Kurdistan Region
Salahaddin University-Erbil
College of Engineering
Mechanic & Mechatronics Engineering Department

Strength of Materials

For Second Stage Students
In Mechanic & Mechatronics Dept.

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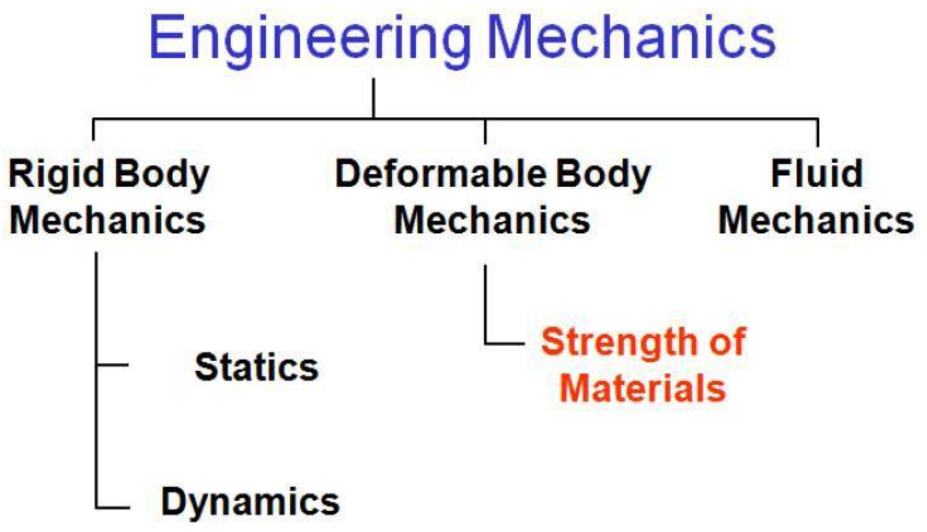
Classification of Engineering Mechanics



<u>Strength of materials</u>

It deals with the relations between externally applied forces or loads and the internal effects in the body.

When materials are loaded, they first deform before actual failure takes place. Hence before selecting any material for engineering purpose, it is important to know the behavior of the material under the action of loads.

<u>Unit</u>: - it is defined as the numerical standard used to measure the qualitative dimension of a physical quantity.

TABLE 1–1 Basic quantities in the SI metric unit system.

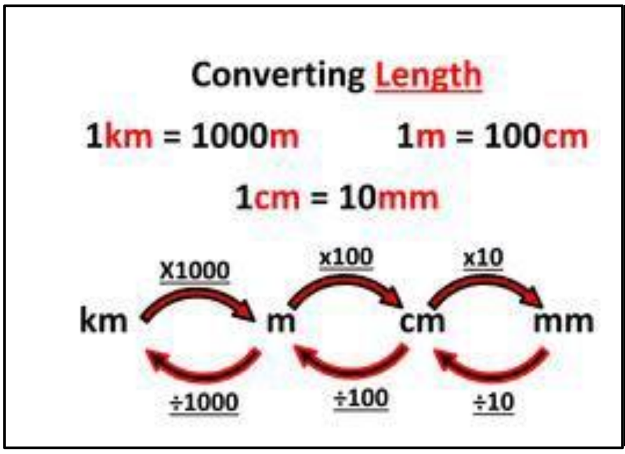
Quantity	SI unit	Other metric units
Length	Meter (m)	Millimeter (mm)
Time	Second (s)	Minute (min), hour (h)
Force	Newton (N)	$kg \cdot m/s^2$
Mass	Kilogram (kg)	$N \cdot s^2/m$
Temperature	Kelvin (K)	Degrees Celsius (°C)
Angle	Radian (rad)	Degree (°)

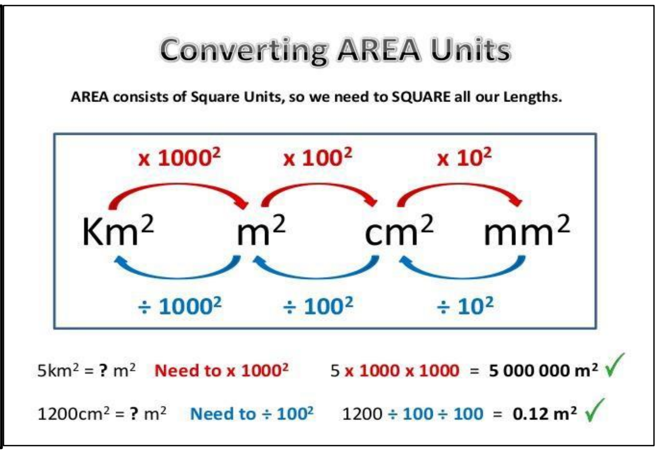
TABLE 1–2 Basic quantities in the U.S. Customary unit system.

Quantity	U.S. Customary unit	Other U.S. units
Length	Foot (ft)	Inch (in.)
Time	Second (s)	Minute (min), hour (h)
Force	Pound (lb)	kip[a]
Mass	Slug	$lb \cdot s^2/ft$
Temperature	Degrees Fahrenheit (°F)	
Angle	Degree (°)	Radian (rad)

TABLE 1–3 Prefixes for SI units.

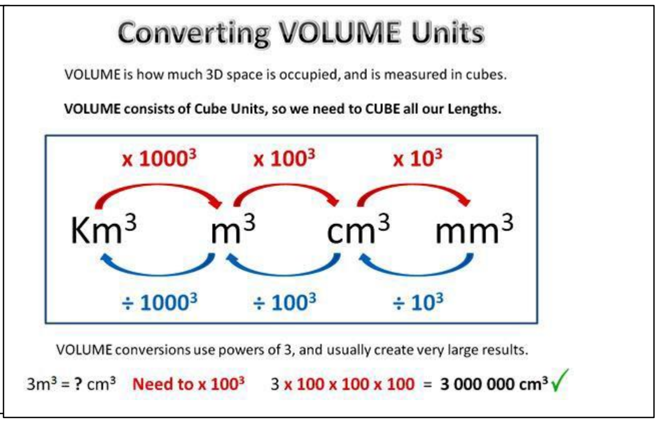
Prefix	SI symbol	Factor
Giga	G	$10^9 = 1\ 000\ 000\ 000$
Mega	M	$10^6 = 1\ 000\ 000$
Kilo	k	$10^3 = 1000$
Milli	m	$10^{-3} = 0.001$
Micro	μ	$10^{-6} = 0.000\ 001$





Customary Units of Length

1 foot (ft) = 12 inches (in)
1 yard (yd) = 3 feet (ft)
1 yard (yd) = 36 inches (in)
1 mile (mi) = 1,760 yards (yd)
1 mile (mi) = 5,280 feet (ft)



Mechanical properties of material

The following are considered as the most important properties of engineering materials

1) Elasticity
2) Plasticity
3) Ductility
4) Malleability
5) Brittleness
6) Toughness
7) Hardness

Any material cannot possess all the above properties because the different properties oppose each other. Hence the engineering Materials can be classified as follows depending upon their Mechanical properties

1)**Elastic Materials: -** These are materials which undergo deformation due to application of forces and once the forces are removed the material regains its original shape.

2)**Plastic materials: -** These are materials which do not regain their original shape even after the external loads acting on the material are removed.

3)**Ductility: -** It is a measure of a material's ability to undergo significant plastic deformation before rupture, which may be expressed as percent elongation or percent area reduction from a tensile test.

4)**Malleable materials: -** These are Materials which can be extended in two directions easily or in simple terms, materials which can be beaten into thin sheets.

5)**Brittle materials: -** these are materials which do not undergo any deformation before failure when external forces act on them.

6)**Tough materials: -** These are materials which can resist sudden loads or shock loads without showing any fracture on failure

7)**Hard material: -** These are materials that have the ability to resist surface abrasion or indentation.

Various tests are carried out on engineering materials to assess their mechanical properties in a material testing laboratory. They are

1) Tension test
2) Compression test
3) Impact test
4) Shear test
5) Torsion test
6) Bending test
7) Fatigue test
8) Hardness test

Stress

Whenever some external forces act on a body it sets up a deformation and the body offer some resistance against deformation. this resistance per unit area to deformation is known as “stress”.

Types of Stresses

The various types of stresses may be classified as:

1- **Simple or Direct Stresses**

i. Tension Stress ii. Compression Stress iii Shear Stress

2- **Indirect Stresses**

i. Bending Stress ii. Torsion Stress

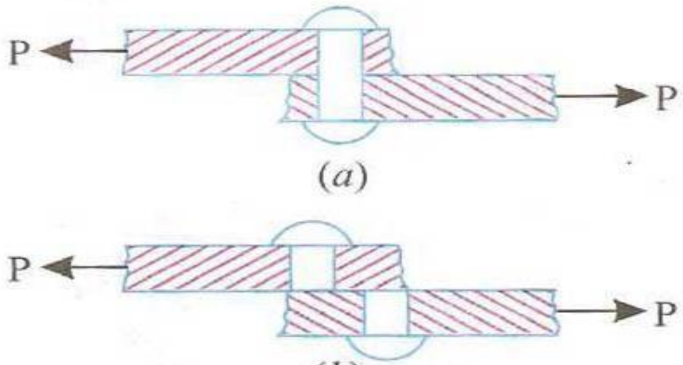
3- **Combined Stresses, any possible combination 1 and 2.**

Simple stresses

Simple stress is offen called direct stress because it developse under direct loading conditions. That is, simple tension and simple compression occur when the applied force or load, is in line with the axis of the member (fig. 1.1 and 1.2) and simple shear occurs, when equal, parallel and opposite forces tend to cause a surface to slide relative to the adjacent surface (fig. 1.3).

Fig. 1.1. Tensile stress

Fig. 1.2. Compressive stress



(b)

Fig. 1.3. (a) Rivet resisting shear.
(b) Rivet failure due to shear.

When simple stress (tension and compression) (σ) (sigma) develops, **we can calculate the magnitude of the tensile or compression stress by,**

$$stress = \frac{force}{area} \quad = \quad \sigma_{t,c} = \frac{p}{A}$$

Where

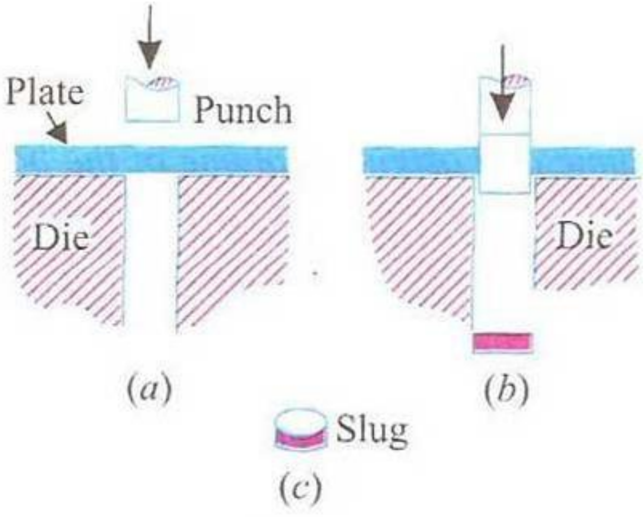
σ = Stress (also called intensity of Stress) kN/m^2 or N/mm^2

P = External Force or Load (kN or N)

A = Cross-Sectional Area of the body (m^2 or mm^2)

It may be noted that in cases of either simple tension or simple compression, the areas which resist the load are perpendicular to the direction of forces. When a member is subjected to simple shear, the resisting area is parallel to the direction of the force. Common situations causing shear stresses are shown in fig. 1.3 and 1.4. **we can calculate the magnitude of the shear stress by,**

$$\tau = \frac{P}{A}$$



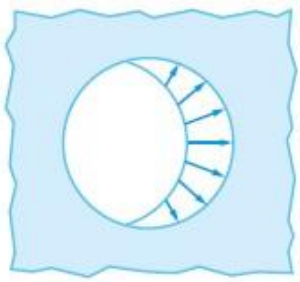


Fig. 1.4. (a) Punch approaching plate;
(b) Punch shearing plate;
(c) Slug showing sheared area.

Stress Units

Psi (Pound per square inch)

Kpsi (Kilo pound per square inch)

In SI Unit

Pa (Pascal = N/m^2)

Kpa (Kilo Pa = 1000 Pa)

Mpa (Mega Pa = 10^6)

Strain

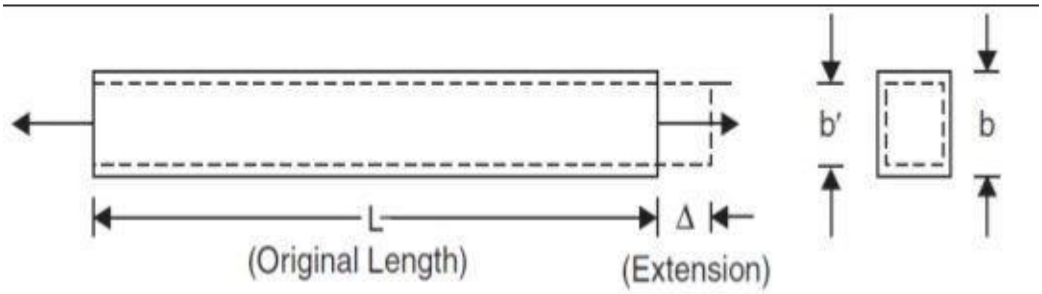
Any element in a material subjected to stress is said to be strained. The strain (ε) is the deformation produced by stress. The various types of strains are explained below:

1. Tensile strain
2. Compressive strain
3. Volumetric strain
4. Shear strain

1- **Tensile Strain**

A piece of material, with uniform cross-section, subjected to a uniform axial tensile stress, will increase its length from (L) to (L + δL), and the increment of length δL is the actual deformation of the material. The fractional deformation or the tensile strain is given by

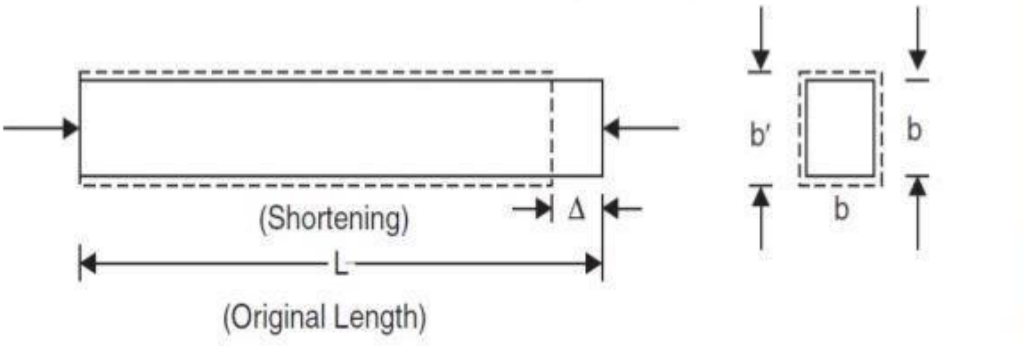
$$\varepsilon_t = \frac{\delta L}{L}$$



2- **Compressive Strain**

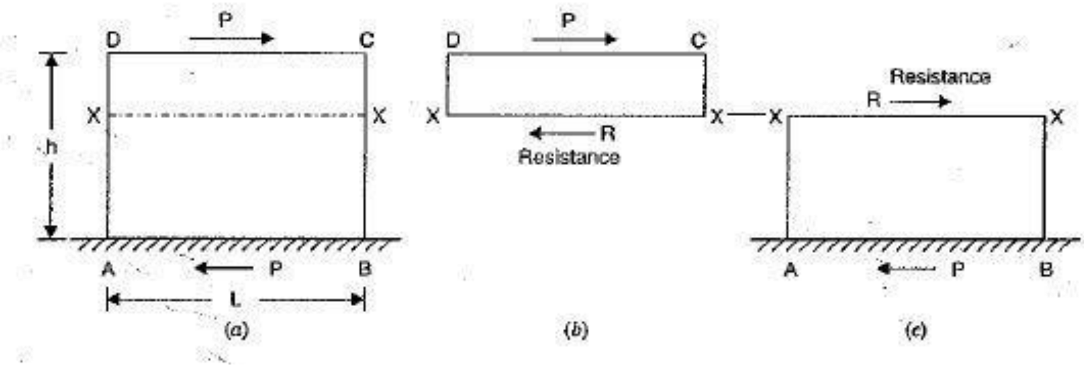
Under compressive forces, a similar piece of material would be reduced in length from L to (L - δL). the fractional deformation again gives the strain ε_c.

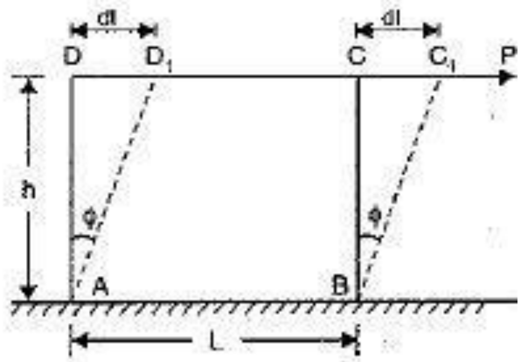
$$\varepsilon_c = \frac{\delta L}{L}$$



3- Shear Strain

In case of a shearing load, a shear strain will be produced which is measured by the angle through which the body distorts.





Note that shear stress is tangential to the are over which it acts. As the bottom face of the block is fixed, the face ABCD will be distorted to ABC_1D_1 through an angle (ϕ) as a result of force P as shown in figure above. And shear strain (ϕ) is given by:

$$\text{Shear Strain} = \boxed{\phi = \frac{DD1}{AD} = \frac{dl}{h}}$$

4- Volumetric Strain

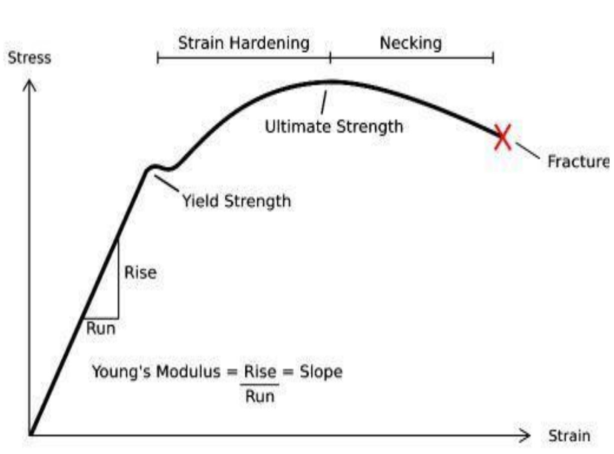
It is defined as the ratio between change in volume and original volume of the body.

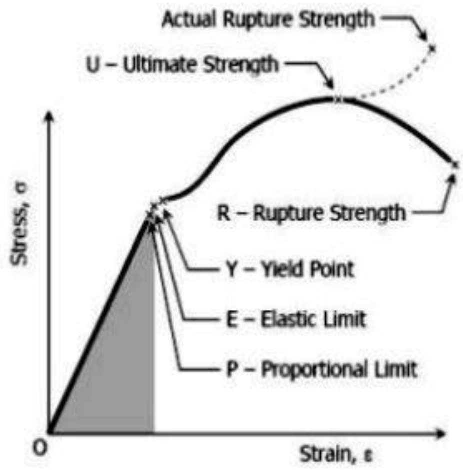
$$\text{Volumetric Strain} \quad \varepsilon_v = \frac{\delta v}{V} = \frac{\text{Change in volume}}{\text{Original volume}}$$

Elastic Limit

We have already discussed that whenever some external system of force acts on a body, it undergoes deformation. If the external forces, causing deformation, are removed the body springs back to its original position.

Beyond the elastic limit, the material gets into plastic stage and in this stage the deformation does not entirely disappear, on the removal of the force. But as a result of this, there is a residual deformation even after the removal of the force.





Hook's Law

Hook's law states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to the corresponding strain is a constant within the elastic limit. This constant is known as **Young's Modulus or Modulus of Elasticity and is denoted by (E)**. But the ratio of shear stress to the corresponding shear strain within the elastic limit, is known as **Modulus of Rigidity or Shear Modulus. This is denoted by (G)**.

$$E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} \quad \text{or} = \frac{\text{Compressive Stress}}{\text{Compresive Strain}} \quad \text{or} = \frac{\sigma}{\varepsilon}, \; (\sigma = E*\varepsilon)$$

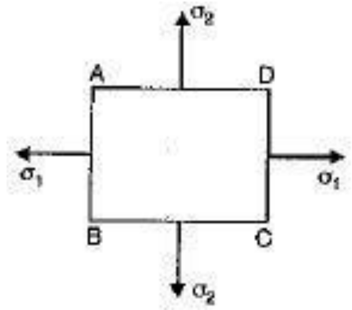
$$\varepsilon = \sigma/E \;,, \quad \varepsilon = P/AE \;,, \quad \varepsilon = \delta L/L \;,, \quad \delta l = \varepsilon * L \;,, \quad \delta L = PL/AE$$

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}} \quad \text{or} = \frac{\tau}{\phi}$$

<u>Poisson's Ratio:</u> the ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and it is generally denoted by (μ). Hence mathematically,

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Two-dimensional figure ABCD, subjected to two mutually perpendicular stress σ_1 and σ_2.



Let σ_1 = normal stress in x-direction

σ_2 = normal stress in y-direction

Consider the strain produced by σ_1

The stress σ_1 will produce strain in the direction of x and also in the direction of y.

the strain in the direction of x will be longitudinal strain and will be equal to

$$\text{Longitudinal Strain} = \frac{\sigma 1}{E}$$

whereas the strain in the direction of y will be **lateral strain** and will be equal to

$$\text{Lateral Strain} = -\mu * \text{longitudinal strain} = \mu * \frac{\sigma 1}{E}$$

Consider the strain produced by σ_2

The stress σ_2 will produce strain in the direction of y and also in the direction of x.

the strain in the direction of y will be longitudinal strain and will be equal to

$$\text{Longitudinal Strain} = \frac{\sigma 2}{E}$$

whereas the strain in the direction of y will be **lateral strain** and will be equal to

$$\text{Lateral Strain} = -\mu * \text{longitudinal strain} = -\mu * \frac{\sigma 2}{E}$$

ε_1 = Total strain in x-direction

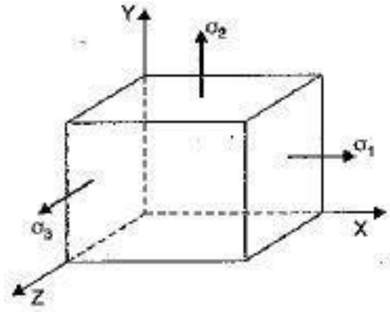
ε_2 = Total strain in y-direction

Now total strain in the direction of x due to stresses σ_1 and σ_2, $\varepsilon_1 = \frac{\sigma 1}{E} - \mu * \frac{\sigma 2}{E}$

Similarity,

total strain in the direction of y due to stresses σ_1 and σ_2, $\varepsilon_2 = \frac{\sigma 2}{E} - \mu * \frac{\sigma 1}{E}$

For three-Dimensional Stress System



The figure shows that a three-dimensional body subjected to three orthogonal normal stresses σ_1, σ_2, σ_3 acting in the directions of x, y and z respectively.

Consider the strains produced by each stress separately.

The stress σ_1 will produce strain in the direction of x and also in the directions of y and z.

The strain in the direction of x will be $\frac{\sigma 1}{E}$, the strains in the direction y and z will be $(- \mu * \frac{\sigma 1}{E})$ Similarity,

The stress σ_2 will produce strain $\frac{\sigma 2}{E}$ in the direction of y and the strain of $(- \mu * \frac{\sigma 2}{E})$ in the direction of x and z each.

The stress σ_3 will produce strain $\frac{\sigma 3}{E}$ in the direction of z and the strain of $(- \mu * \frac{\sigma 3}{E})$ in the direction of x and y each.

total strain in the direction of x due to stresses σ_1, σ_2 and σ_3,

$$\varepsilon_1 = \frac{\sigma 1}{E} - \mu * \frac{\sigma 2}{E} - \mu * \frac{\sigma 3}{E}$$

total strain in the direction of y due to stresses σ_1, σ_2 and σ_3,

$$\varepsilon_2 = \frac{\sigma 2}{E} - \mu * \frac{\sigma 3}{E} - \mu * \frac{\sigma 1}{E}$$

total strain in the direction of z due to stresses σ_1, σ_2 and σ_3,

$$\varepsilon_3 = \frac{\sigma 3}{E} - \mu * \frac{\sigma 1}{E} - \mu * \frac{\sigma 2}{E}$$

Modulus of rigidity G is related to the modulus of elasticity E and Poisson's ratio μ by

$$G = \frac{E}{2(1+\mu)}$$

Problem 1. A rod 150 cm long and of diameter 2.0 cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is $2*10^5$ N/mm^2. Determine:

1- The stress
2- The strain
3- The elongation of the rod.

Solution

Given: length of the rod, L = 150 cm

Dimeter of the rod = 2.0 cm = 20 mm

Area = 3.14/4 (20^2) = 314. 16 mm^2

Axial pull force P = 20 kN = 20 000 N

Modulus of elasticity E = $2 * 10^5$ N/mm^2

1- Stress, $\sigma = P/A$ = 20 000/314.16 = 63.662 N/mm^2 **Ans**

2- Strain, $E = \sigma/\varepsilon$

$\varepsilon = \sigma/E = 63.662/(2*10^5) = 0.000318$ **Ans**

3- Elongation, $\varepsilon = dL/L$

$dL = \varepsilon * L$

$dL = 0.000318 * 150 = 0.0477$ cm. **Ans**

Problem 2. A square steel rod 20 mm * 20 mm in section is to carry an axial load of 100 kN. Calculate the shortening in a length of 50 mm. assume E = $2.14*10^8$ kN/m^2.

Solution:

Area = 0.02 * 0.02 = 0.0004 m^2 = $4 * 10^{-4}$ m^2

Length = L = 50 mm = 0.05 m

Load = P = 100 kN

E = $2.14 * 10^8$ kN/m^2

Shortening of the rod δL = ?

$$\sigma_c = P/A \quad = 100 / 0.0004 = 250\,000 \text{ kN/m}^2$$

$$E = \sigma/\varepsilon \quad = \quad \varepsilon = \sigma/E = 250\,000 / 2.14 * 10^8 = \delta L / L$$

$$\delta L = (250\,000 / 2.14 * 10^8) * L$$

$$\delta L = (250\,000 / 2.14 * 10^8) * 0.05$$

$$\delta L = 0.0000584 \text{ m} \quad \text{or} \quad 0.0584 \text{ mm}$$

Hence, the shortening of the rod = 0.0584 mm. **Ans.**

Problem 3. The following observations were made during a tensile test on mild steel specimen 40 mm in diameter and 200 mm long. Elongation with 40 kN load (within limit of proportionality). δL = 0.0304 mm, Yield load = 161 kN, Maximum load = 242 kN. Length of specimen at fracture = 249 mm

Determine the followings:

1- Young's modulus of elasticity
2- Yield point stress
3- Ultimate stress
4- Percentage elongation.

Solution:

1- E = ?

$\sigma = P / A = 40 / (3.14/4)(0.04^2) = 3.18 * 10^4$ kN/m^2

$\varepsilon = \delta L / L = 0.0304 / 200 = 0.000152$

$E = \sigma / \varepsilon = 3.18 * 10^4 / 0.000152 = 2.09 * 10^8$ kN/m^2 **Ans.**

2- Yield point stress = yield point load / Area

$= 161 / (3.14 / 4) * (0.04^2) = 12.8 * 10^4$ kN/m^2 **Ans.**

3- Ultimate stress = Max. Load / Area $= 242 / (3.14 / 4) * (0.04^2)$

$= 19.2 * 10^4$ kN/m^2 **Ans.**

4- Percentage elongation = Strain = $\delta L / L = (Lf – Lo) / Lo$

$= (249 – 200) / 200$

$= 0.245 * 100 = 24.5$ % **Ana**