General Science :Second stage

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Chapter Two Oscillatory Motion

Oscillations and Mechanical Wave

Periodic Motion

any kinds of motion repeat themselves over and over: the vibration of a quartz crystal in a watch, and the back-and-forth motion of the pistons in a car engine. This kind of motion, called periodic motion or oscillation,

amplitude, Period, Frequency, and angular Frequency

The amplitude of the motion, denoted by *A*, is the maximum magnitude of displacement from equilibrium—that is, the maximum value of x . It is always positive.

The period, *T*, is the time to complete one cycle. It is always positive. The SI unit is the second, but it is sometimes expressed as "seconds per cycle."

The frequency, *f*, is the number of cycles in a unit of time. It is always positive. The SI unit of frequency is the *hert*:

The angular frequency, ω *is* 2π times the frequency:

$$\omega = 2\pi f$$

By definition, period and frequency are reciprocals of each other:

In periodic motion **frequency and period** are reciprocals of each other.

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

(Period, Frequency)

Angular frequency related to frequency and period

$$\omega = 2\pi f = \frac{2\pi}{T}$$

(Frequency, Period)

Example :1 **PERIOD, FREQUENCY, AND ANGULAR FREQUENCY**

An ultrasonic transducer used for medical diagnosis oscillates at 6.7 MHz $= 6.7 \times 10^6$ Hz. How long does each oscillation take, and what is the angular frequency?

SOLUTION

$$T = \frac{1}{f} = \frac{1}{6.7 \times 10^6\ \text{Hz}} = 1.5 \times 10^{-7}\ \text{s} = 0.15\ \mu\text{s}$$

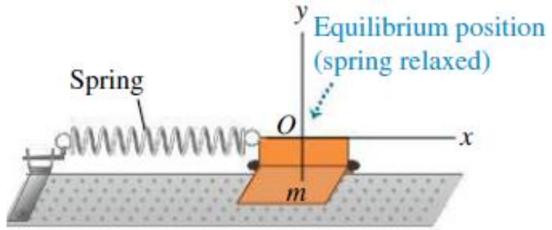
$$\omega = 2\pi f = 2\pi(6.7 \times 10^6\ \text{Hz})$$

$$= (2\pi\ \text{rad/cycle})(6.7 \times 10^6\ \text{cycle/s}) = 4.2 \times 10^7\ \text{rad/s}$$

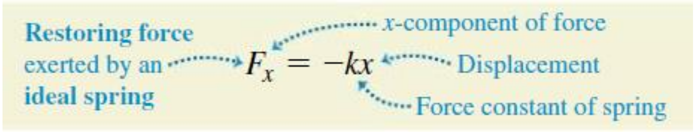
EVALUATE: This is a very rapid vibration, with large f and ω and small T. A slow vibration has small f and ω and large T.

Simple Harmonic motion

The simplest kind of oscillation occurs when the restoring force F_X is *directly proportional* to the displacement from equilibrium *x*. This happens if the spring in this figure is an ideal one that obeys *Hooke's law*. The constant of proportionality between F_X and X is the force constant *k*. On either side of the equilibrium position, F_X and X is always have opposite signs. The force acting *on* a stretched ideal spring as $F_X = -kX$.



The *x*-component of force the spring exerts *on the body* is the negative of this, so



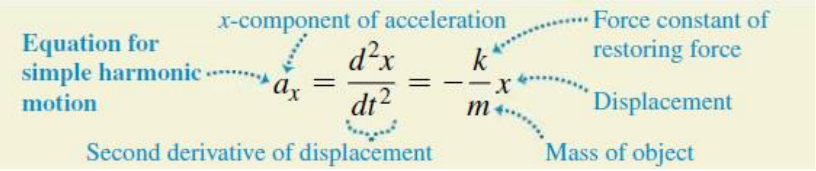
This equation gives the correct magnitude and sign of the force, whether *x* is positive, negative, or zero. The force constant *k* is always positive and has units of $\frac{N}{m}$ (a u. We are assuming that there is no friction.

When the restoring force is directly proportional to the displacement from equilibrium, Applying newton's second law of motion

$F_x = ma_x \Rightarrow a_x = \frac{F_x}{m}$ *the oscillation is called* simple harmonic motion (SHM). The acceleration $a_x = \frac{d^2x}{dt^2} = \frac{F_x}{m}$,

of a body in SHM is

$$a_x = \frac{F_x}{m} = \frac{kx}{m}$$



The minus sign means that, in SHM, the acceleration and displacement always have opposite signs. This acceleration is *not* constant,

Why is simple harmonic motion important? Not all periodic motions are simple harmonic; in periodic motion in general

Displacement, velocity, and acceleration in SHM

We still need to find the displacement *x* as a function of time for a harmonic Oscillator. Equation

Displacement in simple harmonic motion as a function of time: $x = A\cos(\omega t + \phi)$ (Amplitude A, Time t, Phase angle ϕ; Angular frequency $\omega = \sqrt{k/m}$)

Velocity of SHM $\quad v = \dfrac{dx}{dt}$

$$\frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \phi) = -\omega A\sin(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

Acceleration of SHM

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}\text{t}$$

$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \phi) = -\omega^2 A\cos(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

if $\boldsymbol{\omega = 2\pi f = \frac{2\pi}{T}}$ Substitute the value of angular frequency in the equation displacement, velocity and acceleration of SHM we get $\quad \omega = \dfrac{2\pi}{T}$

$$v = \lambda f = \frac{\lambda}{T} = \frac{\lambda}{T}\left(\frac{2\pi}{2\pi}\right) = \frac{2\pi}{T}\left(\frac{\lambda}{2\pi}\right) = \frac{\omega}{\text{K}}$$

Where $\boldsymbol{K = \frac{2\pi}{\lambda}}$ $\qquad \dfrac{1}{K} = \dfrac{\lambda}{2\pi} \qquad v = \lambda f = \dfrac{\omega}{K}$

Displacement of SHM $\quad x = A\cos(\omega t + \emptyset) = A\cos\left(\frac{2\pi}{T}t + \emptyset\right)$

Velocity of SHM $\quad v = -A\omega\sin(\omega t + \emptyset) = -A\omega\sin\left(\frac{2\pi}{T}t + \emptyset\right)$

Acceleration *of SHM* $\quad a = -A\omega^2(\omega t + \emptyset) = -A\omega^2\cos\left(\frac{2\pi}{T}t + \emptyset\right)$

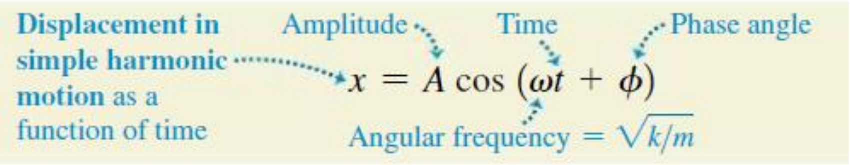
<u>Example determine : Displacement , *Velocity* and Acceleration of SHM at $t = 0$</u>

Solution $\quad x = A\cos\omega t = A\cos\frac{2\pi}{T}t = A\cos\frac{2\pi}{T}(0) = A\cos(0) = A \times 1 = A$

Velocity of SHM $\quad v = -A\omega\sin\frac{2\pi}{T}t = -A\omega\sin\frac{2\pi}{T}(0) = A\omega\sin(0) = A\omega \times 0 = 0$

Acceleration *of SHM* $\quad a = -A\omega^2\cos\frac{2\pi}{T}t = a = -A\omega^2\cos\frac{2\pi}{T}(0) = -A\omega^2\cos(0) = -A\omega^2\cos(0) = -A\omega^2 \times 1 = -A\omega^2$

Graphic diagram for displacement, velocity, and acceleration in SHM in periodic cycle.



$$v = \frac{dx}{dt} = \frac{d}{dt}A\cos(\omega t + \phi) = -\omega A\sin(\omega t + \phi)$$

$$v = -\omega A\sin(\omega t + \phi)$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\{-\omega A\sin(\omega t + \phi)\} = -\omega^2 A\cos(\omega t + \phi)$$

$$a = -\omega^2 A\cos(\omega t + \phi)$$

Displacement, velocity and acceleration in simple Harmonic Motion in one periodi cycle

$T = 2\pi = 360^0$ $\qquad \pi = \frac{T}{2}$

One period cycle is from $t = 0$ sec $to \quad t = 2\pi = T$

Displacement in SHM $\phi = 0$ $\qquad x = A\cos\omega t = A\cos 2\pi f t = A\cos\frac{2\pi}{T}t$ $\qquad x = A\cos\frac{2\pi}{T}t$

Displacement in SHM t $= 0$ $\qquad x = A\cos\frac{2\pi}{T}t = A\cos\frac{2\pi}{T}((0)) = A\cos(0) = A \times 1 = A$ $\qquad \underline{x = A}$

Displacement in SHM t $= \frac{\pi}{2} = \frac{1}{2}\left(\frac{T}{2}\right) = \frac{T}{4}$ $\qquad x = A\cos\frac{2\pi}{T}t = A\cos\frac{2\pi}{T}\left(\frac{T}{4}\right) = A\cos\frac{\pi}{2} = A \times 0 = 0$ $\qquad \underline{x = 0}$

Displacement in SHM t $= \pi = \left(\frac{T}{2}\right)$ $\qquad x = A\cos\frac{2\pi}{T}t = A\cos\frac{2\pi}{T}\left(\frac{T}{2}\right) = A\cos\pi = A \times (-) = -A$ $\qquad x = -A$

Displacement in SHM t $= \frac{3}{2}\pi = \frac{3}{2}\left(\frac{T}{2}\right) = \frac{3}{4}T$ $\qquad x = A\cos\frac{2\pi}{T}t = A\cos\frac{2\pi}{T}\left(\frac{3T}{4}\right) = A\cos\frac{3}{2}\pi = A \times (270) = A \times 0 = 0$ $\qquad \underline{x = 0}$

Displacement in SHM t $= 2\pi = 2\left(\frac{T}{2}\right) = T$ $\qquad x = A\cos\frac{2\pi}{T}t = A\cos\frac{2\pi}{T}T = A\cos 2\pi = A \times (360) = A \times 1 = A$ $\qquad x = A$

velocity in simple Harmonic Motion in one periodi cycle

$$v = -\omega A\sin(\omega t + \phi) = -\omega A\sin\omega t = -\omega A\sin 2\pi f t = -\omega A\sin\frac{2\pi}{T}t$$

$$v - \omega A\sin\frac{2\pi}{T}t$$

velocity in SHM $t = 0$

$$v - \omega A\sin\frac{2\pi}{T}t = -\omega A\sin\frac{2\pi}{T}(0) = -\omega A\sin(0) = \omega A \times (0) = 0$$

$$\boldsymbol{v = 0}$$

velocity in SHM $t = \frac{\pi}{2} = \frac{1}{2}\left(\frac{T}{2}\right) = \frac{T}{4}$

$$v = -\omega A\sin\frac{2\pi}{T}t = -\omega A\sin\frac{2\pi}{T}\left(\frac{T}{4}\right) = -\omega A\sin\left(\frac{\pi}{2}\right) = \omega A \times 1 = -\omega A$$

$$\boldsymbol{v = -\omega A}$$

Velocity in SHM $t = \pi = \left(\frac{T}{2}\right)$

$$v = -\omega A\sin\frac{2\pi}{T}t = -\omega A\sin\frac{2\pi}{T}\left(\frac{T}{2}\right) = -\omega A\sin\pi = \omega A \times 0 = 0$$

$$\boldsymbol{v = 0}$$

velocity in SHM $t = \frac{3}{2}\pi = \frac{3}{2}\left(\frac{T}{2}\right) = \frac{3}{4}T$

$$v = -\omega A\sin\frac{2\pi}{T}t = -\omega A\sin\frac{2\pi}{T}\left(\frac{3}{4}T\right) = -\omega A\sin\frac{3}{2}\pi = -\omega A\sin(270) = -\omega A(-1) = \omega A$$

$$\boldsymbol{v = \omega A}$$

velocity in SHM $t = 2\pi = 2\left(\frac{T}{2}\right) = T$

$$v = -\omega A\sin\frac{2\pi}{T}t = -\omega A\sin\frac{2\pi}{T}(T) = -\omega A\sin 2\pi = -\omega A \times \sin(360) = -\omega A(0) = 0$$

$$\boldsymbol{v = 0}$$

Acceleration *in simple Harmonic Motion in one periodi cycle*

$$a = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 A\cos\omega t = -\omega^2 A\cos\frac{2\pi}{T}t$$

$$a = -\omega^2 A\cos\frac{2\pi}{T}t$$

Acceleration in SHM $t = 0$

$$a = -\omega^2 A\cos\frac{2\pi}{T}t = -\omega^2 A\cos\frac{2\pi}{T}(0) = -\omega^2 A\cos 0 = -\omega^2 A \times 1 = -\omega^2 A$$

$$a = -\omega^2 A$$

Acceleration in SHM $t = \frac{\pi}{2} = \frac{1}{2}\left(\frac{T}{2}\right) = \frac{T}{4}$

$$a = -\omega^2 A\cos\frac{2\pi}{T}t = -\omega^2 A\cos\frac{2\pi}{T}\left(\frac{T}{4}\right) = -\omega^2 A\cos\left(\frac{\pi}{2}\right) = -\omega^2 A\cos(0) = -\omega^2 A \times 0 = 0$$

$$a = 0$$

Acceleration in SHM $t = \pi = \left(\frac{T}{2}\right)$

$$a = -\omega^2 A\cos\frac{2\pi}{T}t = -\omega^2 A\cos\frac{2\pi}{T}\left(\frac{T}{2}\right) = -\omega^2 A\cos(\pi) = -\omega^2 A(-1) = \omega^2 A$$

$$a = \omega^2 A$$

Acceleration in SHM $t = \frac{3}{2}\pi = \frac{3}{2}\left(\frac{T}{2}\right) = \frac{3}{4}T$

$$a = -\omega^2 A\cos\frac{2\pi}{T}t = -\omega^2 A\cos\frac{2\pi}{T}\left(\frac{3T}{4}\right) = -\omega^2 A\cos\left(\frac{3\pi}{2}\right) = -\omega^2 A\cos(270) = -\omega^2 A(0) = 0$$

$$a = 0$$

Acceleration in SHM $t = 2\pi = 2\left(\frac{T}{2}\right) = T$

$$a = -\omega^2 A\cos\frac{2\pi}{T}t = -\omega^2 A\cos\frac{2\pi}{T}(T) = -\omega^2 A\cos(2\pi) = -\omega^2 A\cos(360) = -\omega^2 A(1) = -\omega^2 A$$

$$a = -\omega^2 A$$

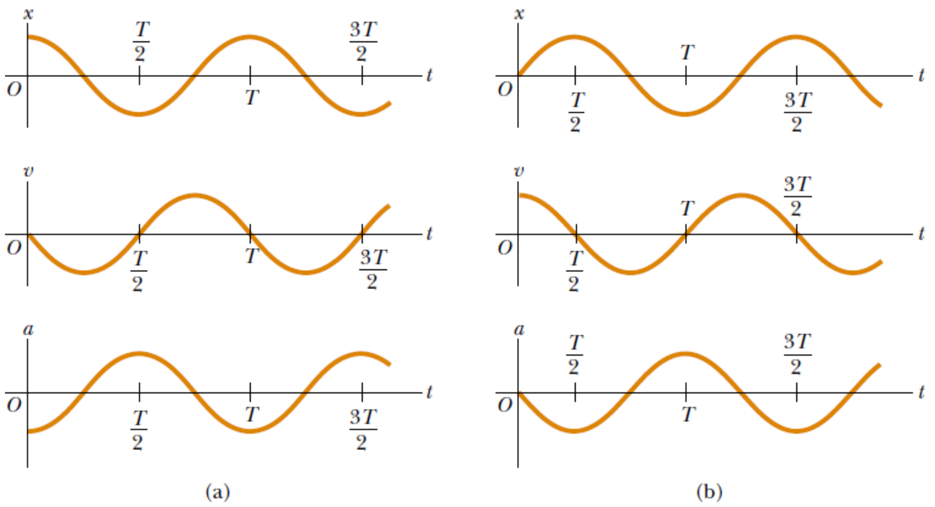


Figure 15.8 (a) Position, velocity, and acceleration versus time for a block undergoing simple harmonic motion under the initial conditions that at $t = 0, x(0) = A$ *and* $v(0) = 0$,.
(b) Position, velocity, and acceleration versus time for a block undergoing simple harmonic motion under the initial conditions that at $t = 0, x(0) = 0$ *and* $v(0) = v_p$.

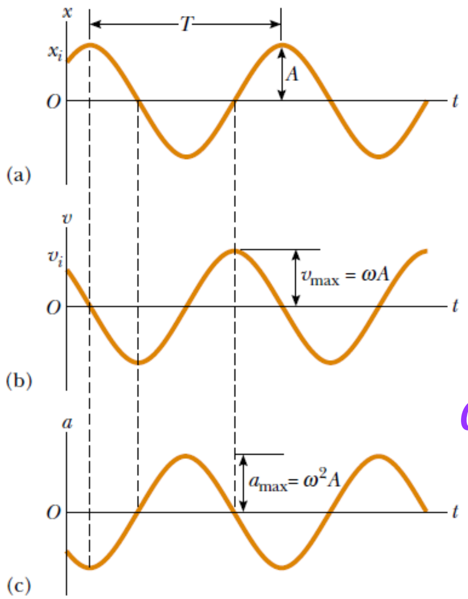
$$T = \frac{2\pi}{\omega}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



$$x_{max} = A$$

$$v_{max} = \omega A$$

$$a_{max} = \omega^2 A$$

figure Two Graphical representation of simple harmonic motion. (a) Position versus time. (b) Velocity versus time. (c) Acceleration versus time. Note that at any specified time the velocity is 90° out of phase with the position and the acceleration is 180° out of phase with the position.

t	x	v	a
0	A	0	$-\omega^2 A$
$T/4$	0	$-\omega A$	0
$T/2$	$-A$	0	$\omega^2 A$
$3T/4$	0	ωA	0
T	A	0	$-\omega^2 A$

That is, the period and frequency depend *only* on the mass of the particle and the force constant of the spring and *not* on the parameters of the motion, such as *A* or f. As we might expect, the frequency is larger for a stiffer spring (larger value of *k*) and decreases with increasing mass of the particle. We can obtain the velocity and acceleration2 of a particle undergoing simple harmonic motion from Equations

Example An object oscillates with simple harmonic motion along the x axis. Its position varies with time according to the equation

$$x = (4.00\text{ m})\cos\left(\pi t + \frac{\pi}{4}\right)$$

where t is in seconds and the angles in the parentheses are in radians.

(A) Determine the amplitude, frequency, and period of the motion.

Solution By comparing this equation with Equation

$$x = (4.00\text{ m})\cos\left(\pi t + \frac{\pi}{4}\right)$$

$x = A\cos(\omega t + \phi)$, we see that $A =$ 4.00 m and $\omega = \pi$ rad/s. Therefore, $f = \omega/2\pi = \pi/2\pi =$ 0.500 Hz and $T = 1/f =$ 2.00 s

(B) Calculate the velocity and acceleration of the object at any time t.

Solution Differentiating x to find v, and v to find a, we obtain

$$x = (4.00\text{ m})\cos\left(\pi t + \frac{\pi}{4}\right)$$

$$v = \frac{dx}{dt} = -(4.00\text{ m/s})\sin\left(\pi t + \frac{\pi}{4}\right)\frac{d}{dt}(\pi t)$$

$$= -(4.00\pi\text{ m/s})\sin\left(\pi t + \frac{\pi}{4}\right)$$

$$a = \frac{dv}{dt} = -(4.00\pi\text{ m/s})\cos\left(\pi t + \frac{\pi}{4}\right)\frac{d}{dt}(\pi t)$$

$$= -(4.00\pi^2\text{ m/s}^2)\cos\left(\pi t + \frac{\pi}{4}\right)$$

(C) Using the results of part (B), determine the position, velocity, and acceleration of the object at $t = 1.00$ s.

Solution Noting that the angles in the trigonometric functions are in radians, we obtain, at $t = 1.00$ s,

$$x = (4.00\text{ m})\cos\left(\pi + \frac{\pi}{4}\right) = (4.00\text{ m})\cos\left(\frac{5\pi}{4}\right)$$

$$= (4.00\text{ m})(-0.707) = -2.83\text{ m}$$

$$v = -(4.00\pi\text{ m/s})\sin\left(\frac{5\pi}{4}\right)$$

$$= -(4.00\pi\text{ m/s})(-0.707) = 8.89\text{ m/s}$$

$$a = -(4.00\pi^2\text{ m/s}^2)\cos\left(\frac{5\pi}{4}\right)$$

$$= -(4.00\pi^2\text{ m/s}^2)(-0.707) = 27.9\text{ m/s}^2$$

(D) Determine the maximum speed and maximum acceleration of the object.

Solution In the general expressions for v and a found in part (B), we use the fact that the maximum values of the sine and cosine functions are unity. Therefore, v varies between $\pm 4.00\pi$ m/s, and a varies between $\pm 4.00\pi^2$ m/s^2. Thus,

$$v_{max} = 4.00\pi\text{ m/s} = 12.6\text{ m/s}$$

$$a_{max} = 4.00\pi^2\text{ m/s}^2 = 39.5\text{ m/s}^2$$

We obtain the same results using the relations $v_{max} = \omega A$ and $a_{max} = \omega^2 A$, where $A = 4.00$ m and $\omega = \pi$ rad/s.

(E) Find the displacement of the object between $t = 0$ and $t = 1.00$ s.

Solution The position at $t = 0$ is

$$x_i = (4.00 \text{ m}) \cos\left(0 + \frac{\pi}{4}\right) = (4.00 \text{ m})(0.707) = 2.83 \text{ m}$$

In part (C), we found that the position at $t = 1.00$ s is -2.83 m; therefore, the displacement between $t = 0$ and $t = 1.00$ s is

$$\Delta x = x_f - x_i = -2.83 \text{ m} - 2.83 \text{ m} = -5.66 \text{ m}$$

Because the object's velocity changes sign during the first second, the magnitude of Δx is not the same as the distance traveled in the first second. (By the time the first second is over, the object has been through the point $x = -2.83$ m once, traveled to $x = -4.00$ m, and come back to $x = -2.83$ m.)

Q1 An object is moving with SHM of amplitude *A* on the end of a spring. If the amplitude is doubled, what happens to the total distance the object travels in one period? What happens to the period? What happens to the maximum speed of the object? Discuss how these answers are related.

SET UP: The frequency f in Hz is the number of cycles per second. The angular frequency ω is $\omega = 2\pi f$ and has units of radians per second. The period T is the time for one cycle of the wave and has units of seconds. The period and frequency are related by $T = \frac{1}{f}$.

EXECUTE: **(a)** $T = \frac{1}{f} = \frac{1}{466 \text{ Hz}} = 2.15\times10^{-3}$ s.

$\omega = 2\pi f = 2\pi(466 \text{ Hz}) = 2.93\times10^{3}$ rad/s.

(b) $f = \frac{1}{T} = \frac{1}{50.0\times10^{-6} \text{ s}} = 2.00\times10^{4}$ Hz. $\omega = 2\pi f = 1.26\times10^{5}$ rad/s.

(c) $f = \frac{\omega}{2\pi}$ so f ranges from $\frac{2.7\times10^{15} \text{ rad/s}}{2\pi \text{ rad}} = 4.3\times10^{14}$ Hz to $\frac{4.7\times10^{15} \text{ rad/s}}{2\pi \text{ rad}} = 7.5\times10^{14}$ Hz. $T = \frac{1}{f}$ so T ranges from $\frac{1}{7.5\times10^{14} \text{ Hz}} = 1.3\times10^{-15}$ s to $\frac{1}{4.3\times10^{14} \text{ Hz}} = 2.3\times10^{-15}$ s.

(d) $T = \frac{1}{f} = \frac{1}{5.0\times10^{6} \text{ Hz}} = 2.0\times10^{-7}$ s and $\omega = 2\pi f = 2\pi(5.0\times10^{6} \text{ Hz}) = 3.1\times10^{7}$ rad/s.

EVALUATE: Visible light has much higher frequency than either sounds we can hear or ultrasound. Ultrasound is sound with frequencies higher than what the ear can hear. Large f corresponds to small T.

Energy of the Simple Harmonic Oscillator

Let us examine the mechanical energy of a system in which a particle undergoes simple harmonic motion, such as the block–spring system illustrated in this Figure. Because the surface is frictionless, the system is isolated and we expect the total mechanical energy of the system to be constant. We assume a massless spring, so the kinetic energy of the system corresponds only to that of the block. We can use Equation to express the kinetic energy of the block as

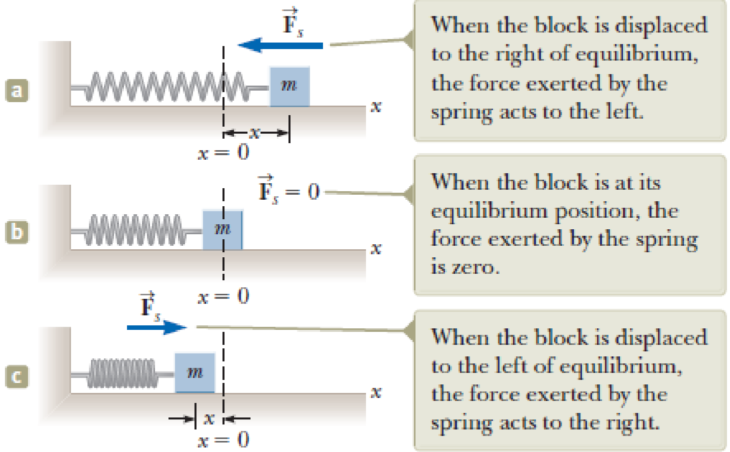


Figure ( ) A block attached to a spring moving on a friction-less surface.

The back and forth movements of such an object are called oscillations. We will focus our attention on a special case of periodic motion called simple harmonic motion. We shall find that all periodic motions can be modeled as combinations of simple harmonic motions. Thus, simple harmonic motion forms a basic building block for more complicated periodic motion.

Simple harmonic motion also forms the basis for our understanding of mechanical waves. Sound waves, seismic waves, waves on stretched strings, and water waves are all produced by some source of oscillation.

Oscillations is a special type of motion called periodic motion. This is a repeating motion of an object in which the object continues to return to a given position after a fixed time interval. Familiar objects that exhibit periodic motion include a pendulum.

Energy in simple Harmonic motion

We can learn even more about simple harmonic motion by using energy considerations. The only horizontal force on the body in SHM in Figs. is the conservative force exerted by an ideal spring. The vertical forces do no work, so the total mechanical energy of the system is *conserved*. We also assume that the mass of the spring itself is negligible.

The kinetic energy of the body is $E_K = \frac{1}{2}mv^2$ **and the potential energy of the spring is** $E_P(U) = \frac{1}{2}kx^2$**, just as in Section**

There are no nonconservative forces that do work, so the total mechanical energy $E_T = E_K + E_P(U)$ **,** E_T **is conserved:**

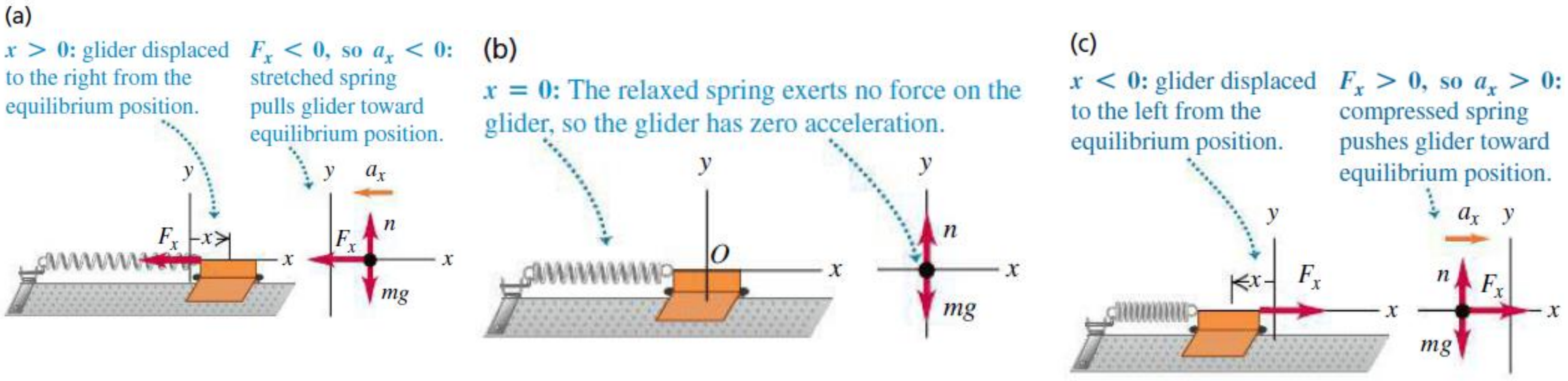
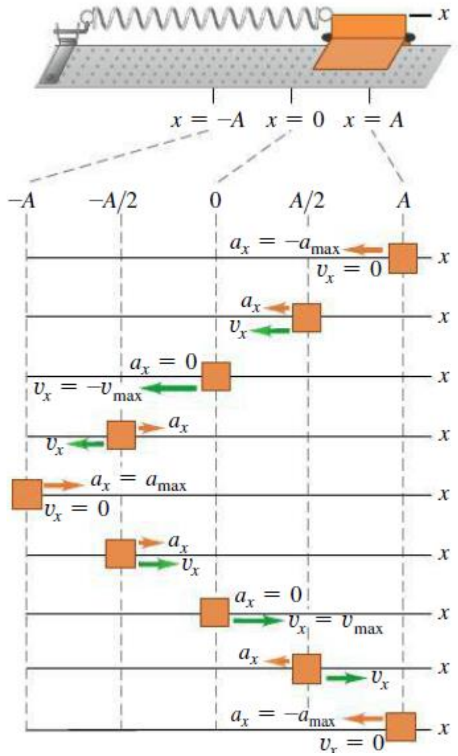


Figure ( )Model for periodic motion. When the body is displaced from its equilibrium position at x = 0, the spring exerts a restoring force back toward the equilibrium position.



How x-velocity vx and x-acceleration ax vary during one cycle of SHM

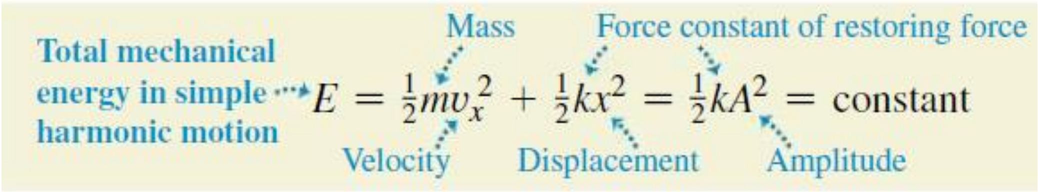
Total energy is constant

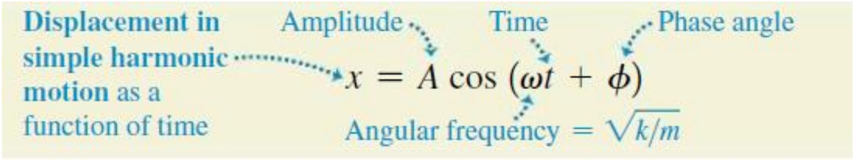
$$E_T = E_K + E_P(U)$$

$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = constant$$

$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

(Since the motion is one-dimensional, $v^2 = v_x^2$ The total mechanical energy E_T is also directly related to the amplitude *A* of the motion. When the body reaches the point *x* = *A*, its maximum displacement from equilibrium, it momentarily stops as it turns back toward the equilibrium position. That is, when *x* = *A* (or -*A*), $v_x = 0$. At this point the energy is entirely potential, and $E_T = \frac{1}{2}kA^2$. Because *E* is constant, it is equal to $\frac{1}{2}kA^2$ at any other point. Combining this expression with, we get





Displacement, velocity, and acceleration in SHM

Velocity (v) $v = \frac{dx}{dt} = \frac{d}{dt}\left(A cos(\omega t + \emptyset)\right)$

$$v = \frac{dx}{dt} = A = -\omega A\left(sin(\omega t + \emptyset)\right)$$

acceleration in SHM a$= \frac{dv}{dt} = -\omega A \frac{d}{dx}\left(sin(\omega t + \emptyset)\right)$

a$= \frac{dv}{dt} = -\omega^2 A\left(cos(\omega t + \emptyset)\right)$

a$= \frac{dv}{dt} = -\omega^2 x$

We can verify this equation (total energy) by substituting the equation of displacement *x* and the equation of velocity v_x *from equation of total energy*

$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E_T = \frac{1}{2}mA^2\frac{K}{m}\left(sin(\omega t + \emptyset)\right)^2 + \frac{1}{2}kA^2\{cos(\omega t + \emptyset)\}^2$$

By substitute the equation of angle frequency for simple harmonic motion from the above equation we get the

Angular frequency for simple harmonic motion → $\omega = \sqrt{\frac{k}{m}}$ ← Force constant of restoring force; ← Mass of object

$$\omega^2 = \frac{k}{m}$$

$$E_T = \frac{1}{2}m\{-\omega A sin(\omega t + \emptyset)\}^2 + \frac{1}{2}k\{Acos(\omega t + \emptyset)\}^2$$

$$E_T = \frac{1}{2}mA^2\omega^2\left(sin(\omega t + \emptyset)\right)^2 + \frac{1}{2}k\{Acos(\omega t + \emptyset)\}^2$$

$$E_T = \frac{1}{2}mA^2\frac{K}{m}\left(sin(\omega t + \emptyset)\right)^2 + \frac{1}{2}kA^2\{cos(\omega t + \emptyset)\}^2$$

$$E_T = \frac{1}{2}kA^2\left(sin(\omega t + \emptyset)\right)^2 + \frac{1}{2}kA^2\{cos(\omega t + \emptyset)\}^2$$

$$\left(sin(\omega t + \emptyset)\right)^2 + \{cos(\omega t + \emptyset)\}^2 = 1$$

$$E_T = \frac{1}{2}kA^2\left[\left(sin(\omega t + \emptyset)\right)^2 + \{cos(\omega t + \emptyset)\}^2\right] = \frac{1}{2}kA^2$$

$$E_T = \frac{1}{2}kA^2$$

Total mechanical energy in simple harmonic motion → $E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \text{constant}$ (Mass, Velocity, Force constant of restoring force, Displacement, Amplitude)

$$v_x = \mp\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

That is, the total mechanical energy of a simple harmonic oscillator is a constant of the motion and is proportional to the square of the amplitude. The total mechanical energy is equal to the maximum potential energy stored in the spring when $x = \mp A$ because $v = 0$ at these points and there is no kinetic energy. At the equilibrium position, where $E_P(U) = 0$ because $x = 0$, the total energy, all in the form of kinetic energy, is again $\frac{1}{2}kA^2$.

Hence our expressions for displacement and velocity in SHM are consistent with energy conservation, as they must be. We can use above equation to solve for the velocity v_x of the body at a given displacement x:

$$E_T = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$\frac{1}{2}mv_x^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

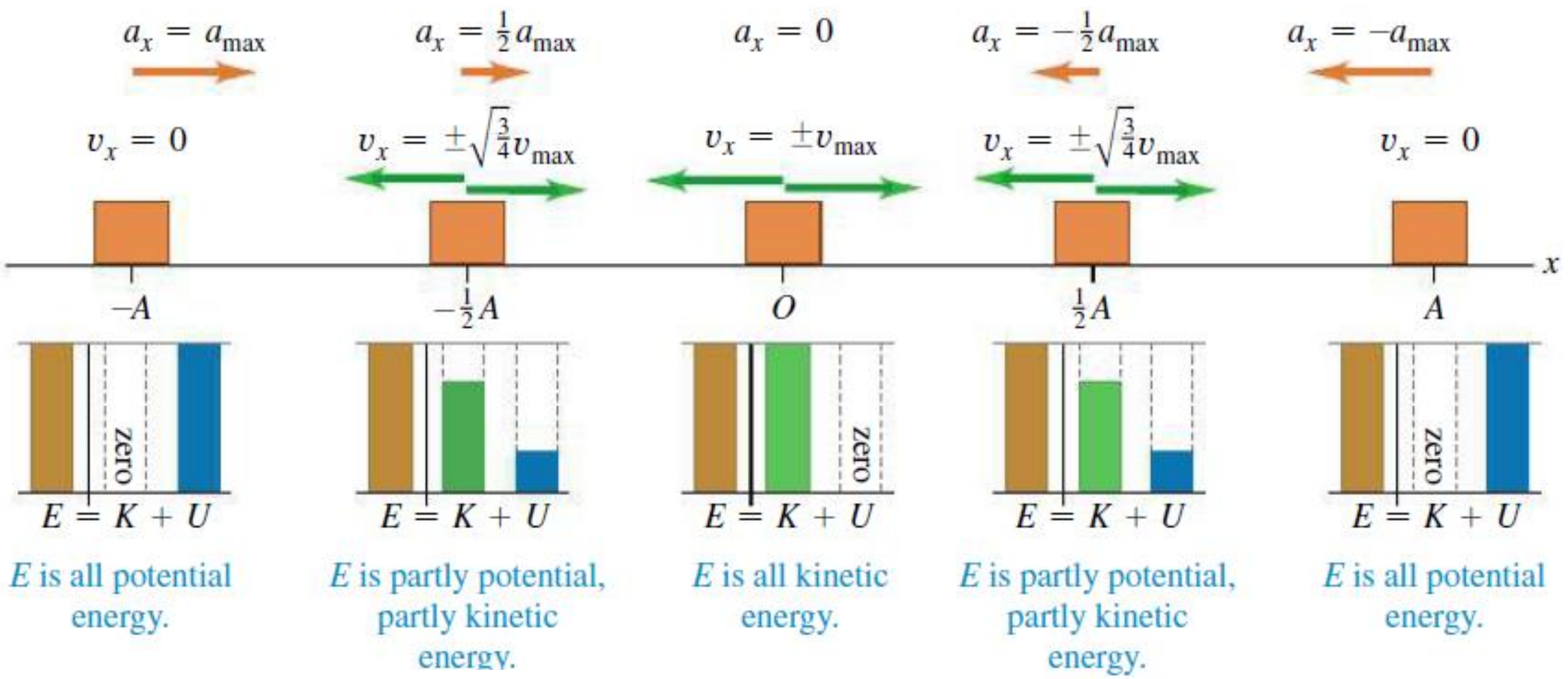
$$mv_x^2 = kA^2 - kA^2$$

$$v_x^2 = \frac{kA^2}{m} - \frac{kx^2}{m}$$

$$v_x^2 = \frac{k}{m}A^2 - \frac{k}{m}x^2$$

$$v_x^2 = \frac{k}{m}(A^2 - x^2)$$

The sign means that at a given value of x the body can be moving in either direction. For example, when $x = \mp\frac{A}{2}$,



Graphs of *E*, *K*, and *U* versus displacement in SHM. The velocity of the body is *not* constant, so these images of the body at equally spaced positions are *not* equally spaced in time.

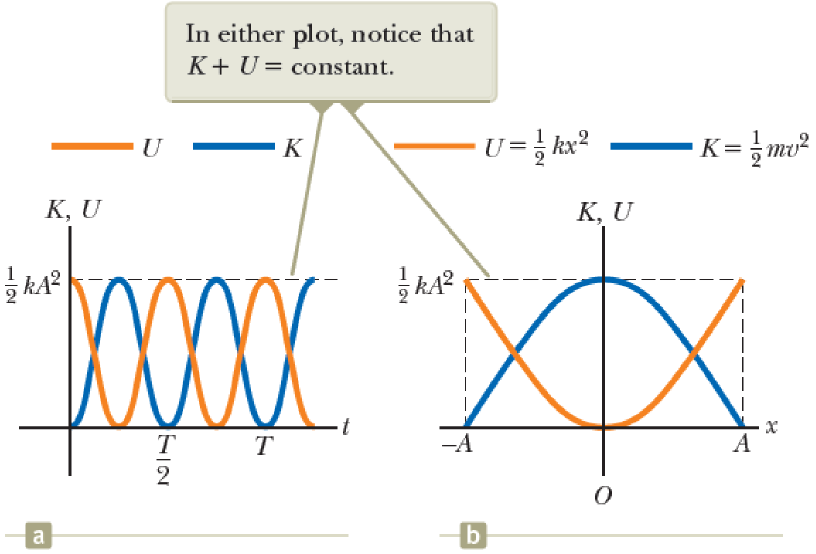
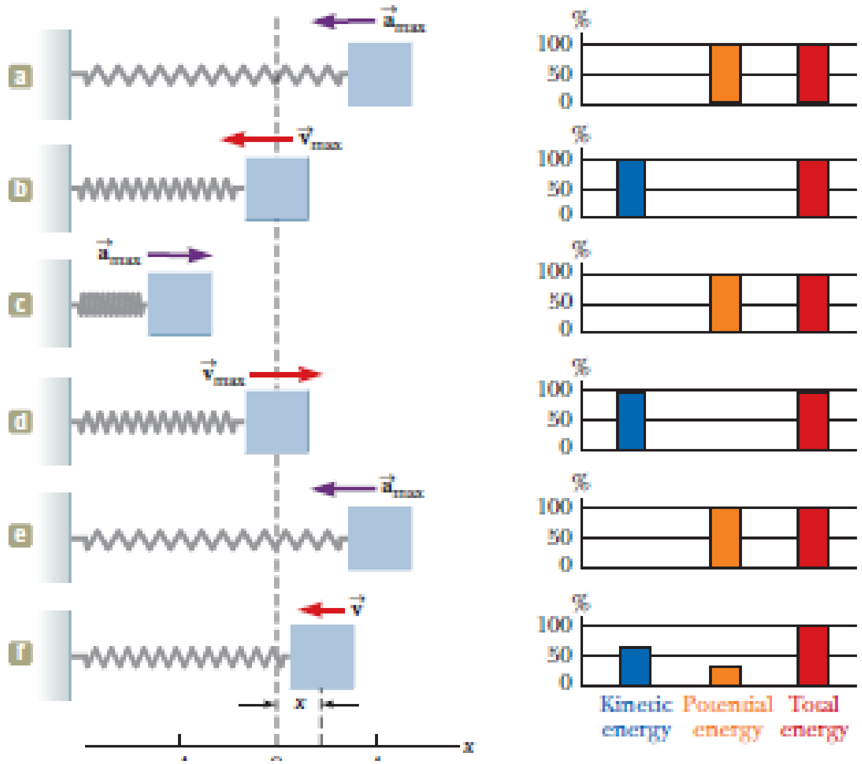


Figure (a) Kinetic energy and potential energy versus time for a simple harmonic oscillator with $\emptyset = 0$. (b) Kinetic energy and potential energy versus position for a simple harmonic oscillator.



t	x	v	a	K	U
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
$\frac{T}{4}$	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
$\frac{T}{2}$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$\frac{3T}{4}$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
t	x	v	$-\omega^2 x$	$\frac{1}{2}mv^2$	$\frac{1}{2}kx^2$

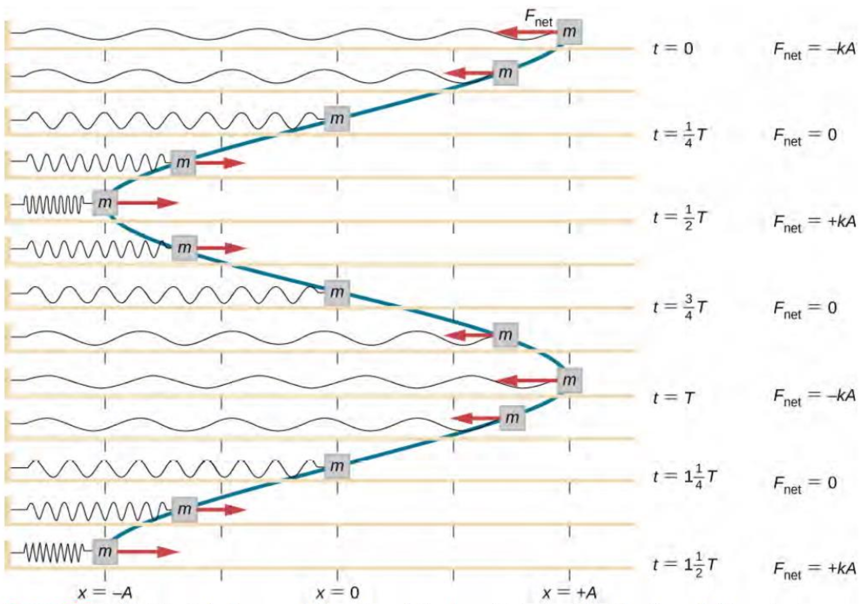


Figure 15.5 A block is attached to one end of a spring and placed on a frictionless table. The other end of the spring is anchored to the wall. The equilibrium position, where the net force equals zero, is marked as $x = 0$ m. Work is done on the block, pulling it out to $x = +A$, and the block is released from rest. The block oscillates between $x = +A$ and $x = -A$. The force is also shown as a vector.

Figure (a) through (e) Several instants in the simple harmonic motion for a block–spring system. Energy bar graphs show the distri-bution of the energy of the system at each instant. The parameters in the table at the right refer to the block–spring system, assuming at $t = 0$, $x = A$; hence, $x = A \cos \omega t$. For these five special instants, one of the types of energy is zero. (f) An arbitrary point in the motion of the oscilla-tor. The system possesses both kinetic energy and potential energy at this instant as shown in the bar graph.

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