λ associated with the change in n_s and B_{in}, respectively, are shown. (a) Type I superconductor, with $\xi > \lambda$, and (b) Type II superconductor, with $\lambda > \xi$, and $B_{app} < B_{c1}$.

Type-I: Meissner state $B = H + M = 0$ for $H < H_c$; normal state at $H > H_c$

Type-II: Meissner state $B = H + M = 0$ for $H < H_{c1}$; partial flux penetration for $H_{c1} < H < H_{c2}$; normal state for $H > H_{c2}$

Type I Superconductors

- These are pure and soft metals like lead and indium.
- The critical field of these conductors are low
- Hence these materials are not suitable for high field applications.
- They are called soft superconductors (or ideal)
- They have low melting point .
- These materials obeys silsbee's rule
- These materials shows meissner effect .the transition from normal to superconducting state is sharp.
- They have low value of H_c and T_c .

Type II Superconductors

- These are alloy and hard metals like lead and indium.
- The critical field of these conductors are high
- Hence these materials are suitable for high field applications.
- They are called hard superconductors
- They have high melting point .
- These materials breaks silsbee's rule
- These materials shows incomplete meissner effect , has broad transition region.
- They have high value of H_c and T_c .

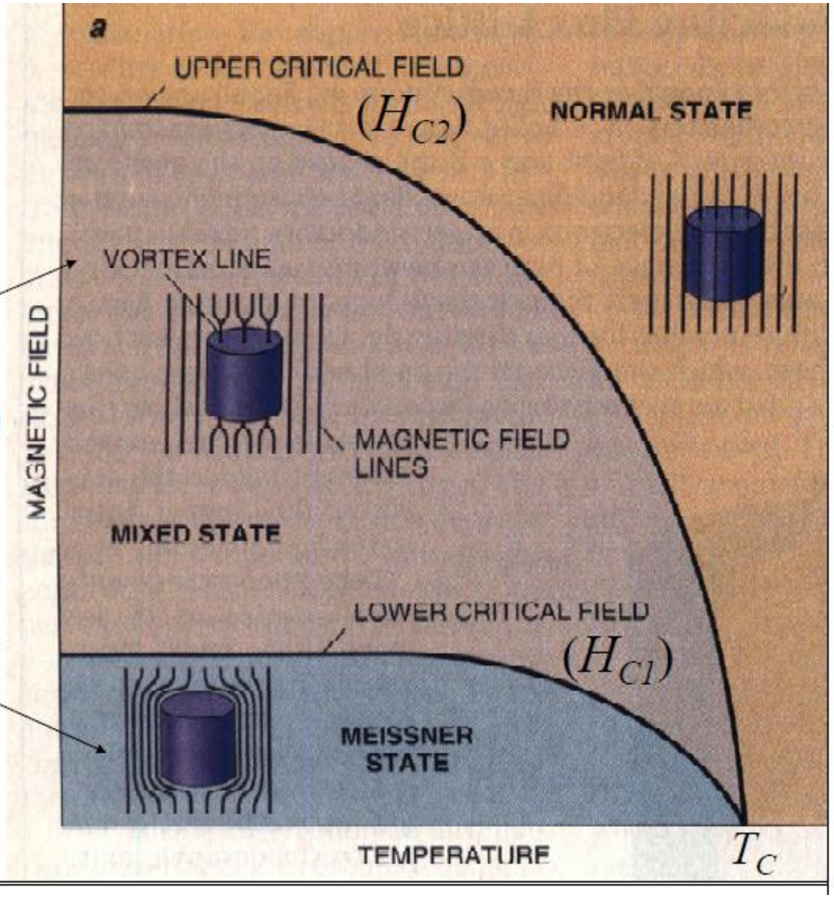
For Type II superconductor, there are two critical fields: $H_{C1}(T)$ & $H_{C2}(T)$

mixed state

($H_{C1} < H < H_{C2}$) is neither fully superconducting nor fully normal.

Meissner state

($0 < H < H_{C1}$) complete Meissner state



Scattering mechanisms, mean free path, dependence on temperature

- **In a perfect crystal, electrons will travel unhindered due to their wave nature: the wave is diffracted by the lattice and reforms unchanged**
- **The lattice is not perfect, but is always distorted due to**
  - **thermal vibrations of the lattice ions (phonons)**
  - **Structural defects (missing atoms, dislocation planes, grain boundaries)**
  - **Chemical impurities (foreign atoms, for example interstitial gas or substitutions with atoms having the same valence but different atomic radius)**

- **As the temperature is decreased, the first contribution fades out**
- **The conductivity remains finite due to defects and impurities**
- **The ratio of 300 K to low temperature (for instance, 10 K) resistivity is called RRR and is a gauge of the sample purity and its deviations from a perfect crystal**

The electrical resistivity of most metals is dominated at room temperature (300K) by collisions of the conduction electrons with lattice phonons and at liquid helium temperature (4K) by collisions with impurity atoms and mechanical imperfections

<u>Two dominant scattering mechanism</u>

1. **Phonon or lattice scattering**

2. **Ionized scattering**

1. <u>Lattice scattering or phonon scattering</u>

At temperature, T > 0 K, atoms randomly vibrate. This thermal vibrations cause a disruption of the periodic potential function. This resulting in an interaction between carrier and the vibrating lattice atoms.

Mobility due to lattice scattering, μ_L

$$\mu_L \propto T^{-3/2}$$

As temperature decreases, the probability of a scattering event decreases. Thus, mobility increases

Temperature ↓, Scattering Probability ↓, Mobility ↑,diffusion current density ↑
Temperature ↑, Scattering Probability ↑, Mobility ↓, diffusion current density ↓

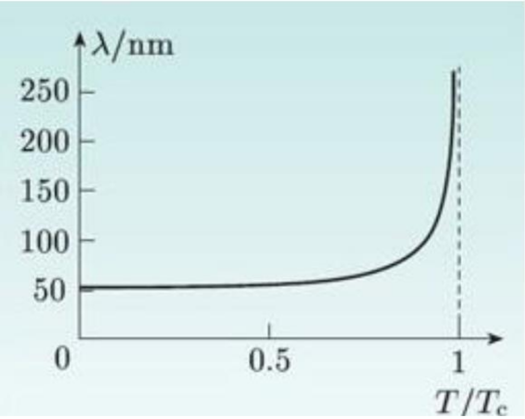


Figure : The penetration depth λ as a function of temperature for tin

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}}$$

2. Ionized scattering

Coulomb interaction between carriers and ionized impurities produces scattering or collusion. This alter the velocity characteristics of the carriers.

Mobility due to ionized ion scattering, μ_I

$$\mu_I \propto \frac{T^{3/2}}{N_I}$$

← Total ionized impurity concentration

- If temperature increases, the random thermal velocity of a carrier increases, reducing the time the carrier spends in the vicinity of the ionized impurity center. This causes the scattering effect decreases and mobility increases.

Temperature ↑, Thermal velocity↑, Time around ionized impurity↓, scattering effect decreases ↓**, Mobility ↑, diffusion current density ↑**

- If the number of ionized impurity centers increases, then the probability of a carrier encountering an ionized impurity centers increases, thus reducing mobility

Ionized Impurity ↑, Scattering Probability ↑, Mobility ↓, diffusion current density ↓

Normally, more than one source of scattering is present, for example both impurities and lattice phonons. It is normally a very good approximation to combine their influences using "Matthiessen's Rule" (developed from work by Augustus Matthiessen in 1864):

The net mobility is given by

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

Due to phonon scattering (μ_L)

Due to ionized ion scattering (μ_I)

$$\frac{1}{\mu_{Total}} = \frac{1}{\mu_{Lattice}} + \frac{1}{\mu_{ion}}$$

Due to phonon scattering

Due to ionized ion scattering

Q/ If a penetration depth λ for mercury equal 75nm at 3.5 K find: 1) The penetration depth at 0K , 2) The concentration of super conducting electrons n_S at 0K

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}} \qquad \lambda_0 = \lambda\left(\sqrt{1-\left(\frac{T}{T_C}\right)^4}\right) \qquad \lambda_0 = 75\ nm\left(\sqrt{1-\left(\frac{3.5}{4.12}\right)^4}\right) \qquad \lambda_0 = 75\ nm\left(\sqrt{1-0.5208}\right)$$

$$\lambda_0 = 75\ nm\left(\sqrt{0.4792}\right) \qquad \lambda_0 = 75\ nm \times 0.6922 = 51.918\ nm$$

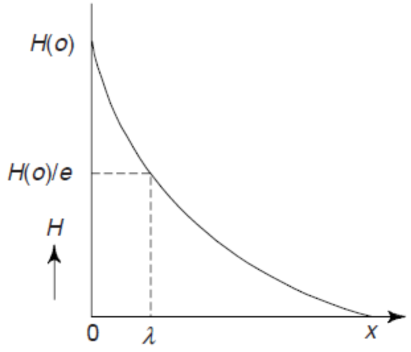
Q/2 Calculate the penetration depth λ for material when $T = T_C$.

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}} \qquad \lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T}\right)^4}} \qquad \lambda = \frac{\lambda_0}{\sqrt{1-1^4}} \qquad \lambda = \frac{\lambda_0}{\sqrt{1-1}} \qquad \lambda = \frac{\lambda_0}{\sqrt{0}} \qquad \lambda = \infty$$

Q3 :**Calculate the London penetration depth at 0 K, if the critical temperatures of lead is 7.193 K the London penetration depth 43.4 nm at 5.2 K.**

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}} \qquad \lambda_0 = \lambda\left(\sqrt{1-\left(\frac{T}{T_C}\right)^4}\right) \qquad \lambda_0 = 43.4\ nm\left(\sqrt{1-\left(\frac{5.2}{.7.193}\right)^4}\right) \qquad \lambda_0 = 43.4\ nm\left(\sqrt{1-(0.7229)^4}\right)$$

$$\lambda_0 = 43.4\ nm\left(\sqrt{1-0.2731}\right) \qquad \lambda_0 = 43.4\ nm\left(\sqrt{0.7269}\right) \qquad \lambda_0 = 43.4\ nm \times 0.85258 = 37\ nm$$



x is the x is the distance for H to fall from H(0) to H(0)/e e

Q5 If magnetic field at zero kelvin is $25\, Tesla$ London penetrate depth $20\, nm\ and$ x is the distance for H to fall from H(0) to H(0)/e have these values (2 , 12, 20 ,25, 30, 35) determine magnetic fields.

$$1- \quad H_{(X)} = H_C(0)e^{-\frac{x}{\lambda}} \qquad H_{(X)} = 25 \times e^{-\frac{2}{20}} = 25 \times e^{-0.1} = 25 \times 0.9048 = 22.62$$

$$2- \quad H_{(12)} = 25 \times e^{-\frac{12}{20}} = 25 \times e^{-0.6} = 25 \times 0.5488 = 13.72$$

$$3- \quad H_{(20)} = 25 \times e^{-\frac{20}{20}} = 25 \times e^{-1} = 25 \times 0.3678 = 9.197\ \text{T}$$

$$4- \quad H_{(12)} = 25 \times e^{-\frac{30}{20}} = 25 \times e^{-1.5} = 25 \times 0.223 = 5.5\, T$$

$$5- \quad H_{(12)} = 25 \times e^{-\frac{35}{20}} = 25 \times e^{-1.75.} = 25 \times 0.17377 = 4.344$$

Q5 If magnetic field at zero kelvin is $25\, Tesla$ London penetrate depth $20\, nm\ and$ x is the distance for H to fall from H(0) to H(0)/e have these values (5 , 10, 20 , 25,) determine magnetic fields.

$$H_{(X)} = H_C(0)e^{-\frac{x}{\lambda}} \qquad \frac{H_{(X)}}{H_C(0)} = e^{-\frac{x}{\lambda}} \qquad \frac{H_C(0)}{H_{(X)}} = e^{\frac{x}{\lambda}}$$

Take ln for both sides

$$Ln\left(\frac{H_C(0)}{H_{(X)}}\right) = Ln\left(e^{\frac{x}{\lambda}}\right) \qquad \frac{x}{\lambda} = Ln\left(\frac{H_C(0)}{H_{(X)}}\right) \qquad \mathbf{x = \lambda Ln\left(\frac{H_C(0)}{H_{(X)}}\right)}$$

$$(1):\quad \mathbf{x = \lambda Ln\left(\frac{H_C(0)}{H_{(X)}}\right)} \qquad \mathbf{x = 20\ nm \times Ln\left(\frac{25\ Tes}{5}\right)} \qquad \mathbf{x = 20\ nm \times Ln\,(5)} \qquad \mathbf{x = 20\ nm \times 1.609 = 32.18\ nm}$$

$$(2):\quad \mathbf{x = \lambda Ln\left(\frac{H_C(0)}{H_{(X)}}\right)} \qquad \mathbf{x = 20\ nm \times Ln\left(\frac{25\ Tes}{10}\right)} \qquad \mathbf{x = 20\ nm \times Ln\,(2.5)} \qquad \mathbf{x = 20\ nm \times 0.916 = 18.33\ nm}$$

$$(3):\quad \mathbf{x = \lambda Ln\left(\frac{H_C(0)}{H_{(X)}}\right)} \qquad \mathbf{x = 20\ nm \times Ln\left(\frac{25\ Tes}{20}\right)} \qquad \mathbf{x = 20\ nm \times Ln\,(1.25)} \qquad \mathbf{x = 20\ nm \times 0.223 = 4.46\ nm}$$

$$(4):\quad \mathbf{x = \lambda Ln\left(\frac{H_C(0)}{H_{(X)}}\right)} \qquad \mathbf{x = 20\ nm \times Ln\left(\frac{25\ Tes}{25}\right)} \qquad \mathbf{x = 20\ nm \times Ln\,(1)} \qquad \mathbf{x = 20\ nm \times 0 = 0}$$

Phase Diagram

The superconducting state is defined by three very important factors, namely, critical temperature T_C, critical field B_C and critical current density J_C. Each of these parameters is very much dependant on the other two parameters. Maintaining the superconducting state requires that both the magnetic field and the current density, as well as the temperature, remain below the critical values, all of which depend on the material. The phase diagram in this Fig. demonstrates the relationship between T_C, B_C and J_C. The highest values for B_C and J_C occur at 0 K, while the highest value for T_C occurs when B_C and J_C are zero.

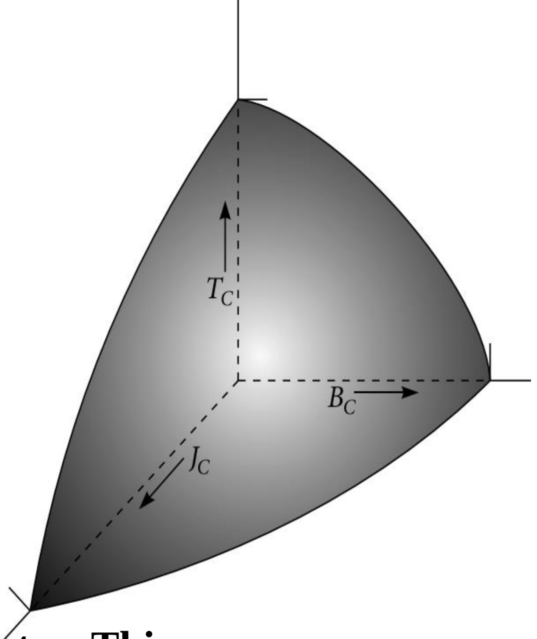


Figure ( ):Phase diagram of a superconductor. This is a 3D figure obtained by plotting critical temperature T_C, critical magnetic field *BC* and critical current density J_C.

When considering all three parameters, the plot represents a critical surface. From this surface, and moving toward the origin, the material is superconducting. The material is normal in regions outside this surface. When electrons form Cooper pairs, they can share the same quantum wave function or energy state. This results in a lower energy state for the superconductor. T_C and B_C are values where it becomes favourable for the electron pairs to break apart.

Current density larger than the critical value is forced to flow through a normal material. This flow through the normal material of the mixed state is connected with the motion of the magnetic field lines past pinning sites. For most practical applications, superconductors must be able to carry high currents and withstand high magnetic field without reverting to its normal state.

Higher B_C and J_C values depend upon two important parameters which influence energy minimization, penetration depth and coherence length (distance upto which Cooper pairs moves without breaking). Penetration depth is the characteristic length of the decrease of a magnetic field due to surface currents.

Penetration Depth

As we have seen, magnetic fields are expelled from the interior of a type I superconductor by the formation of surface currents. In reality, these currents are not formed in an infinitesimally thin layer on the surface. Instead, they penetrate the surface to a small extent. Within this thin layer, which is about 00 nm thick, the magnetic field B decreases exponentially from its external value to zero, according to the expression

$$B(x) = B_0 e^{-x/\lambda}$$

where it is assumed that the external magnetic field is parallel to the surface of the sample. In this equation, *B*0 is the value of the magnetic field at the surface, *x* is the distance from the surface to some interior point, and is a parameter called the penetration depth. The variation of magnetic field with distance inside a type I superconductor is plotted in Figure 12.12. The superconductor occupies the region on the positive side of the *x* axis. As you can see, the magnetic field becomes very small at depths a few times below the surface. Values for are typically in the range 10 to 100 nm. Penetration depth varies with temperature according to the empirical expression

where 0 is the penetration depth at From this expression we see that 0 K. becomes infinite as *T* approaches *Tc*. Furthermore, as *T* approaches *Tc*, while the sample is in the superconducting state, an applied magnetic field penetrates more and more deeply into the sample. Ultimately, the field penetrates the entire sample (becomes infinite), and the sample becomes normal.