General Science second stage

Chapter one

Wave

A Basic Introduction

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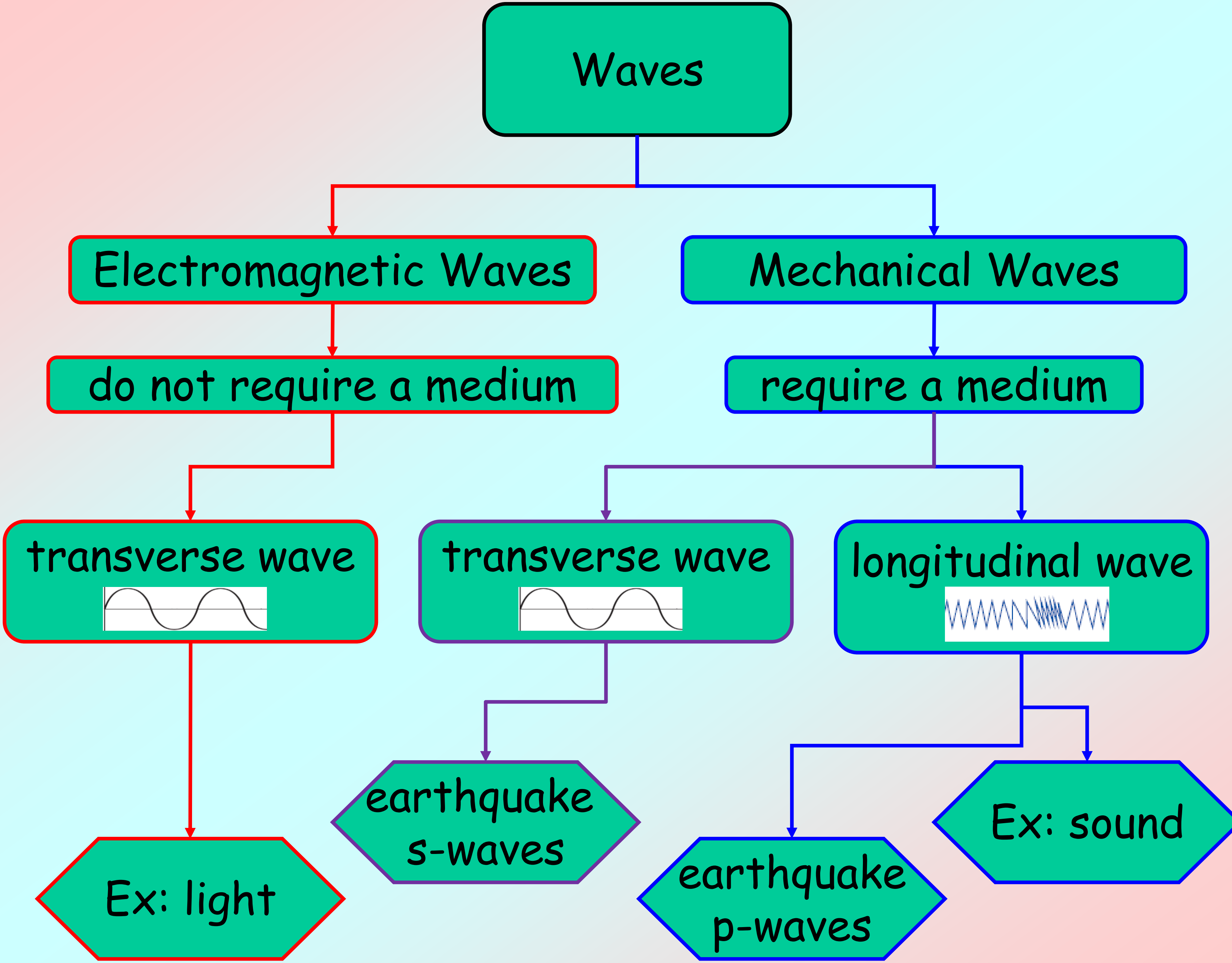
Waves

Ripples on a pond, musical sounds, seismic tremors triggered by an earthquake— all these are *wave* phenomena. Waves can occur whenever a system is disturbed from equilibrium and when the disturbance can travel, or *propagate*, from one region of the system to another. As a wave propagates, it carries energy. The energy in light waves from the sun warms the surface of our planet; the energy in seismic waves can crack our planet's crust.

Vibrations and Waves

- A wave is a disturbance that carries energy through matter (a medium) or space
- The source of all wave motion is a vibration.
- Waves transmit energy, not matter.
- Some of a wave's energy is always being dissipated as heat. In time, this will reduce the wave's amplitude (displacement).
- A wave cannot exist in one place but must extend from one place to another.
- Light and sound are both examples of waves.


Waves
Electromagnetic Waves
Mechanical Waves
do not require a medium
require a medium
transverse wave
transverse wave
longitudinal wave
earthquake s-waves
Ex: light
earthquake p-waves
Ex: sound

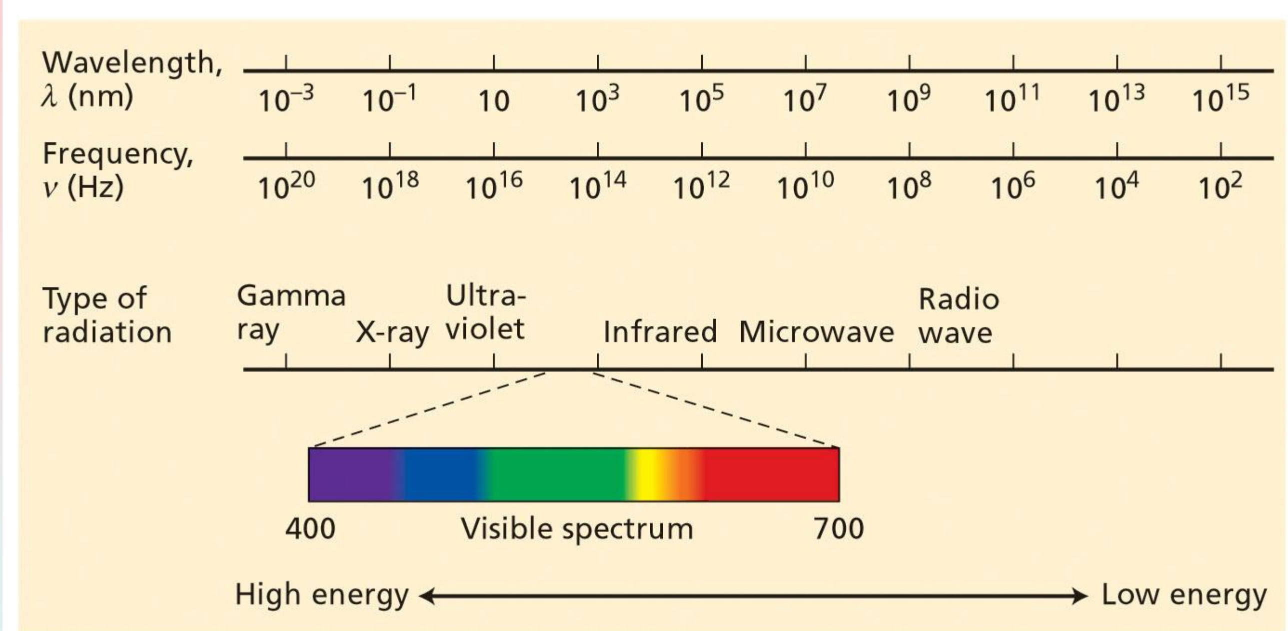
Electromagnetic Waves

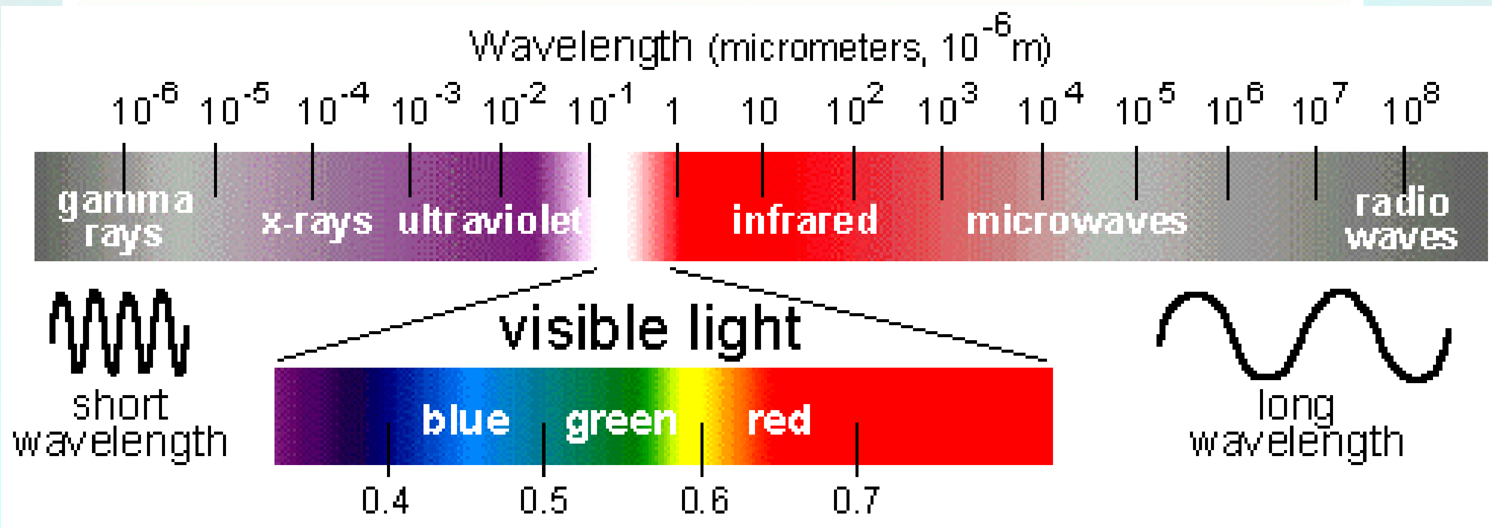
- EM waves are waves that can travel through empty space. They do <u>not</u> require a medium.
- EM waves are transverse waves.
- The EM spectrum consists of waves including gamma rays, x-rays, UV light, visible light, IR waves, microwaves and radio waves.
- EM waves travel through space at the speed of light.

Mechanical Waves

- Mechanical waves are waves that must travel through some form of matter to carry their energy.
- The matter that carries a wave is called the medium and includes matter such as air, water, rock, metal, etc.
- Mechanical waves can be either transverse or longitudinal waves.
- In a given medium, the speed of waves is constant. Waves travel very quickly through solids, less quickly through liquids and slowest through gases.

Electromagnetic Spectrum



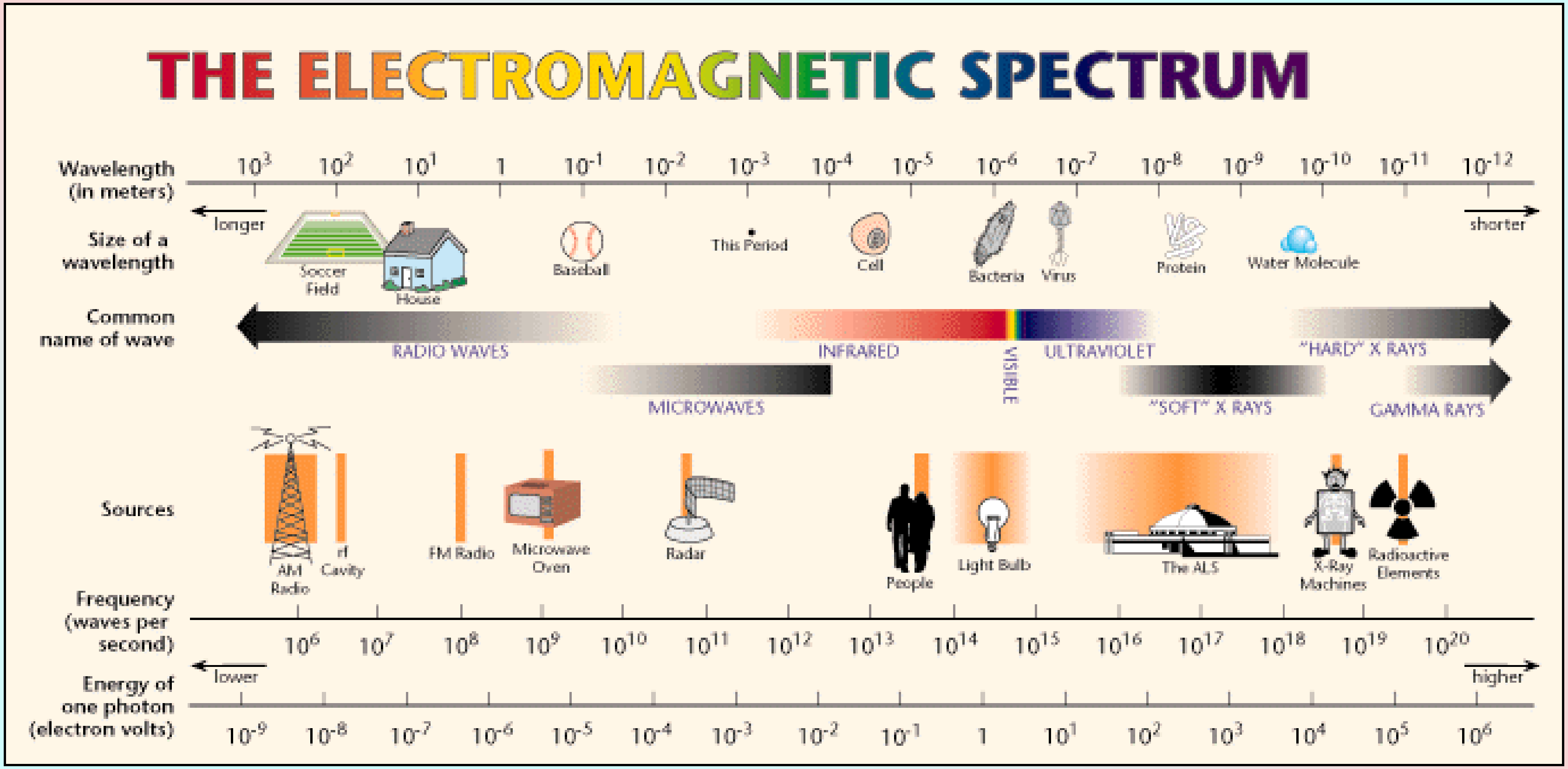


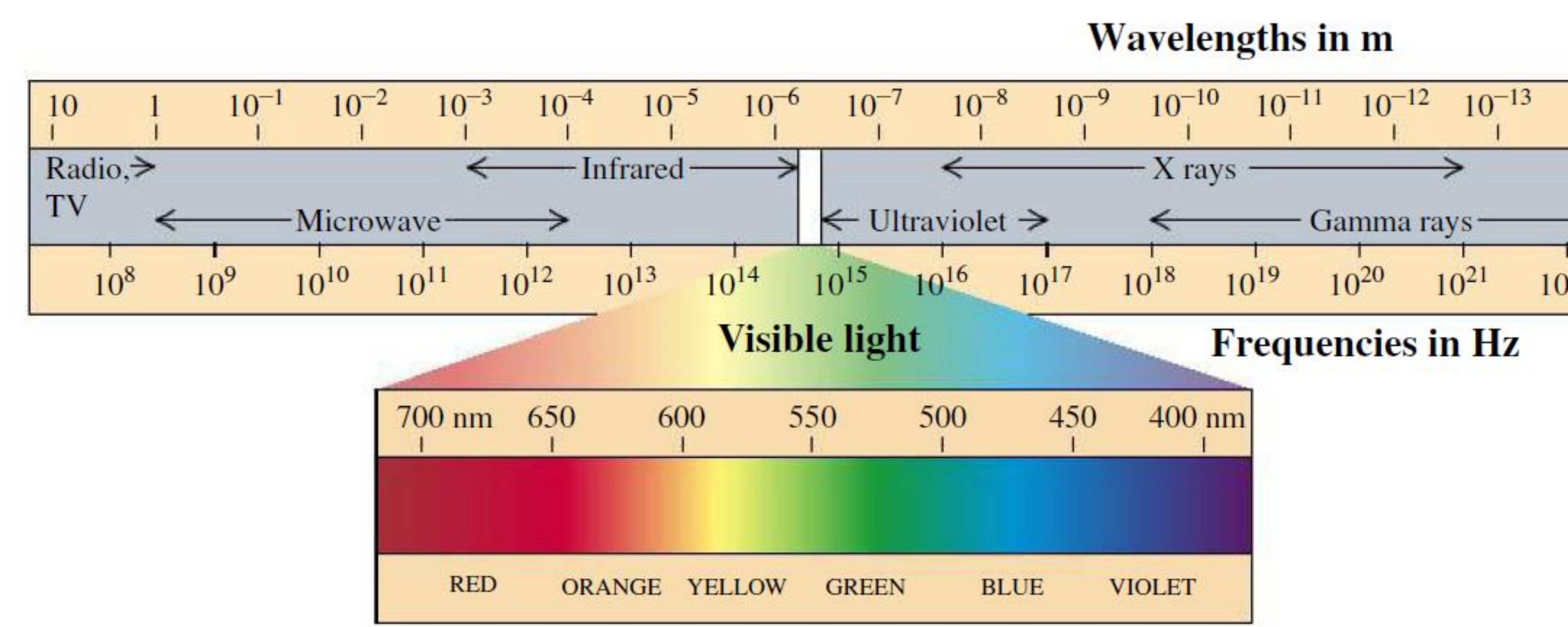
Electromagnetic radiation travels through space without requiring a medium

Hot bodies emit short wave radiation Solar radiation is short-wave radiation

Electromagnetic Spectrum

The electromagnetic spectrum illustrates the range of wavelengths and frequencies of electromagnetic waves.

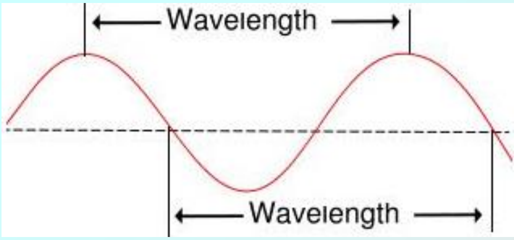




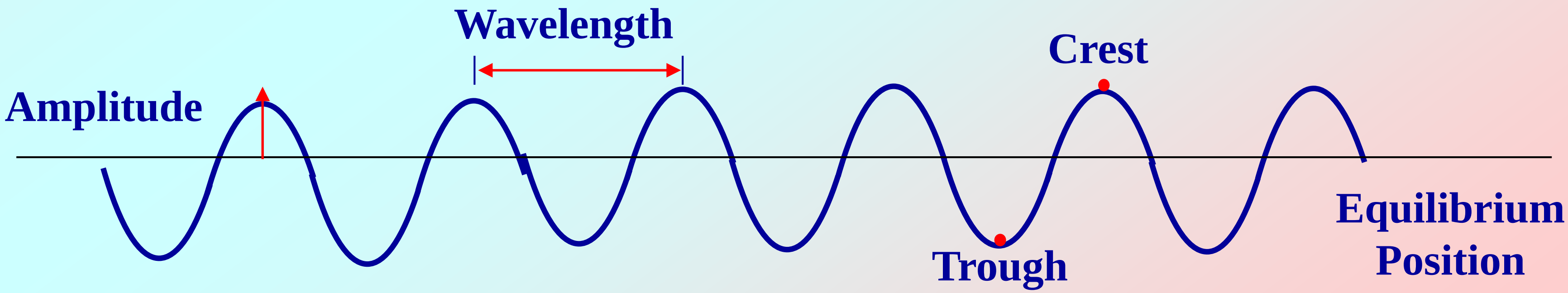
Wavelengths in m
10
1
10⁻¹ 10⁻² 10⁻³ 10⁻⁴ 10⁻⁵ 10⁻⁶ 10⁻⁷ 10⁻⁸ 10⁻⁹ 10⁻¹⁰ 10⁻¹¹ 10⁻¹² 10⁻¹³
Radio, TV
Infrared
X rays
Microwave
Ultraviolet
Gamma rays
10⁸ 10⁹ 10¹⁰ 10¹¹ 10¹² 10¹³ 10¹⁴ 10¹⁵ 10¹⁶ 10¹⁷ 10¹⁸ 10¹⁹ 10²⁰ 10²¹
Visible light
Frequencies in Hz
700 nm
650
600
550
500
450
400 nm
RED
ORANGE
YELLOW
GREEN
BLUE
VIOLET

Properties of Waves

- **Amplitude** – Maximum displacement from equilibrium. The larger the amplitude, the more energy the wave carries.
- **Wavelength** – The distance between successive identical parts of a wave. Represented by lambda, **λ.**



- **Crest** – The high point of a wave.
- **Trough** – The low point of a wave.
- **Period** – The time needed for a wave to make one complete cycle (wavelength) of motion. Represented by ***T***. Units of seconds.
- **Frequency** – Number of cycles (wavelengths) per unit time. Represented by ***f***. Units of hertz (Hz).



Frequency The *frequency* f of periodic, or oscillatory, motion is the number of oscillations per second. In the SI system, it is measured in hertz:

$$1 \text{ hertz} = 1 \text{ Hz} = 1 \text{ oscillation per second} = 1 \text{ s}^{-1}.$$

Period The *period* T is the time required for one complete oscillation, or **cycle.** It is related to the frequency by

$$T = \frac{1}{f}.$$

Simple Harmonic Motion In *simple harmonic motion* (SHM), the displacement $x(t)$ of a particle from its equilibrium position is described by the equation

$$x = x_m \cos(\omega t + \phi) \quad \text{(displacement)},$$

in which x_m is the **amplitude** of the displacement, $\omega t + \phi$ is the **phase** of the motion, and ϕ is the **phase constant.** The **angular frequency** ω is related to the period and frequency of the motion by

$$\omega = \frac{2\pi}{T} = 2\pi f \quad \text{(angular frequency)}.$$

$$v = -\omega x_m \sin(\omega t + \phi) \quad \text{(velocity)}$$

$$a = -\omega^2 x_m \cos(\omega t + \phi) \quad \text{(acceleration)}.$$

Displacement of Wave

$$x(t) = A\cos(\omega t + \phi)$$

Velocity of wave function

$$\frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \phi) = -\omega A\sin(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

Acceleration (a) of wave function

$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \phi) = -\omega^2 A\cos(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

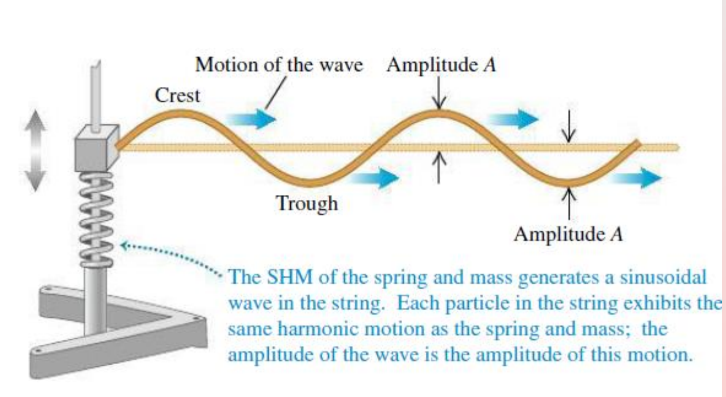
$$\omega = 2\pi f = \frac{2\pi}{T} \qquad f = \frac{1}{T} \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{\omega}{2\pi}$$

$$k \equiv \frac{2\pi}{\lambda}$$

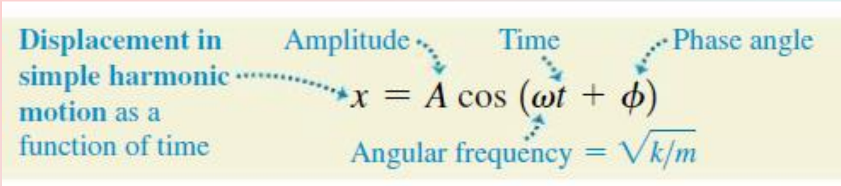
The value $2\pi\ \lambda$ is defined as the **wave number**. The symbol for the wave number is k and has units of inverse meters

Velocity of the wave is

$$v = \frac{\lambda}{T} = \frac{\lambda}{T}\left(\frac{2\pi}{2\pi}\right) = \frac{\omega}{k}.$$



Example :draw Graphs of (a) *x* versus *t*, (b) *vx* versus *t*, and (c) *ax* versus *t* for a body in SHM. For the motion depicted in these graphs, when phase constant is zero. At on complete cycle.



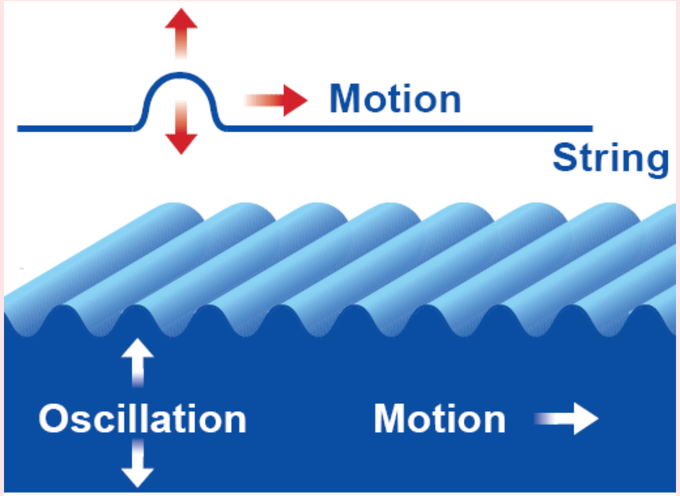
$$x = Acos\left(\frac{2\pi}{T}\,t + \emptyset\right)$$

$$x = Acos\left(\frac{2\pi}{T}\,\mathrm{t}\right)$$

Transverse Waves

- In transverse waves, particles vibrate at right angles to the direction the wave travels.
- Transverse waves can be modeled by using a sine wave

- Types of transverse waves include EM waves and ocean waves.
- Depending on the type of wave, transverse waves may or may not require a medium.

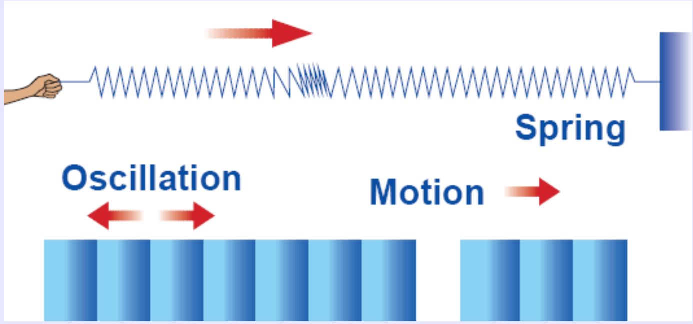


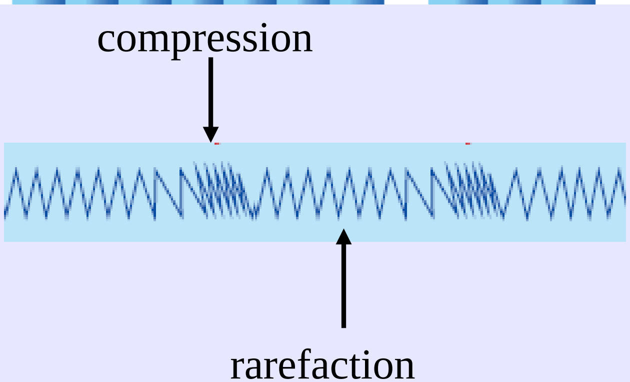
Longitudinal Waves

In longitudinal waves, particles vibrate back and forth in the same direction that the wave travels.

- Longitudinal waves can be modeled by using a spring.

- Sound waves are longitudinal waves.
- Longitudinal waves are mechanical waves – they require a medium.





Wave function for a sinusoidal Wave

Wave function for **a sinusoidal wave propagating in** $+x$**-direction**

Amplitude, Position, Time

$$y(x,t) = A\cos\left[\omega\left(\frac{x}{v} - t\right)\right]$$

Angular frequency $= 2\pi f$, Wave speed

The displacement $y(x,t)$ is a function of both the location x of the point and the time t. We could make Eq. ab0ve more general by allowing for different values of the phase angle, a We can rewrite the wave function in several different but useful forms. We can express it in terms of the period $T =$

Wave function for **a sinusoidal wave propagating in** $+x$**-direction**

Amplitude, Position, Time

$$y(x,t) = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

Wavelength, Period

It's convenient to define a quantity k, called the **wave number:**

$$k = \frac{2\pi}{\lambda} \quad \text{(wave number)} \qquad \omega = vk \quad \text{(periodic wave)}$$

Substituting $\lambda = 2\pi/k$ and $f = \omega/2\pi$ into Eq.

For a **periodic wave:** $v = \lambda f$ (Wave speed, Wavelength, Frequency)

Wave function for **a sinusoidal wave propagating in**

Amplitude, Position, Time

$$y(x,t) = A\cos(kx - \omega t)$$

Wave function for a sinusoidal Wave

Wave function for **a sinusoidal wave propagating in +x-direction**

$$y(x, t) = A\cos(kx - \omega t)$$

Amplitude: A; Position: x; Time: t; Wave number $= 2\pi/\lambda$; Angular frequency $= 2\pi f$

$$\omega = vk \qquad \text{(periodic wave)}$$

$$k = \frac{\omega}{v}$$

Wave function for **a sinusoidal wave propagating in +x-direction**

$$y(x, t) = A\cos\left[\omega\left(\frac{x}{v} - t\right)\right]$$

Amplitude: A; Position: x; Time: t; Angular frequency $= 2\pi f$; Wave speed: v

$$k = \frac{2\pi}{\lambda} \qquad \text{(wave number)}$$

Wave function for **a sinusoidal wave propagating in +x-direction**

$$y(x, t) = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

Amplitude: A; Position: x; Time: t; Wavelength: λ; Period: T

The one man holds one end of the clothesline taut and wiggles it up and down sinusoidally with frequency 2.00 Hz and amplitude 0.075 m. The wave speed on the clothesline is $v = 12\ \frac{m}{s}$. At $t = 0$ Throcky's end has maximum positive displacement and is instantaneously at rest. Assume that no wave bounces back from the far end. (a) Find the wave amplitude A, angular frequency ω, period T, wavelength λ, and wave number k. (b) Write a wave function describing the wave. (c) Write equations for the displacement, as a function of time, of Throcky's end of the clothesline and of a point 3.00 m from that end.

SOLUTION

EXECUTE: (a) The wave amplitude and frequency are the same as for the oscillations of Throcky's end of the clothesline, $A = 0.075$ m and $f = 2.00$ Hz. Hence

$$\omega = 2\pi f = \left(2\pi \frac{\text{rad}}{\text{cycle}}\right)\left(2.00 \frac{\text{cycles}}{\text{s}}\right) = 4.00\pi \text{ rad/s} = 12.6 \text{ rad/s}$$

The period is $T = 1/f = 0.500$ s, and from Eq. (15

$$\lambda = \frac{v}{f} = \frac{12.0 \text{ m/s}}{2.00 \text{ s}^{-1}} = 6.00 \text{ m}$$

We find the wave number

$$k = \frac{\omega}{v} = \frac{4.00\pi \text{ rad/s}}{12.0 \text{ m/s}} = 1.05 \text{ rad/m}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{6.00 \text{ m}} = 1.05 \text{ rad/m}$$

(b) We write the wave function using below equation and the values of A, T, and λ from part (a):

Wave function for a sinusoidal wave propagating in $+x$-direction (Amplitude, Position, Time, Wavelength, Period):

$$y(x,t) = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

$$y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

$$= (0.075\ \text{m})\cos 2\pi\left(\frac{x}{6.00\ \text{m}} - \frac{t}{0.500\ \text{s}}\right)$$

$$= (0.075\ \text{m})\cos[(1.05\ \text{rad/m})x - (12.6\ \text{rad/s})t]$$

We can also get this same expression from from below equation by using the values of ω and k from part (a).

Wave function for a sinusoidal wave propagating in $+x$-direction (Amplitude, Position, Time; Wave number $= 2\pi/\lambda$; Angular frequency $= 2\pi f$):

$$y(x,t) = A\cos(kx - \omega t)$$

(c) We can find the displacement as a function of time at x = 0 and x = +3.00 m by substituting these values into the wave function from part (b):

$$y(x=0,t) = (0.075\ \text{m})\cos 2\pi\left(\frac{0}{6.00\ \text{m}} - \frac{t}{0.500\ \text{s}}\right)$$

$$= (0.075\ \text{m})\cos(12.6\ \text{rad/s})t$$

$$y(x=+3.00\ \text{m},t) = (0.075\ \text{m})\cos 2\pi\left(\frac{3.00\ \text{m}}{6.00\ \text{m}} - \frac{t}{0.500\ \text{s}}\right)$$

$$= (0.075\ \text{m})\cos[\pi - (12.6\ \text{rad/s})t]$$

$$= -(0.075\ \text{m})\cos(12.6\ \text{rad/s})t$$

Q2 **Transverse waves on a string have wave speed $v = 8\frac{m}{s}$, amplitude 0.0700 m, and wavelength 0.320 m. The waves travel in the −*x*-direction, and at $t = 0$ the $x = 0$ end of the string has its maximum upward displacement. (a) Find the frequency, period, and wave number of these waves. (b) Write a wave function describing the wave. (c) Find the transverse displacement of a particle at $x = 0.360$ m at time $t = 0.150$ s. (d) How much time must elapse from the instant in part (c) until the particle at $x = 0.360$ m next has maximum upward displacement**

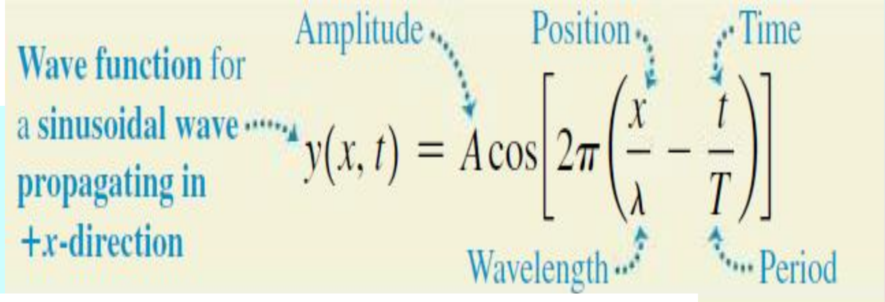
IDENTIFY: Use $v = f\lambda$ to calculate v. $T = 1/f$ and k is defined by $k = 2\pi/\lambda$. The general form of the wave function is given by $y(x, t) = A\cos 2\pi(x/\lambda + t/T)$, which is the equation for the transverse displacement.

SET UP: $v = 8.00$ m/s, $A = 0.0700$ m, $\lambda = 0.320$ m

EXECUTE: (a) $v = f\lambda$ so $f = v/\lambda = (8.00\text{ m/s})/(0.320\text{ m}) = 25.0\text{ Hz}$

$T = 1/f = 1/25.0\text{ Hz} = 0.0400\text{ s}$

$k = 2\pi/\lambda = 2\pi\text{ rad}/0.320\text{ m} = 19.6\text{ rad/m}$



(b) For a wave traveling in the −*x*-direction,

$y(x, t) = A\cos 2\pi(x/\lambda + t/T)$

At $x = 0$, $y(0, t) = A\cos 2\pi(t/T)$, so $y = A$ at $t = 0$. This equation describes the wave specified in the problem.

Substitute in numerical values:

$y(x, t) = (0.0700\text{ m})\cos[2\pi(x/(0.320\text{ m}) + t/(0.0400\text{ s}))]$.

Or, $y(x, t) = (0.0700\text{ m})\cos[(19.6\text{ m}^{-1})x + (157\text{ rad/s})t]$.

(c) From part (b), $y = (0.0700\text{ m})\cos[2\pi(x/0.320\text{ m} + t/0.0400\text{ s})]$.

Plug in $x = 0.360$ m and $t = 0.150$ s:

$y = (0.0700\text{ m})\cos[2\pi(0.360\text{ m}/0.320\text{ m} + 0.150\text{ s}/0.0400\text{ s})]$

$y = (0.0700\text{ m})\cos[2\pi(4.875\text{ rad})] = +0.0495\text{ m} = +4.95\text{ cm}$

(d) In part (c) $t = 0.150$ s.

$y = A$ means $\cos\left[2\pi(x/\lambda + t/T)\right] = 1$

$\cos\theta = 1$ for $\theta = 0,\ 2\pi,\ 4\pi, \ldots = n(2\pi)$ or $n = 0, 1, 2, \ldots$

So $y = A$ when $2\pi(x/\lambda + t/T) = n(2\pi)$ or $x/\lambda + t/T = n$

$t = T(n - x/\lambda) = (0.0400\ \text{s})(n - 0.360\ \text{m}/0.320\ \text{m}) = (0.0400\ \text{s})(n - 1.125)$

For $n = 4$, $t = 0.1150$ s (before the instant in part (c))

For $n = 5$, $t = 0.1550$ s (the first occurrence of $y = A$ after the instant in part (c)). Thus the elapsed time is $0.1550\ \text{s} - 0.1500\ \text{s} = 0.0050$ s.

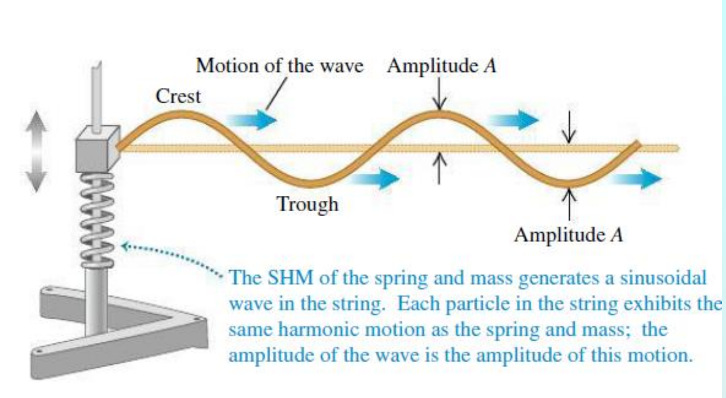
EVALUATE: Part (d) says $y = A$ at 0.115 s and next at 0.155 s; the difference between these two times is 0.040 s, which is the period. At $t = 0.150$ s the particle at $x = 0.360$ m is at $y = 4.95$ cm and traveling upward. It takes $T/4 = 0.0100$ s for it to travel from $y = 0$ to $y = A$, so our answer of 0.0050 s is reasonable.

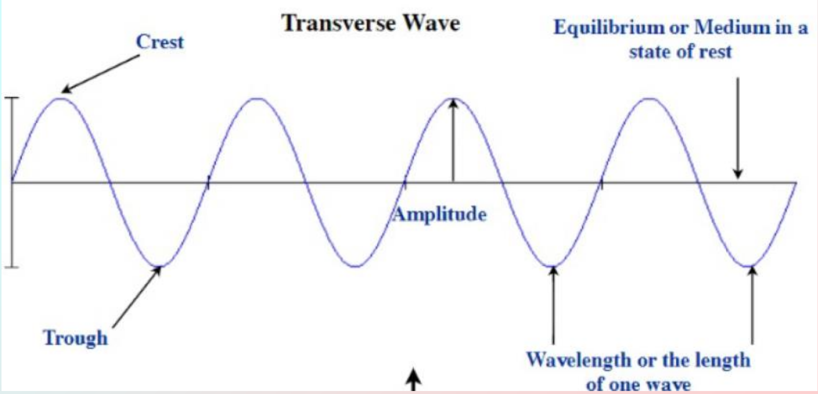
Transverse Waves

In transverse waves, the medium moves perpendicularly (90°) to the direction that the wave moves. The most commonly observed wave that has transverse wave properties is that of a water ripple that moves outward from a source of disturbance that acted at some point on the surface of the water. For these types of waves, the water roughly moves in an up and down pattern, which one observes as the waves moving outward. The energy of a transverse wave correlates to the square of the amplitude of the movement of the medium

This means that if the water surface is displaced 1 metre in causing a transverse wave motion, the energy of the waves that move away from the disturbance are 1/4 the energy carried by waves created if the water surface was displaced by 2 metres. Quantifying this relationship we get:

The Length of a Transverse wave is the Distance between the Crests (or Troughs) is called the Wavelength (λ)





Earthquakes

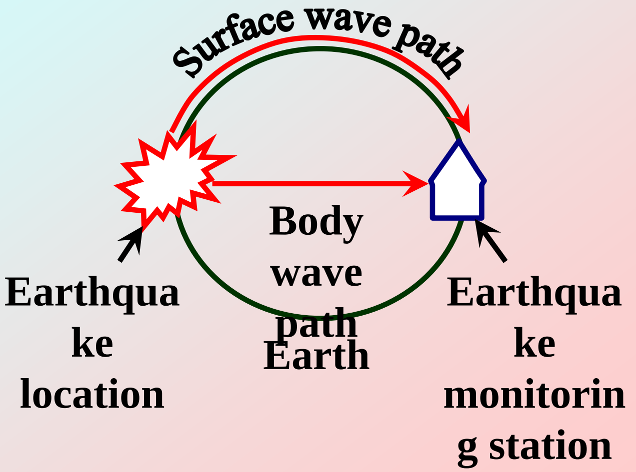
- Vibrations provide the energy for waves. The bigger the vibrations, the bigger the waves they create.
- The vibrations from earthquakes can cause waves in the ocean and in the ground.
- Tsunamis are giant ocean waves caused by undersea earthquakes.
- Earthquakes cause waves in the ground called seismic waves. Different types of seismic waves travel at different speeds:
  - P-waves are longitudinal waves; they travel the fastest.
  - S-waves are transverse waves.

- Earthquakes release energy that travels through and around the Earth in seismic waves.
- The two main types of seismic waves are body waves and surface waves.
  - Body waves can travel through Earth’s inner layers.
  - Surface waves move only along the surface.

Earthquake Waves

- The two types of body waves are primary (P) waves and secondary (S) waves.
  - P waves vibrate parallel to the direction they are traveling in a push-pull motion. P waves travel at a velocity of 4 to 6 km/s.
  - S waves vibrate perpendicular to their direction of travel. S waves travel at a velocity of 3 to 4 km/s.
- The two types of surface waves are the Love wave and the Rayleigh wave.
  - Love waves move the surface of the ground from side to side.
  - Rayleigh waves roll along the surface in a circular motion, like an ocean wave. Most of the shaking felt from an earthquake is caused by Rayleigh waves.

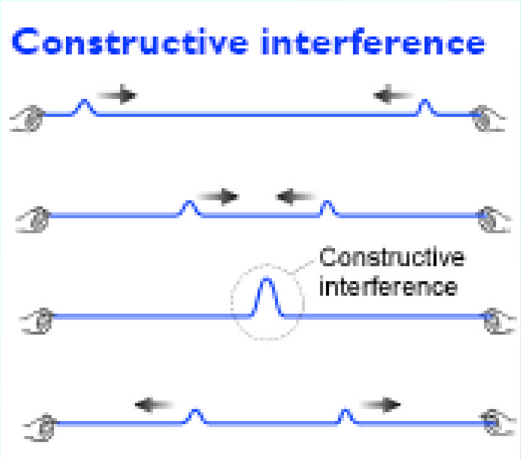
  - The Love wave is slightly faster than the Rayleigh wave, but both move at about 4 km/s.



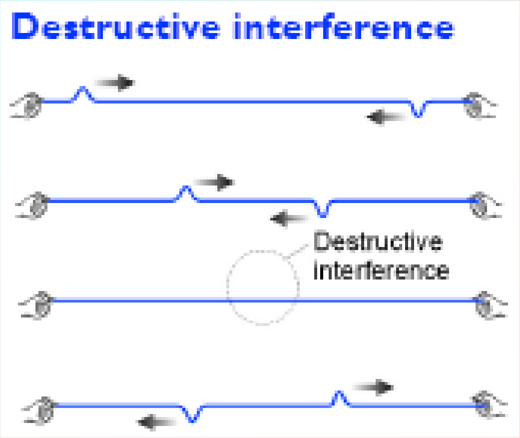
Wave Interactions – Interference

- When two or more waves are at the same place at the same time, the resulting effect is called interference.
- Waves overlap to form an interference pattern.
- The waves combine to form a single wave.

- Constructive interference - when the crest of one wave overlaps the crest of another, their individual effects add together. The result is a wave of increased amplitude.

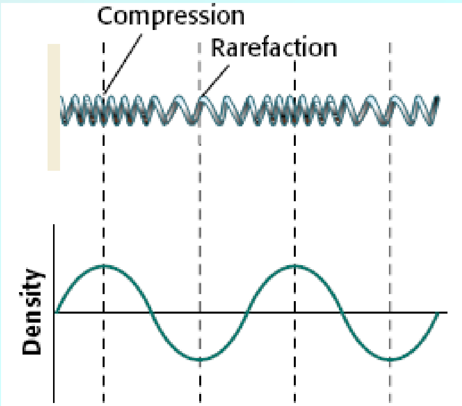


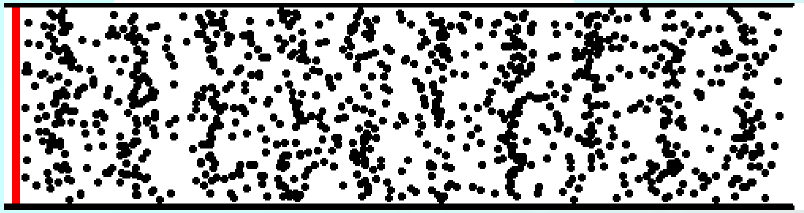
- Destructive interference – when the crest of one wave overlaps the trough of another, their individual effects are reduced. The result is a wave of decreased amplitude.

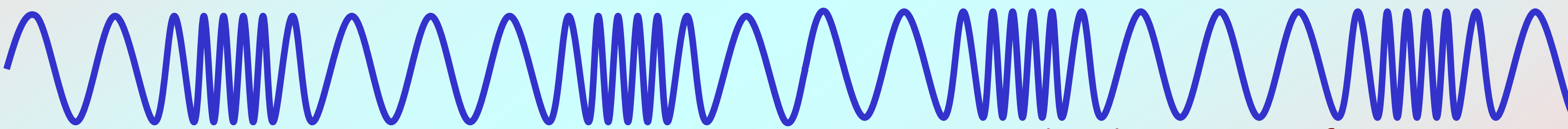


Sound in Air

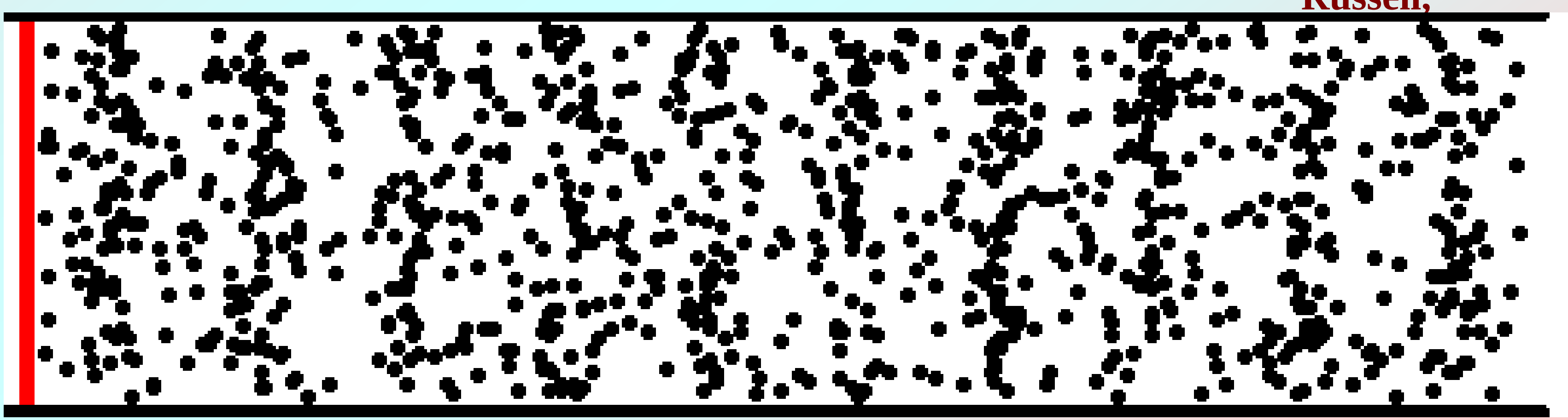
- Compression waves travel through air or along springs.
- These waves travel with areas of compressions and rarefactions.
- Sound is a mechanical wave – remember that it is not the medium that travels from one place to another, but the pulse that travels.







Animation courtesy of Dr. Dan Russell,



Natural Frequency

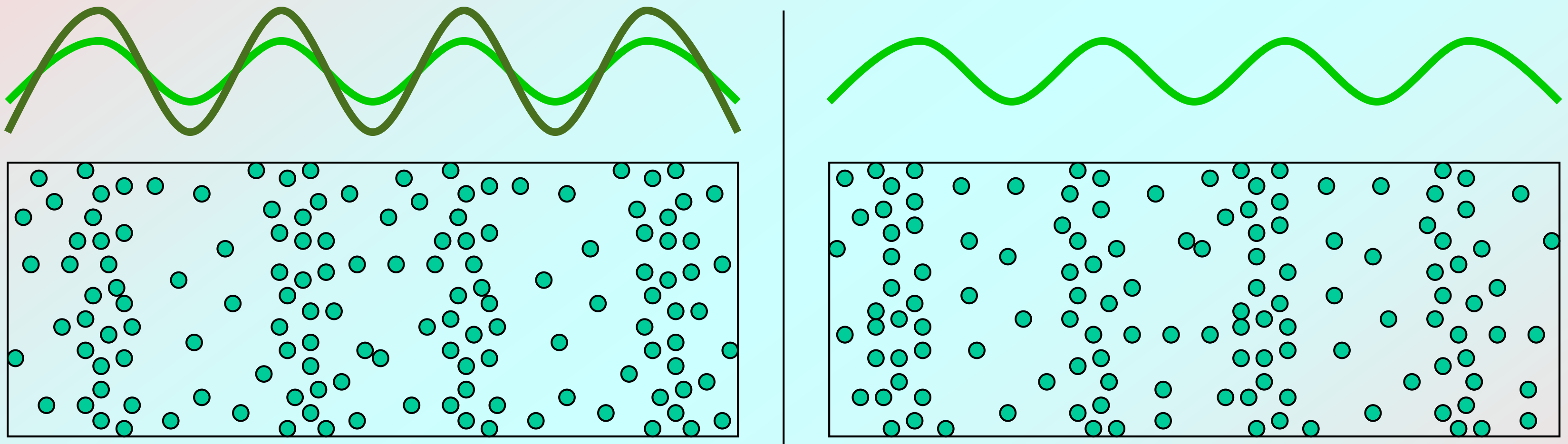
- **Everything vibrates, from planets and stars to atoms and almost everything in between.**
- **A natural frequency is one at which minimum energy is required to produce forced vibrations.**
- **It is also the frequency that requires the least amount of energy to continue this vibration.**
- **Natural frequencies depend on factors such as the elasticity and shape of the object.**

Resonance

- Every object vibrates at a characteristic frequency – this is its resonant, or natural, frequency.
- Resonance is a condition that exists when the frequency of a force applied to a system matches the natural frequency of the system.
- Examples of resonance are:
  - Pushing a swing
  - Tuning a radio station
  - Voice-shattered glass.
  - **Tacoma Narrows Bridge Collapse in 1940**.—High winds set up standing waves in the bridge, causing the bridge to oscillate at one of its natural frequencies.

Interference

- **Sound waves interfere with each other in the same way as all waves.**
- **Constructive interference - augmentation**



- **Destructive interference - cancellation**