Kurdistan Region
Salahaddin University-Erbil
College of Engineering
Mechanic & Mechatronics Engineering Department

Strength of Materials

For Second Stage Students
In Mechanic & Mechatronics Dept.

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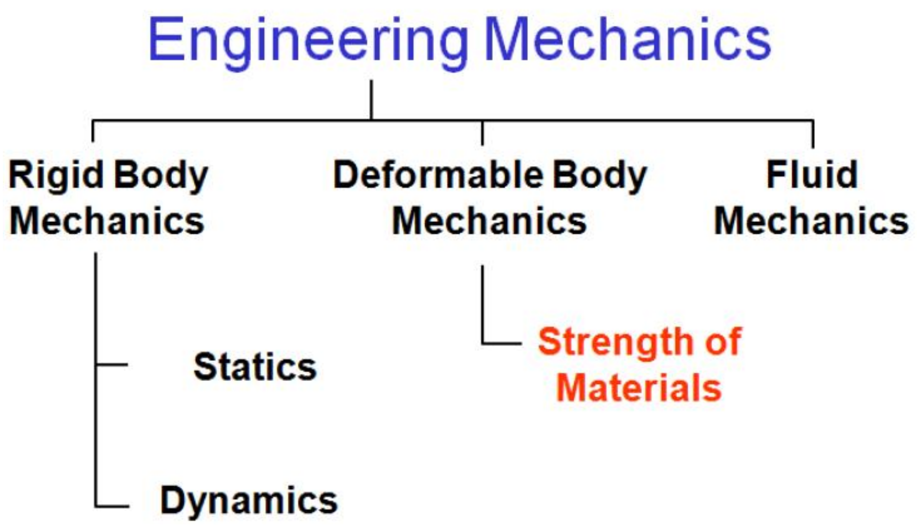
Classification of Engineering Mechanics



Strength of materials

It deals with the relations between externally applied forces or loads and the internal effects in the body.

When materials are loaded, they first deform before actual failure takes place. Hence before selecting any material for engineering purpose, it is important to know the behavior of the material under the action of loads.

Unit: - it is defined as the numerical standard used to measure the qualitative dimension of a physical quantity.

TABLE 1–1 Basic quantities in the SI metric unit system.

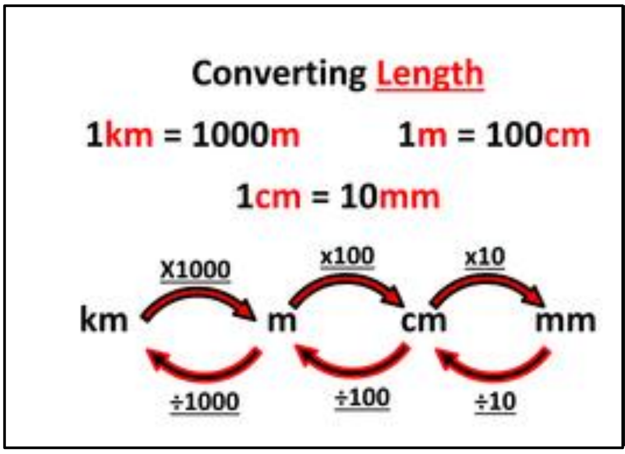
Quantity	SI unit	Other metric units
Length	Meter (m)	Millimeter (mm)
Time	Second (s)	Minute (min), hour (h)
Force	Newton (N)	$kg \cdot m/s^2$
Mass	Kilogram (kg)	$N \cdot s^2/m$
Temperature	Kelvin (K)	Degrees Celsius (°C)
Angle	Radian (rad)	Degree (°)

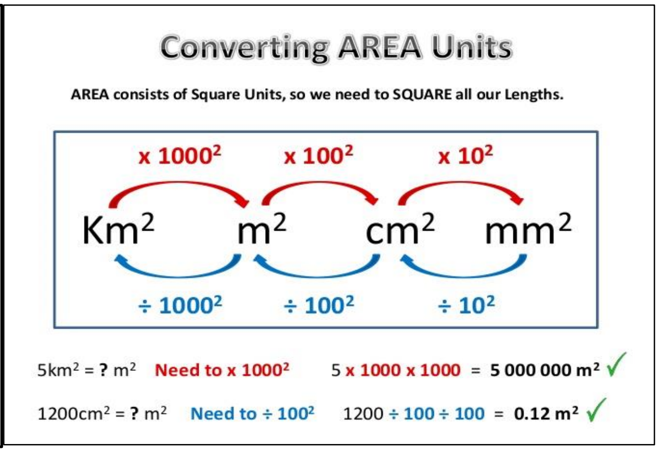
TABLE 1–2 Basic quantities in the U.S. Customary unit system.

Quantity	U.S. Customary unit	Other U.S. units
Length	Foot (ft)	Inch (in.)
Time	Second (s)	Minute (min), hour (h)
Force	Pound (lb)	kip[a]
Mass	Slug	$lb \cdot s^2/ft$
Temperature	Degrees Fahrenheit (°F)	
Angle	Degree (°)	Radian (rad)

TABLE 1–3 Prefixes for SI units.

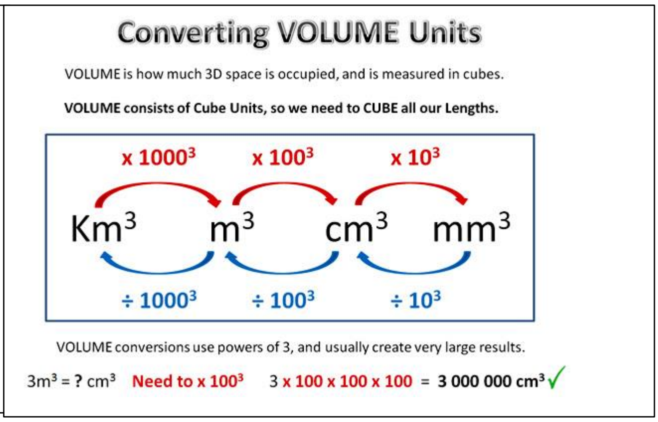
Prefix	SI symbol	Factor
Giga	G	$10^9 = 1\,000\,000\,000$
Mega	M	$10^6 = 1\,000\,000$
Kilo	k	$10^3 = 1000$
Milli	m	$10^{-3} = 0.001$
Micro	μ	$10^{-6} = 0.000\,001$





Customary Units of Length

1 foot (ft) = 12 inches (in)
1 yard (yd) = 3 feet (ft)
1 yard (yd) = 36 inches (in)
1 mile (mi) = 1,760 yards (yd)
1 mile (mi) = 5,280 feet (ft)



Mechanical properties of material

The following are considered as the most important properties of engineering materials

1) Elasticity
2) Plasticity
3) Ductility
4) Malleability
5) Brittleness
6) Toughness
7) Hardness

Any material cannot possess all the above properties because the different properties oppose each other. Hence the engineering Materials can be classified as follows depending upon their Mechanical properties

1) Elastic Materials: - These are materials which undergo deformation due to application of forces and once the forces are removed the material regains its original shape.

2) Plastic materials: - These are materials which do not regain their original shape even after the external loads acting on the material are removed.

3) Ductility: - It is a measure of a material's ability to undergo significant plastic deformation before rupture, which may be expressed as percent elongation or percent area reduction from a tensile test.

4) Malleable materials: - These are Materials which can be extended in two directions easily or in simple terms, materials which can be beaten into thin sheets.

5) Brittle materials: - these are materials which do not undergo any deformation before failure when external forces act on them.

6) Tough materials: - These are materials which can resist sudden loads or shock loads without showing any fracture on failure

7) Hard material: - These are materials that have the ability to resist surface abrasion or indentation.

Various tests are carried out on engineering materials to assess their mechanical properties in a material testing laboratory. They are

1) Tension test
2) Compression test
3) Impact test
4) Shear test
5) Torsion test
6) Bending test
7) Fatigue test
8) Hardness test

Stress

Whenever some external forces act on a body it sets up a deformation and the body offer some resistance against deformation. this resistance per unit area to deformation is known as “stress”.

Types of Stresses

The various types of stresses may be classified as:

1- **Simple or Direct Stresses**

   i. Tension Stress   ii. Compression Stress   iii Shear Stress

2- **Indirect Stresses**

   i. Bending Stress   ii. Torsion Stress

3- **Combined Stresses, any possible combination 1 and 2.**

Simple stresses

Simple stress is offen called direct stress because it develope under direct loading conditions. That is, simple tension and simple compression occur when the applied force or load, is in line with the axis of the member (fig. 1.1 and 1.2) and simple shear occurs, when equal, parallel and opposite forces tend to cause a surface to slide relative to the adjacent surface (fig. 1.3).

Fig. 1.1. Tensile stress

Fig. 1.2. Compressive stress

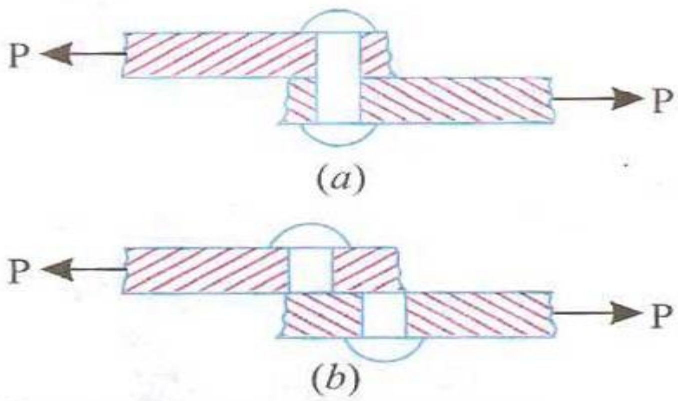


Fig. 1.3. (*a*) Rivet resisting shear.
(*b*) Rivet failure due to shear.

When simple stress (tension and compression) (σ) (sigma) develops, **we can calculate the magnitude of the tensile or compression stress by,**

$$\boxed{stress = \frac{force}{area}} = \boxed{\sigma_{t,c} = \frac{p}{A}}$$

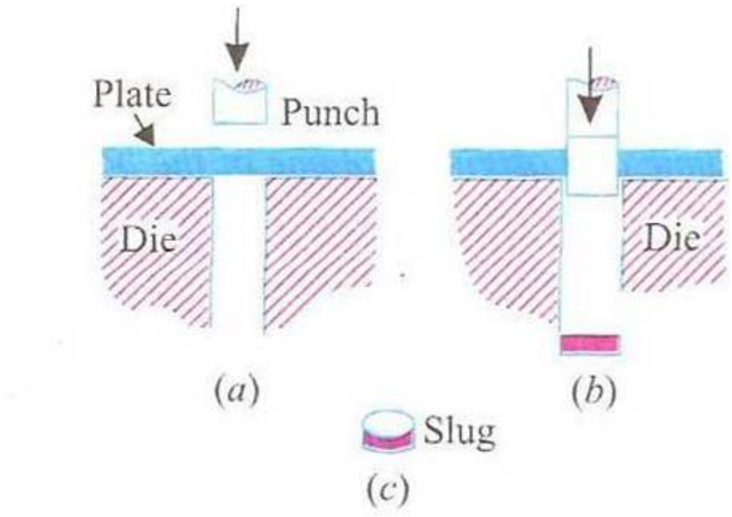
Where σ = Stress (also called intensity of Stress) kN/m^2 or N/mm^2

P = External Force or Load (kN or N)

A = Cross-Sectional Area of the body (m^2 or mm^2)

It may be noted that in cases of either simple tension or simple compression, the areas which resist the load are perpendicular to the direction of forces. When a member is subjected to simple shear, the resisting area is parallel to the direction of the force. Common situations causing shear stresses are shown in fig. 1.3 and 1.4. **we can calculate the magnitude of the shear stress by,**

$$\boxed{\tau = \frac{P}{A}}$$



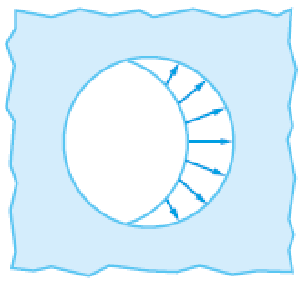


Fig. 1.4.(a) Punch approaching plate;
(b) Punch shearing plate;
(c) Slug showing sheared area.

Stress Units

Psi (Pound per square inch)

Kpsi (Kilo pound per square inch)

In SI Unit

Pa (Pascal = N/m^2)

Kpa (Kilo Pa = 1000 Pa)

Mpa (Mega Pa = 10^6)

Strain

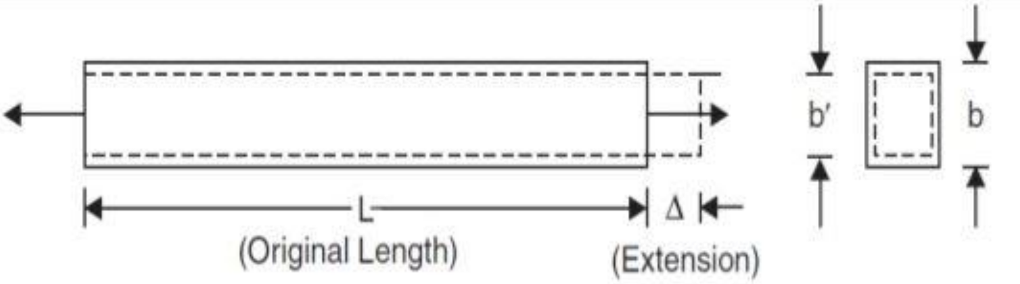
Any element in a material subjected to stress is said to be strained. The strain (ε) is the deformation produced by stress. The various types of strains are explained below:

1. Tensile strain
2. Compressive strain
3. Volumetric strain
4. Shear strain

1- **<u>Tensile Strain</u>**

A piece of material, with uniform cross-section, subjected to a uniform axial tensile stress, will increase its length from (L) to (L + δL), and the increment of length δL is the actual deformation of the material. The fractional deformation or the tensile strain is given by

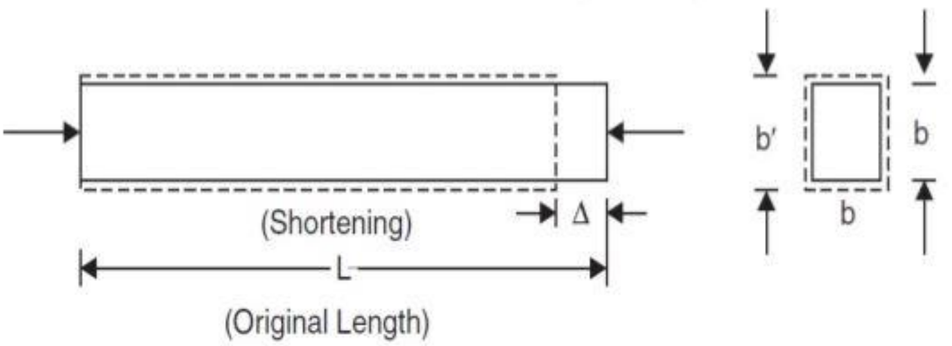
$$\boxed{\varepsilon_t = \frac{\delta L}{L}}$$



2- **<u>Compressive Strain</u>**

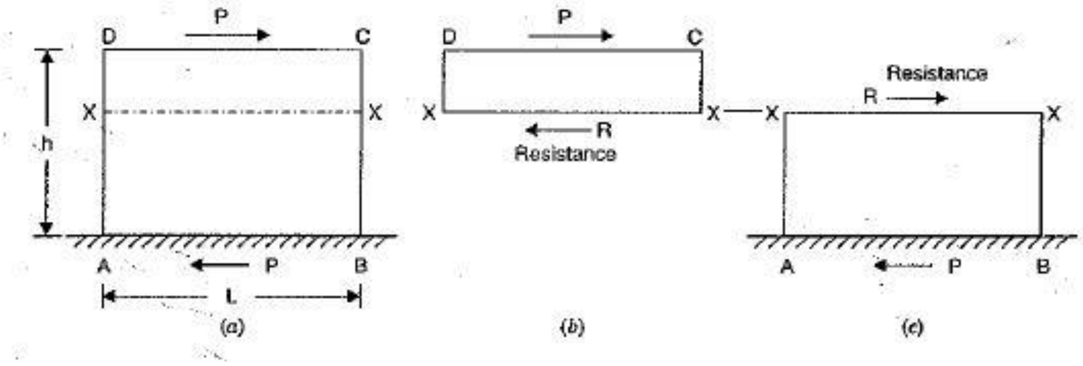
Under compressive forces, a similar piece of material would be reduced in length from L to (L - δL). the fractional deformation again gives the strain ε_c.

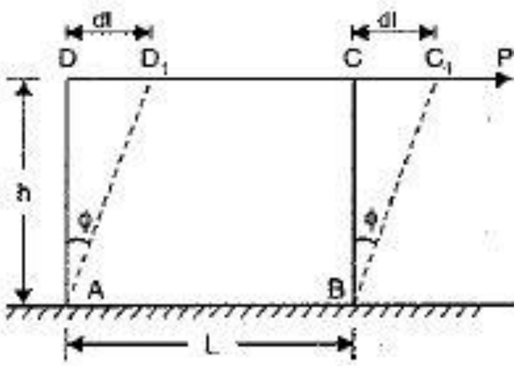
$$\boxed{\varepsilon_c = \frac{\delta L}{L}}$$



3- Shear Strain

In case of a shearing load, a shear strain will be produced which is measured by the angle through which the body distorts.





Note that shear stress is tangential to the are over which it acts. As the bottom face of the block is fixed, the face ABCD will be distorted to ABC_1D_1 through an angle (ϕ) as a result of force P as shown in figure above. And shear strain (ϕ) is given by:

$$\text{Shear Strain} = \boxed{\phi = \frac{\mathrm{DD1}}{\mathrm{AD}} = \frac{dl}{h}}$$

4- Volumetric Strain

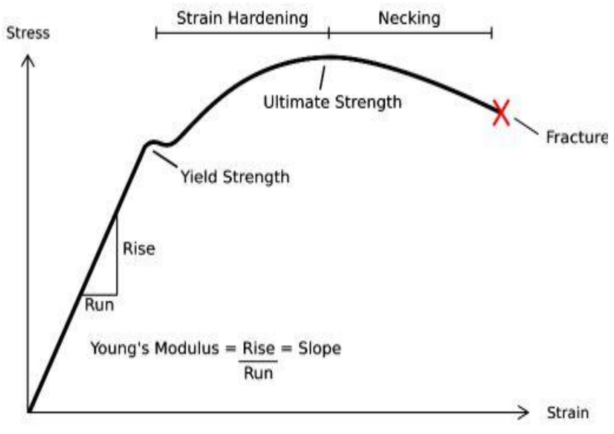
It is defined as the ratio between change in volume and original volume of the body.

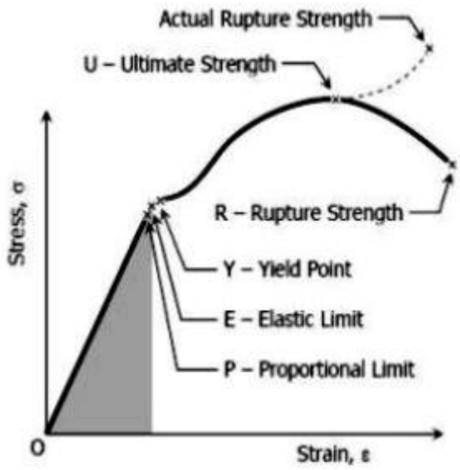
$$\text{Volumetric Strain} \quad \varepsilon_v = \frac{\delta v}{v} = \frac{\text{Change in volume}}{\text{Orignal volume}}$$

Elastic Limit

We have already discussed that whenever some external system of force acts on a body, it undergoes deformation. If the external forces, causing deformation, are removed the body springs back to its original position.

Beyond the elastic limit, the material gets into plastic stage and in this stage the deformation does not entirely disappear, on the removal of the force. But as a result of this, there is a residual deformation even after the removal of the force.





Hook's Law

Hook's law states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to the corresponding strain is a constant within the elastic limit. This constant is known as **Young's Modulus or Modulus of Elasticity and is denoted by (E)**. But the ratio of shear stress to the corresponding shear strain within the elastic limit, is known as **Modulus of Rigidity or Shear Modulus. This is denoted by (G)**.

$$E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} \quad \text{or} = \frac{\text{Compressive Stress}}{\text{Compresive Strain}} \quad \text{or} = \frac{\sigma}{\varepsilon} \quad , (\sigma = E*\varepsilon)$$

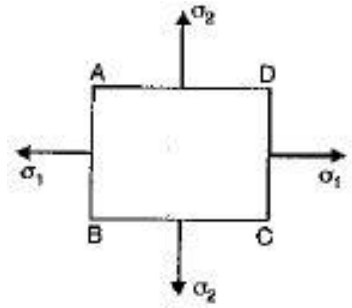
$$\varepsilon = \sigma/E \quad ,, \quad \varepsilon = P/AE \ ,, \quad \varepsilon = \delta L/L \ ,, \quad \delta l = \varepsilon * L \ ,, \quad \delta L = PL/AE$$

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}} \quad \text{or} = \frac{\tau}{\phi}$$

<u>Poisson's Ratio:</u> the ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and it is generally denoted by (μ). Hence mathematically,

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Two-dimensional figure ABCD, subjected to two mutually perpendicular stress σ_1 and σ_2.



Let σ_1 = normal stress in x-direction

σ_2 = normal stress in y-direction

Consider the strain produced by σ_1

The stress σ_1 will produce strain in the direction of x and also in the direction of y.

the strain in the direction of x will be longitudinal strain and will be equal to

$$\text{Longitudinal Strain} = \frac{\sigma 1}{E}$$

whereas the strain in the direction of y will be **lateral strain** and will be equal to

$$\text{Lateral Strain} = -\ \mu * \text{longitudinal strain} = \mu * \frac{\sigma 1}{E}$$

Consider the strain produced by σ_2

The stress σ_2 will produce strain in the direction of y and also in the direction of x.

the strain in the direction of y will be longitudinal strain and will be equal to

$$\text{Longitudinal Strain} = \frac{\sigma 2}{E}$$

whereas the strain in the direction of y will be **lateral strain** and will be equal to

$$\text{Lateral Strain} = -\ \mu * \text{longitudinal strain} = -\ \mu * \frac{\sigma 2}{E}$$

ε_1 = Total strain in x-direction

ε_2 = Total strain in y-direction

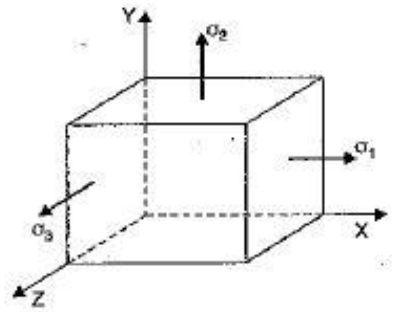
Now total strain in the direction of x due to stresses σ_1 and σ_2, $\varepsilon_1 = \frac{\sigma 1}{E} - \mu * \frac{\sigma 2}{E}$

Similarity,

total strain in the direction of y due to stresses σ_1 and σ_2, $\varepsilon_2 = \frac{\sigma 2}{E} - \mu * \frac{\sigma 1}{E}$

For three-Dimensional Stress System

The figure shows that a three-dimensional body subjected to three orthogonal normal stresses σ_1, σ_2, σ_3 acting in the directions of x, y and z respectively. Consider the strains produced by each stress separately.



The stress σ_1 will produce strain in the direction of x and also in the directions of y and z.

The strain in the direction of x will be $\frac{\sigma 1}{E}$, the strains in the direction y and z will be $(- \mu * \frac{\sigma 1}{E})$ Similarity,

The stress σ_2 will produce strain $\frac{\sigma 2}{E}$ in the direction of y and the strain of $(- \mu * \frac{\sigma 2}{E})$ in the direction of x and z each.

The stress σ_3 will produce strain $\frac{\sigma 3}{E}$ in the direction of z and the strain of $(- \mu * \frac{\sigma 3}{E})$ in the direction of x and y each.

total strain in the direction of x due to stresses σ_1, σ_2 and σ_3,

$$\varepsilon_1 = \frac{\sigma 1}{E} - \mu * \frac{\sigma 2}{E} - \mu * \frac{\sigma 3}{E}$$

total strain in the direction of y due to stresses σ_1, σ_2 and σ_3,

$$\varepsilon_2 = \frac{\sigma 2}{E} - \mu * \frac{\sigma 3}{E} - \mu * \frac{\sigma 1}{E}$$

total strain in the direction of z due to stresses σ_1, σ_2 and σ_3,

$$\varepsilon_3 = \frac{\sigma 3}{E} - \mu * \frac{\sigma 1}{E} - \mu * \frac{\sigma 2}{E}$$

Modulus of rigidity G is related to the modulus of elasticity E and Poisson's ratio μ by

$$\boxed{G = \frac{E}{2(1+\mu)}}$$

Problem 1. A rod 150 cm long and of diameter 2.0 cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is $2*10^5$ N/mm^2. Determine:

1- The stress
2- The strain
3- The elongation of the rod.

Solution

Given: length of the rod, L = 150 cm

Dimeter of the rod = 2.0 cm = 20 mm

Area = 3.14/4 (20^2) = 314. 16 mm^2

Axial pull force P = 20 kN = 20 000 N

Modulus of elasticity E = $2 * 10^5$ N/mm^2

1- Stress, $\sigma = P/A$ = 20 000/314.16 = 63.662 N/mm^2 **Ans**

2- Strain, $E = \sigma/\varepsilon$

$\varepsilon = \sigma/E = 63.662/(2*10^5) = 0.000318$ **Ans**

3- Elongation, $\varepsilon = dL/L$

$dL = \varepsilon * L$

$dL = 0.000318 * 150 = 0.0477$ cm. **Ans**

Problem 2. A square steel rod 20 mm * 20 mm in section is to carry an axial load of 100 kN. Calculate the shortening in a length of 50 mm. assume E = $2.14*10^8$ kN/m^2.

<u>Solution:</u>

Area = 0.02 * 0.02 = 0.0004 m^2 = $4 * 10^{-4}$ m^2

Length = L = 50 mm = 0.05 m

Load = P = 100 kN

E = $2.14 * 10^8$ kN/m^2

Shortening of the rod $\delta L = ?$

$$\sigma_c = P/A \quad = 100 / 0.0004 = 250\,000 \text{ kN/m}^2$$

$$E = \sigma/\varepsilon \quad = \quad \varepsilon = \sigma/E = 250\,000 / 2.14 * 10^8 = \delta L / L$$

$$\delta L = (250\,000 / 2.14 * 10^8) * L$$

$$\delta L = (250\,000 / 2.14 * 10^8) * 0.05$$

$$\delta L = 0.0000584 \text{ m} \quad \text{or} \quad 0.0584 \text{ mm}$$

Hence, the shortening of the rod = 0.0584 mm. **Ans.**

Problem 3. The following observations were made during a tensile test on mild steel specimen 40 mm in diameter and 200 mm long. Elongation with 40 kN load (within limit of proportionality). $\delta L = 0.0304$ mm, Yield load = 161 kN, Maximum load = 242 kN. Length of specimen at fracture = 249 mm

Determine the followings:

1- Young's modulus of elasticity

2- Yield point stress

3- Ultimate stress

4- Percentage elongation.

Solution:

1- E = ?

$\sigma = P / A = 40 / (3.14/4)(0.04^2) = 3.18 * 10^4 \text{ kN/m}^2$

$\varepsilon = \delta L / L = 0.0304 / 200 = 0.000152$

$E = \sigma / \varepsilon = 3.18 * 10^4 / 0.000152 = 2.09 * 10^8 \text{ kN/m}^2$ **Ans.**

2- Yield point stress = yield point load / Area

$= 161 / (3.14 / 4) * (0.04^2) = 12.8 * 10^4 \text{ kN/m}^2$ **Ans.**

3- Ultimate stress = Max. Load / Area $= 242 / (3.14 / 4) * (0.04^2)$

$= 19.2 * 10^4 \text{ kN/m}^2$ **Ans.**

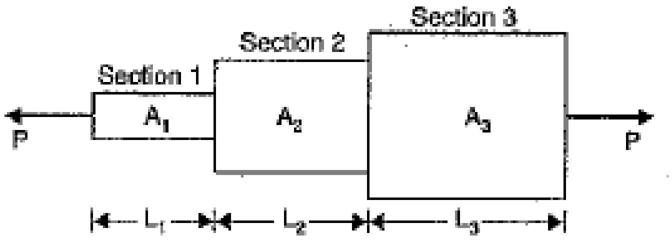
4- Percentage elongation = Strain = $\delta L / L = (Lf – Lo) / Lo$

$= (249 – 200) / 200$

$= 0.245 * 100 = 24.5\ \%$ **Ans.**

ANALYSIS OF BARS OF VARYING SECTIONS

A bar of different length and different diameters as shown in figure. This rod is subjected to an external load P.



Though each section is subjected to the same axial load P, yet the stress, strains and change in length will be different. The total change in length will be obtained by adding the changes in length of individual section.

<u>Let:</u>

P = axial load , **L1** = length of section 1, **A1** = cross-sectional area of section 1

L2, A2 = length and cross-sectional area of section 2

L3, A3 = length and cross-sectional area of section 3

E = young's modulus or modulus of elasticity for the bar or rod material.

Then the stress for section 1,

$$\sigma_1 = \frac{\text{Load}}{\text{Area section 1}} = \frac{P}{A1}$$

Similarly, the stresses for section 2 and section 3 are given as,

$$\sigma_2 = \frac{\text{Load}}{\text{Area section 2}} = \frac{P}{A2}$$

$$\sigma_3 = \frac{\text{Load}}{\text{Area section 3}} = \frac{P}{A3}$$

Strain for section 1 equal to,

$$\varepsilon_1 = \frac{\sigma 1}{E} = \frac{P}{A1 * E}$$

Similarly, the strains of section 2 and section 3 are,

$$\varepsilon_2 = \frac{\sigma 2}{E} = \frac{P}{A2 * E} ,,,, \qquad \varepsilon_3 = \frac{\sigma 3}{E} = \frac{P}{A3 * E}$$

But strain in section 1 equal to,

$$\varepsilon_1 = \frac{\delta L1}{L1} \;,,, \qquad \delta L1 = \varepsilon_1 * L1$$

$$\delta L1 = \frac{P * L1}{A1 * E}$$

Change of the length for section 2 and section 3

$$\delta L2 = \frac{P*L2}{A2*E} \quad ,,,, \quad \delta L3 = \frac{P*L3}{A3*E}$$

Total change in the length of the bar or rod, when the young's modulus of different section is same

$$\text{Total } \delta L = \delta L1 + \delta L2 + \delta L3 = \frac{P*L1}{A1*E} + \frac{P*L2}{A2*E} + \frac{P*L3}{A3*E} = \frac{P}{E} * [\frac{L1}{A1} + \frac{L2}{A2} + \frac{L3}{A3}]$$

If the young's modulus of different section is different, then total change in length of the rod equal to,

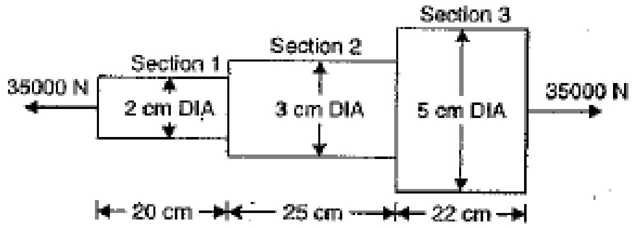
$$\text{Total } \delta L = \delta L1 + \delta L2 + \delta L3 = \frac{P*L1}{A1*E1} + \frac{P*L2}{A2*E2} + \frac{P*L3}{A3*E3} =$$

$$p* [\frac{L1}{E1*A1} + \frac{L2}{E2*A2} + \frac{L3}{E3*A3}]$$

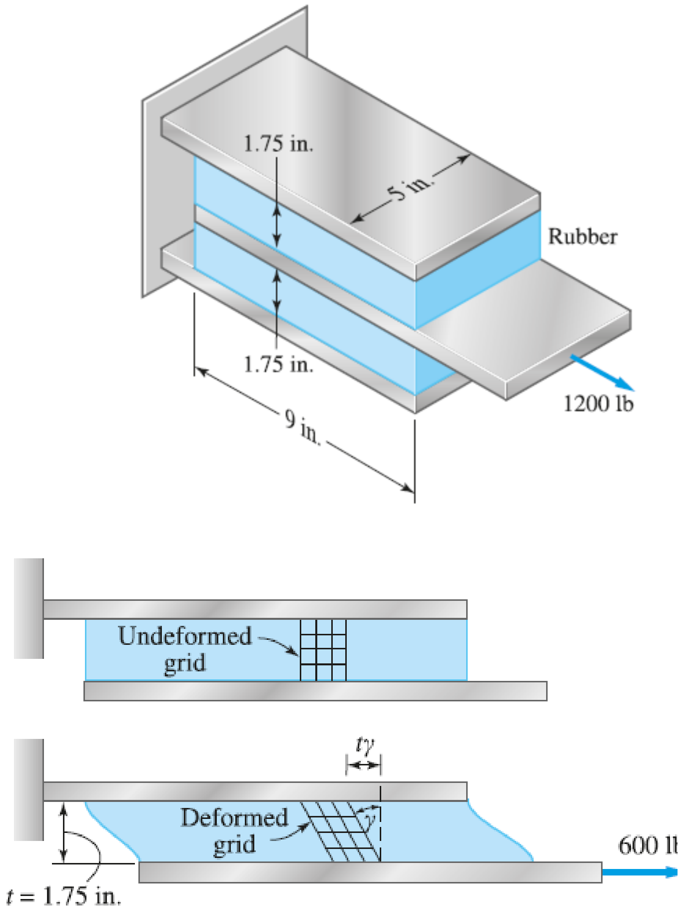
Problem 4. An axial pull of 35000 N is acting on a bar consisting of three length as shown in figure. If the young's modulus = 2.1 * 10^5 N/mm^2, determine:

1- Stresses in each section and

2- Total extension of the bar (δL).



Problem 5. Two 1.75-in.-thick rubber pads are bonded to three steel plates to form the shear mount shown in Fig. Find the displacement of the middle plate when the 1200-lb load is applied. Consider the deformation of rubber only. Use E = 500 psi and μ = 0.48 for rubber.



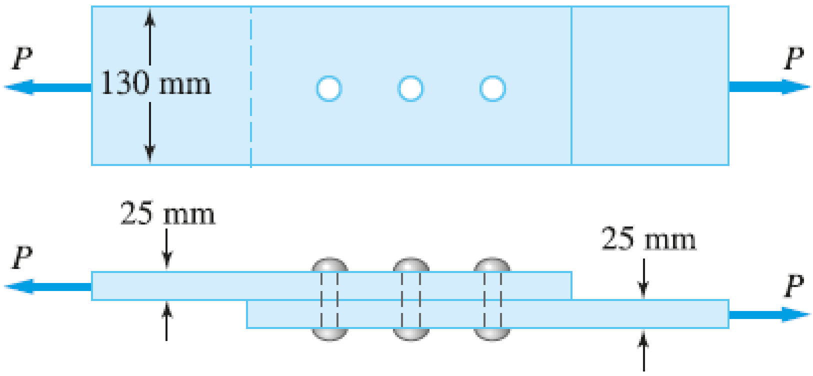
(b)

Problem 6. The lap joint is connected by three 20-mm-diameter rivets. Assuming that the axial load P = 50 kN is distributed equally among the three rivets, find

(a) the shear stress in a rivet;

(b) the bearing stress between a plate and a rivet; and

(c) the maximum average tensile stress in each plate.

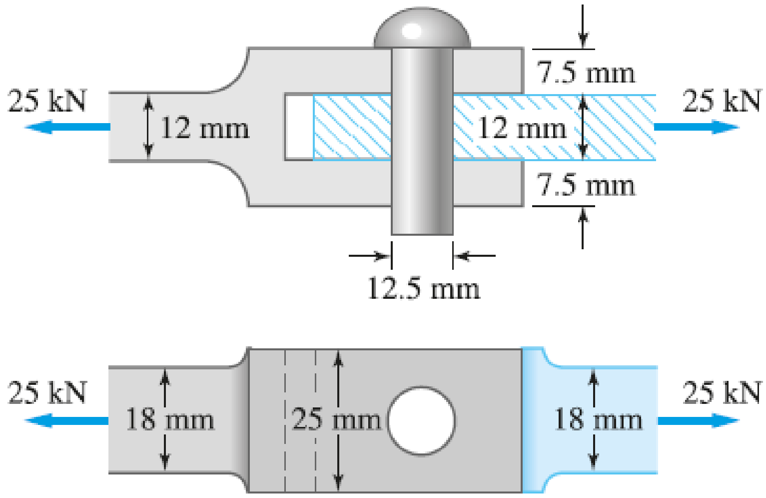


Problem 7. For the joint shown in the figure, calculate

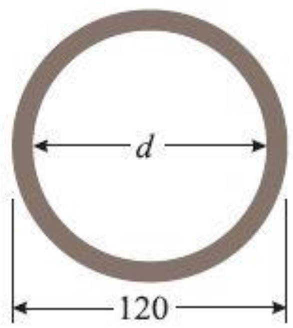
(a) the largest bearing stress between the pin and the members;

(b) the average shear stress in the pin; and

(c) the largest average normal stress in the members.

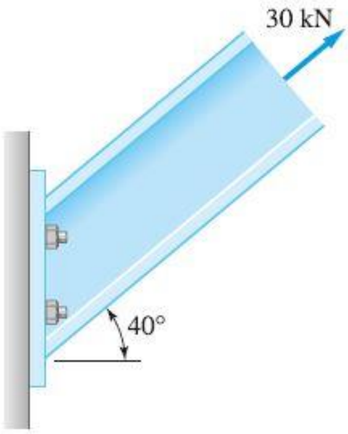


Problem 8: A hollow steel tube 3.5 m long has external diameter of 120 mm. in order to determine the internal diameter, the tube subjected to a tensile load of 400 kN and extension was measured to be 2 mm. if the modulus of elasticity for the tube material is 200 Gpa, determine the internal diameter of the tube.



Solution:

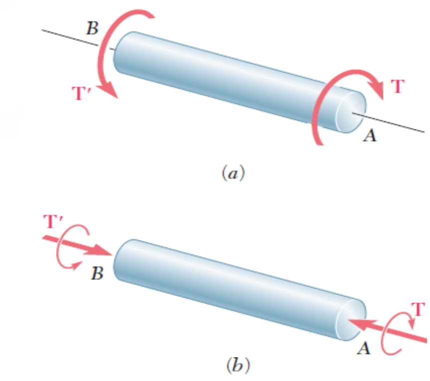
Problem 9: The plate welded to the end of the I-beam is fastened to the support with four 10-mm-diameter bolts (two on each side). Assuming that the load is equally divided among the bolts, determine the normal and shear stresses in a bolt.



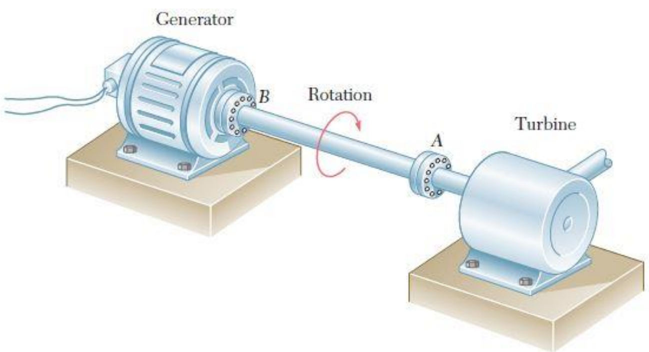
Torsion

Introduction

A shaft said to be in torsion, when equal and opposite torque are applied at the two ends of the shaft. **The torque is equal to the product of the force applied (tangentially to the end of a shaft) and radius of the shaft.** Due to the application of the torques at the two ends, the shaft is subjected to a twisting moment. This causes the shear stresses and strains in the material of the shaft. As shown in figure.



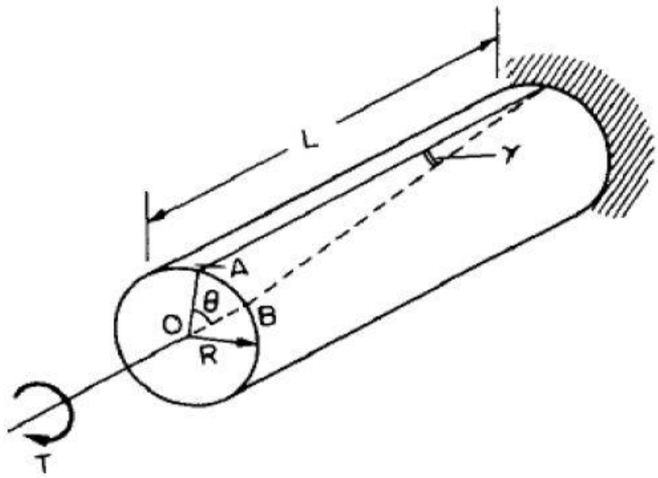
<u>Shafts:</u> are structural members with length significantly greater than the largest cross-sectional dimension used in transmitting torque from one plane to another.



<u>Torsion:</u> twisting of a structural member, when it is loaded by couples that produce rotation about its longitudinal axis.

Derivation of Shear Stress Produced in A Circular Shaft Subjected to Torsion

When a circular shaft is subjected to torsion, shear stresses are set up in the material of the shaft. To determine the magnitude of shear stress at any point on the shaft, consider a shaft fixed at one end and free at the other end as shown in the figure.



Now let the shaft is subjected to a torque T at the end (A). as a result of this torque T, the shaft at the end (A) will rotate clockwise and every cross-section of the shaft will be subjected to shear stresses. Under the action of this torque a radial line at the free end of the shaft twists through an angle θ, point (A) moves to (B) and (AB) subtends an angle ɣ at the fixed end. This is then the angle of distortion of the shaft, i.e. the shear strain. As shown in figure.

Let:

R = Radius of the shaft
L = Length of shaft
T = Torque applied at the end (A)
τ = Shear stress induced at the surface of the shaft due to torque T
G = Modulus of rigidity of the material of the shaft
ɣ = Shear strain
θ = Angle of twist

Now distortion at the outer surface due to torque T

$$= AB$$

Shear strain at outer surface = Distortion per unit length

$$= \frac{\text{Distortion at the outer surface (AB)}}{\text{Length of shaft (L)}} = \mathbf{\tan \gamma = \gamma} \quad \text{(if } \gamma \text{ is very small then } \tan \gamma = \gamma\text{)}$$

Shear strain at outer surface, $\gamma = \tan^{-1} \frac{AB}{L}$ ………………………………….. (i)

Arc AB = OA * θ = Rθ (OA = R = Radius of shaft)

Substituting the value of AB in equation (i), we get

Shear strain at outer surface, $\gamma = \tan^{-1} \frac{R*\theta}{L}$ …………………………..…. (ii)

Now the modulus of rigidity (G) of the material of the shaft is given as

$$G = \frac{\text{Shear stress at the outer surface}}{\text{Shear strain at the outer surface}} = \frac{\tau}{\frac{R\theta}{L}} = \frac{\tau * L}{R*\theta} = \quad \frac{G*\theta}{L} = \frac{\tau}{R}$$

$$\boldsymbol{\tau = \frac{R*G*\theta}{L}}$$

For a given shaft subjected to a given torque (T), the values of G, θ and L are constant. Hence shear stress produced is proportional to the radius R.

$\tau \propto R$ or τ / R = Constant …………………………………….(iii)

If (q) is the shear stress induced at any radius 'r' from the center of the shaft then

$$\frac{\tau}{R} = \frac{q}{r}$$

Therefore, $\frac{\tau}{R} = \frac{q}{r} = \frac{G*\theta}{L}$

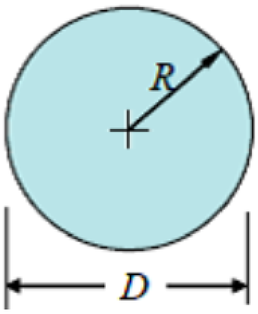
Maximum Torque Transmitted by A Circular Shaft

The maximum torque transmitted by a circular shaft, is obtained from the maximum shear stress induced at the outer surface of the shaft.

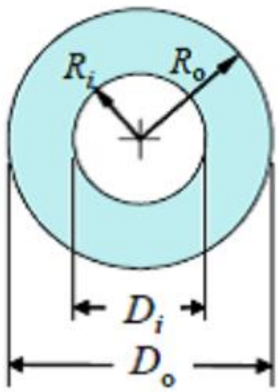
for a **solid or a hollow shaft**, maximum torque transmitted by the shafts can be determined.

For a solid shaft ……………………………$T_{max} = \frac{\pi}{16} * \tau * D^3$

But for a hollow shaft ……………………..$T_{max} = \frac{\pi}{16} * \tau * (Do^4 – Di^4)/ Do$



Solid Shaft



Hollow Shaft

Power Transmitted by Shafts

Once the expression for torque (T) for a solid or a hollow shaft is obtained, power transmitted by the shafts can be determined.

Let:

N = revolution per minute (r.p.m) of the shaft

T = Mean torque transmitted in N.m

ω = angular speed of the shaft

Then

$$\textbf{Power} = \frac{2 * \pi * N * T}{60} \textbf{ watt}$$

$$\textbf{Power} = \omega * T \qquad \omega = \frac{2 * \pi * N}{60}$$

Problem 1: A solid shaft of 150 mm diameter is used to transmit torque. Find the maximum torque transmitted by the shaft if the maximum shear induced to the shaft is 45 N/mm^2

Solution:

Problem 2: The shearing stress is a solid shaft is not to exceed 40 N/mm^2 when the torque transmitted is 20000 N.m. Determine the minimum diameter of the shaft.

Solution:

Problem 3: Two shafts of the same material and of same lengths are subjected to the same torque, if the first shaft is of a solid circular section and the second shaft is of hollow circular section, whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the shaft.

Solution:

TORQUE IN TERMS OF POLAR MOMENT OF INERTIA

Polar moment of inertia of a plane area is defined as the moment of inertia of the area about an axis perpendicular to the plane of the figure and passing through the center of gravity of the area. Or polar moment of inertia is the resistance of any object for twisting moment and **it is denoted by symbol J.**

Polar moment inertia for circular shaft is

For solid shafts: $J = \frac{\pi}{32} * D^4$

For hollow shafts: $J = \frac{\pi}{32} * (Do^4 - Di^4)$

$$\theta = \frac{T*L}{G*J}$$

Where

θ : Angle of twist of the shaft in radian
T : torque transmitted by the shaft
L : length of the shaft
G : modulus of rigidity
J : Polar moment of inertia

POLAR MODULUS

Polar modulus is defined as the ratio of the polar moment of inertia to the radius of the shaft. It is also called torsional section modulus. It is denoted by Z_p. Mathematically.

$$Z_p = \frac{J}{R}$$

For a solid shaft: $Z_p = \frac{\pi}{16} * D^3$

For Hollow shaft: $Z_p = \frac{\pi}{16 * Do} * (Do^4 - Di^4)$

STRENGTH OF A SHAFT AND TORSIONAL RIGIDITY

The strength of a shaft means the maximum torque or maximum power the shaft can transmit. Torsional rigidity or stiffness of the shaft is defined as the product of modulus of rigidity (G) and polar moment of inertia of the shaft (J). Hence mathematically, the torsional rigidity is given as,

Torsional rigidity = G * J

Torsional rigidity is also defined as the torque required to produce a twist of one radian per unit length of the shaft.

Let a twisting moment (**T**) produces a twist of (**θ**) radians in a shaft of length (**L**).

$$\theta = \frac{T * L}{G * J} \Longrightarrow \textbf{Torsional rigidity} = \frac{T * L}{\theta}$$

If

L = one meter and

θ = one radian

The torsional rigidity = Torque

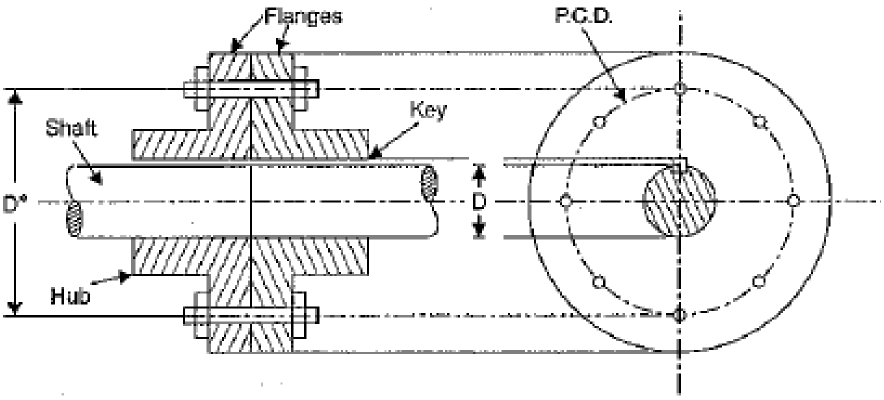
Problem 4: (H.W)

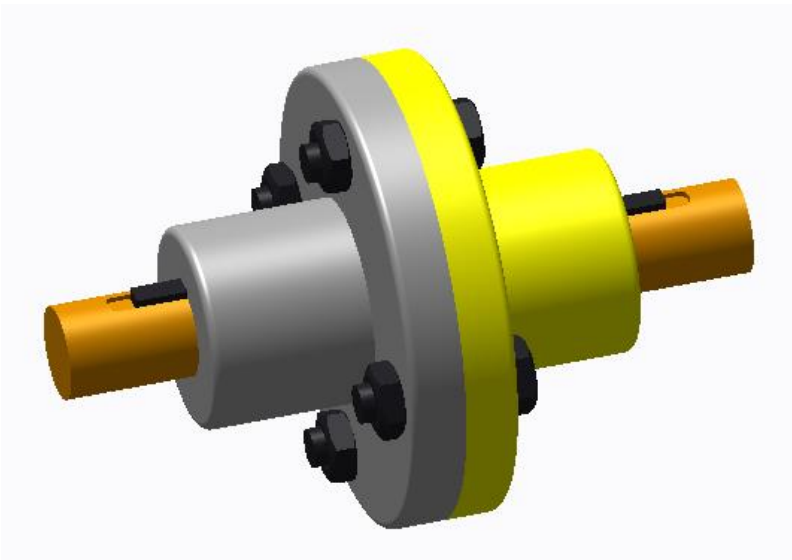
Determine the diameter of a solid steel shaft which will transmit 90 kW at 160 r.p.m. Also determine the length of the shaft if the twist must not exceed 1^o over the entire length. The maximum shear stress is limited to 60 N/mm^2. Take the value of modulus of rigidity = 8 * 10^4 N/mm^2

Problem 5: A hollow shaft of diameter ratio 3/8 (internal dia. To outer dia.) is to transmit 375 kW power at 100 r.p.m. the maximum torque being 20% greater than the mean. The shear stress is not to exceed 60 N/mm^2 and twist in a length of 4 m not to exceed 2^o. Calculate its external and internal diameters which would satisfy both the above conditions. Assume modulus of rigidity, $G = 0.85 * 10^5$ N/mm^2.

FLANGED COUPLING

A flange coupling is used to connect two shafts as shown in figure.





The flanges of the two shafts are joined together by bolts and nuts (or rivets) and torque is then transferred from one shaft to another through the bolts. Connection between each shaft and coupling is provided by the key. The bolts are arranged along a circle called the pitch circle. The bolts are subjected to shear stress when torque is transmitted from one shaft to another.

Let:

τ = Shear stress in the shaft

q = Shear stress in the bolt

d = diameter of bolt

D = diameter of shaft

D* = Diameter of bolt pitch circle

n = number of bolts

Maximum load that can be resisted by one bolt = Stress in bolt * Area of one bolt

$$= \mathbf{q * \frac{\pi}{4} * d^2}$$

Torque resisted by one bolt = load resisted by one bolt * Radius of pitch circle

$$= \mathbf{q * \frac{\pi}{4} * d^2 * \frac{D*}{2}}$$

Total torque resisted by n bolts

$$= \mathbf{n * q * \frac{\pi}{4} * d^2 * \frac{D*}{2} \quad = \quad n * q * \frac{\pi * d^2 * D*}{8}} \quad \ldots\ldots\ldots\text{(i)}$$

But the torque transmitted by the shaft,

$$\mathbf{T = \frac{\pi}{16} * \tau * D^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Since the torque resisted by the bolts should be equal to the torque transmitted by the shaft, therefore, equating (i) and (ii), we get.

$$\mathbf{n * q * \frac{\pi * d^2 * D*}{8} = \frac{\pi}{16} * \tau * D^3}$$

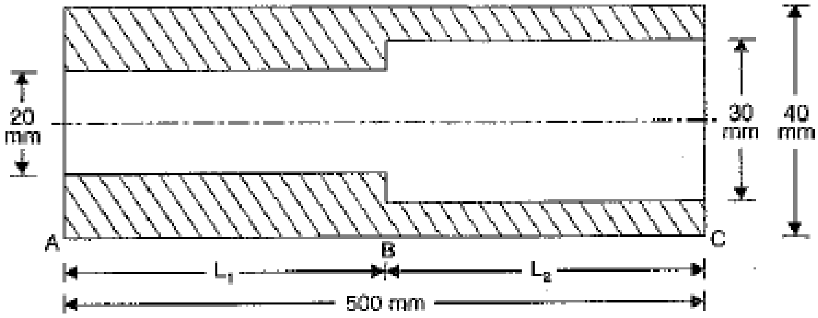
from the above equation the unknown value of any parameter (say number of bolts or diameter of bolt) can be calculated.

Problem 6: Two shafts are connected end to end by means of a flanged coupling in which there are 12 bolts, the pitch circle diameter being 25 cm. the maximum shear stress is limited to 55 N/mm^2 in the shafts and 20 N/mm^2 in the bolts. If one shaft is solid of 5 cm diameter and the other is hollow of 10 cm external diameter. Calculate the internal diameter of the hollow shaft and the bolt diameter so that both shafts and the coupling are all equally strong in torsion.

STRENGTH OF A SHAFT OF VARYING SECTION

When a shaft is made up of different length of different diameters, the torque transmitted by individual sections should be calculated first. The strength of such a shaft is the minimum value of these torque.

Problem 7: A shaft of ABC of 500 mm length and 40 mm external diameter is bored, for a part of its length AB, to a 20 mm diameter and for the remaining length BC to a 30 mm diameter bore. If the shear stress is not to exceed 80 N/mm^2, find the maximum power, the shaft can transmit at a speed of 200 r.p.m. if the angle of twist in the length of 20 mm diameter bore is equal to that in the 30 mm diameter bore, find the length of the shaft that has been bored to 20 mm and 30 mm diameter.



Shear Force and Bending Moment

The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force. It is briefly written as S.F. the algebraic sum of the moment of all the forces acting to the right or left of the section is known as bending moment. It is written as B.M.

Shear Force and Bending Moment Diagram

A shear force diagram is one which shows the variation of the shear force along the length of the beam. And a bending moment diagram is one which shows the variation of the bending moment along the length of the beam.

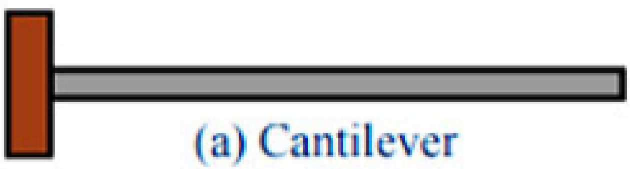
Before drawing the force and bending moment diagrams, we must know the different types of beams and different types of load acting on the beam.

Types of Beams

The following are the important types of beams:

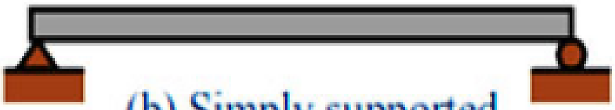
1- Cantilever beam.
2- Simply supported beam.
3- Overhanging beam.
4- Continuous beam.
5- Fixed beams.

1- Cantilever Beam. A beam which is fixed at one end and free at the other end, is known as cantilever beam. Such beam is shown in figure (a).



(a) Cantilever

2- **Simply Supported Beam.** A beam supported or resting freely on the supports at its both ends, is known as simply supported beam. Such beam is shown in figure (b).



(b) Simply supported

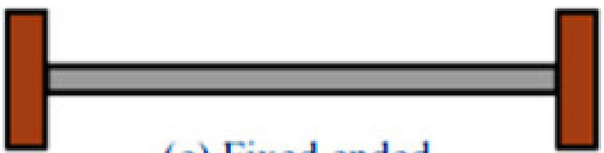
3- **Overhanging Beam.** If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam. Overhanging beam is shown in figure (c).

(c) Overhanging

4- **Continuous beam.** A beam which is provided more than two supports as shown in figure (d), is known as continuous beam.

(d) continuous

5- **Fixed Beam.** A beam whose both ends are fixed or built-in walls, is known as fixed beam. Such beam is shown in figure.



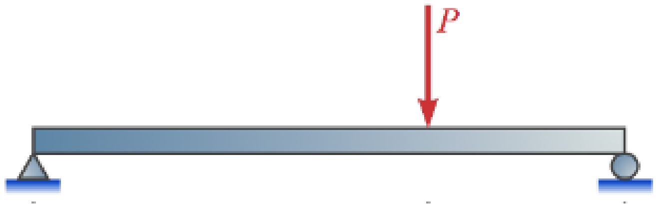
(e) Fixed ended

Types of Load

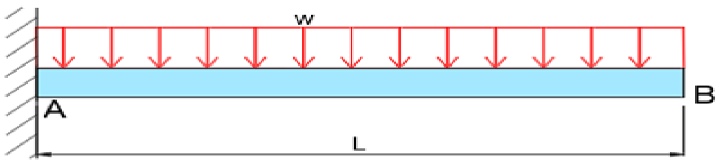
A beam is normally horizontal and the loads acting on the beams are generally vertical. The following are the important types of load acting on a beam.

1- Concentrated or Point Load.
2- Uniformly Distributed Load, and
3- Uniformly Varying Load.

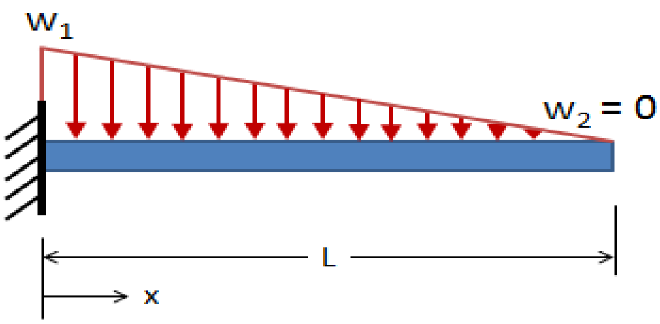
1- Concentrated or Point Load. A concentrated load is one which is considered to act at a point, although in practice it must really be distributed over a small area. As shown in figure



2- Uniformly Distributed Load. A uniformly distributed load is one which is spread over a beam in such a manner that rate of loading is uniform along the length (each unit length is loaded to the same rate) as shown in figure.

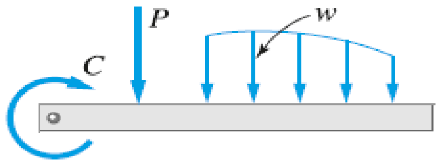
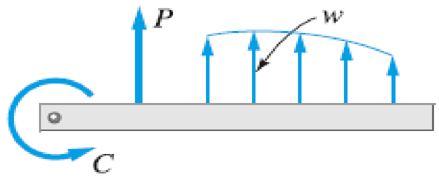
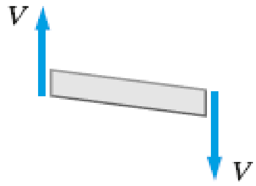
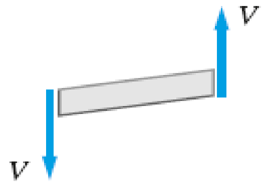
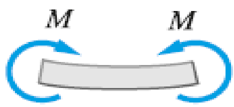
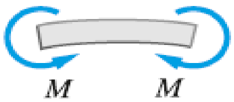


3- Uniformly Varying Load. A uniformly varying load is one which is spread over a beam in such a manner that rate of loading varies from point to point along the length as shown in figure, in which the load is zero at one end and increases uniformly to the other end such load is known as triangular load.



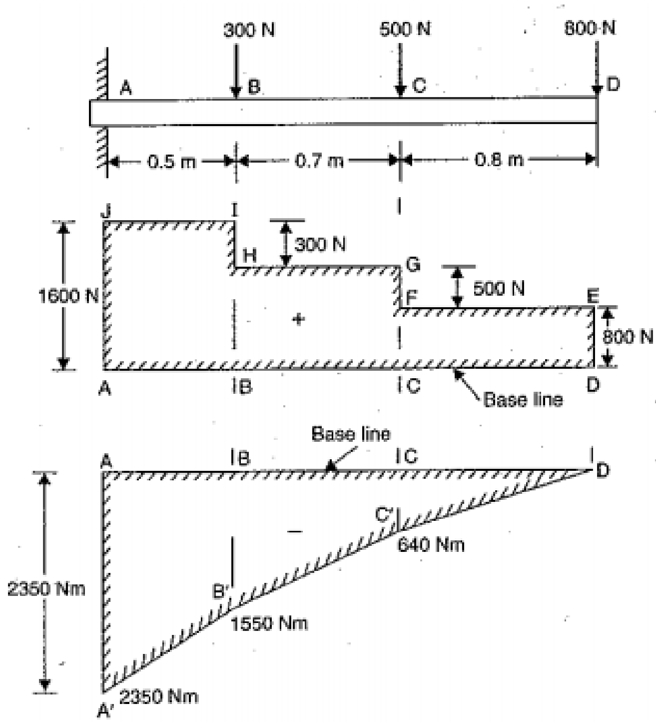
For solving numerical problems, the total load is equal to the area of the triangle and this total load is assumed to be acting at the center of gravity of the triangle at a distance of 2/3 of total length of beam.

Sign Conventions for Shear Force and Bending Moment

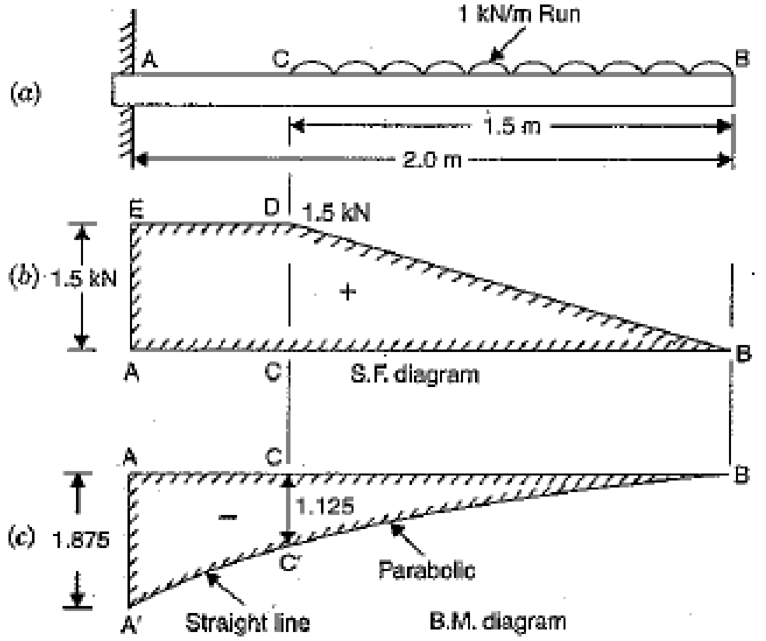
	Positive	Negative
External loads		
Shear force		
Bending moment		

Problem 1. A cantilever beam of length 2 m carries the point loads as shown in figure. Draw the shear force and bending moment diagrams for the cantilever beam.

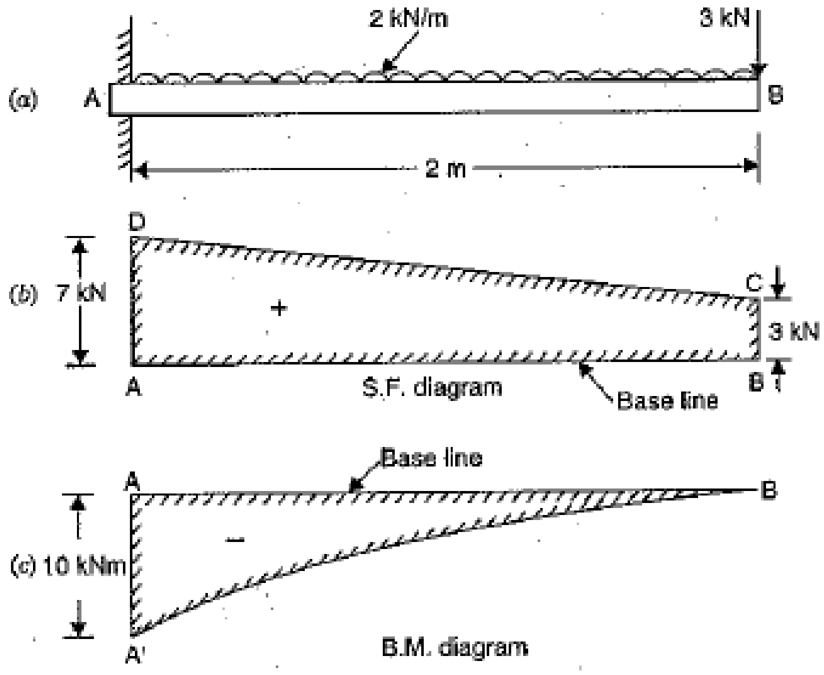
Solution:



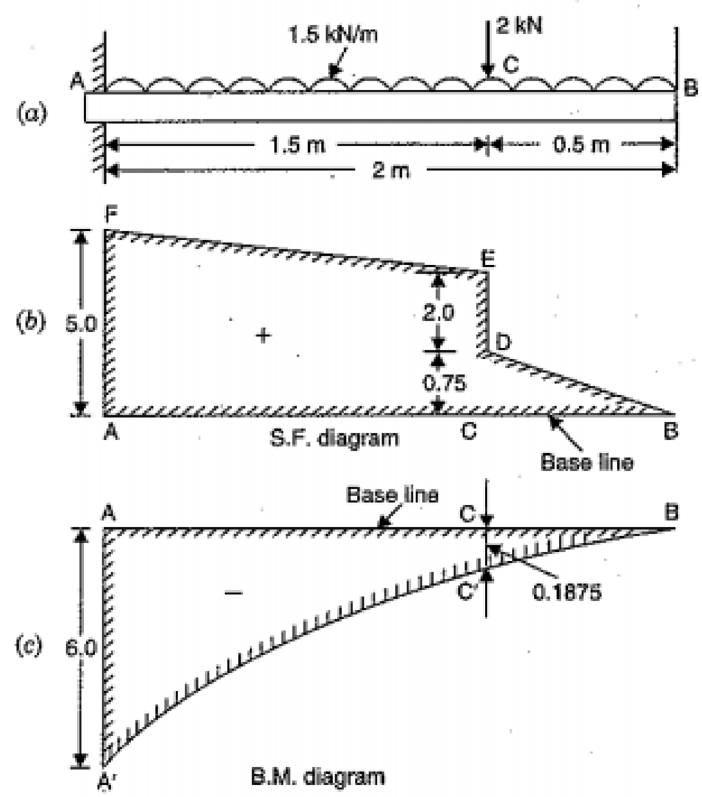
Problem 2. A cantilever of length 2 m carries a uniformly distributed load of 1 kN/m run over a length of 1.5 m from the free end. Draw the shear force and bending moment diagrams for the cantilever.



Problem 3. A cantilever of length 2 m carries a uniformly distributed load of 2 kN/m length over the whole length and a point load of 3 kN at the free end. Draw the shear force and bending moment diagrams for the cantilever.



Problem 4. A cantilever of length 2 m carries a uniformly distributed load of 1.5 kN/m run over the whole length and a point load of 2 kN at a distance of 0.5 m from the free end. Draw the shear force and bending moment diagrams for the cantilever.



Problem 5. A cantilever of length 4 m carries a gradually varying load, zero at the free end to 2 kN/m at the fixed end. Draw the shear force and bending moment diagrams for cantilever.