Kurdistan Region
Salahaddin University-Erbil
College of Engineering
Mechanic & Mechatronics Engineering Department

Strength of Materials

For Second Stage Students
In Mechanic & Mechatronics Dept.

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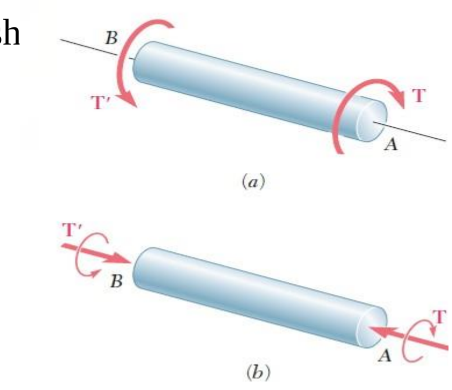
Torsion

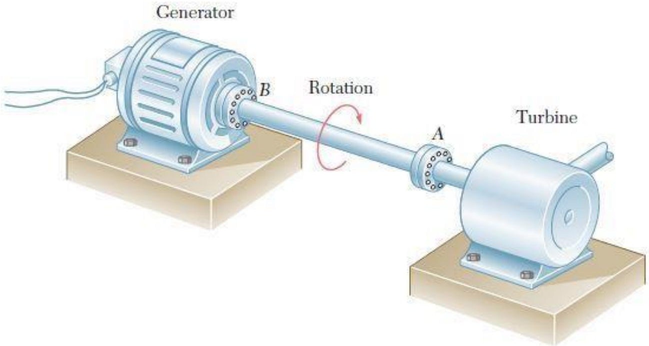
Introduction

A shaft said to be in torsion, when equal and opposite torque are applied at the two ends of the shaft. **The torque is equal to the product of the force applied (tangentially to the end of a shaft) and radius of the shaft.** Due to the application of the torques at the two ends, the shaft is subjected to a twisting moment. This causes the shear stresses and strains in the material of the sh



<u>Shafts:</u> are structural members with length significantly

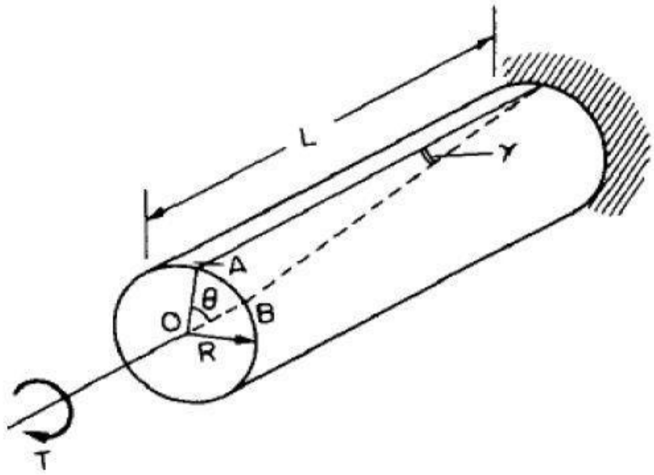
greater than the largest cross-sectional dimension used in transmitting torque from one plane to another.



<u>Torsion:</u> twisting of a structural member, when it is loaded by couples that produce rotation about its longitudinal axis.

Derivation of Shear Stress Produced in A Circular Shaft Subjected to Torsion

When a circular shaft is subjected to torsion, shear stresses are set up in the material of the shaft. To determine the magnitude of shear stress at any point on the shaft, consider a shaft fixed at one end and free at the other end as shown in the figure.



Now let the shaft is subjected to a torque T at the end (A). as a result of this torque T, the shaft at the end (A) will rotate clockwise and every cross-section of the shaft will be subjected to shear stresses. Under the action of this torque a radial line at the free end of the shaft twists through an angle θ, point (A) moves to (B) and (AB) subtends an angle ɣ at the fixed end. This is then the angle of distortion of the shaft, i.e. the shear strain. As shown in figure.

Let:

R = Radius of the shaft
L = Length of shaft
T = Torque applied at the end (A)
τ = Shear stress induced at the surface of the shaft due to torque T
G = Modulus of rigidity of the material of the shaft
ɣ = Shear strain
θ = Angle of twist

Now distortion at the outer surface due to torque T

= AB

Shear strain at outer surface = Distortion per unit length

$$= \frac{\text{Distortion at the outer surface (AB)}}{\text{Length of shaft (L)}} = \tan\gamma = \gamma$$ (if γ is very small then $\tan\gamma = \gamma$)

Shear strain at outer surface, $\gamma = \tan^{-1}\frac{AB}{L}$……………………………………. (i)

Arc AB = OA * θ = Rθ (OA = R = Radius of shaft)

Substituting the value of AB in equation (i), we get

Shear strain at outer surface, $\gamma = \tan^{-1}\frac{R*\theta}{L}$……………………………….…. (ii)

Now the modulus of rigidity (G) of the material of the shaft is given as

$$G = \frac{\text{Shear stress at the outer surface}}{\text{Shear strain at the outer surface}} = \frac{\tau}{\frac{R\theta}{L}} = \frac{\tau * L}{R*\theta} = \frac{G*\theta}{L} = \frac{\tau}{R}$$

$$\tau = \frac{R*G*\theta}{L}$$

For a given shaft subjected to a given torque (T), the values of G, θ and L are constant. Hence shear stress produced is proportional to the radius R.

$\tau \propto R$ or τ / R = Constant ……………………………………(iii)

If (q) is the shear stress induced at any radius 'r' from the center of the shaft then

$$\frac{\tau}{R} = \frac{q}{r}$$

Therefore, $$\frac{\tau}{R} = \frac{q}{r} = \frac{G*\theta}{L}$$

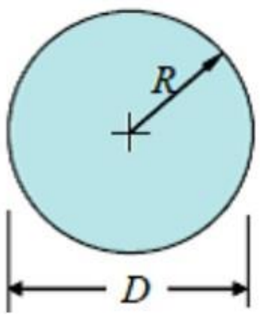
Maximum Torque Transmitted by A Circular Shaft

The maximum torque transmitted by a circular shaft, is obtained from the maximum shear stress induced at the outer surface of the shaft.

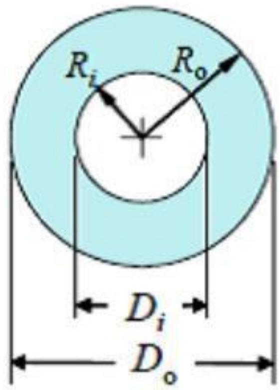
for a **solid or a hollow shaft**, maximum torque transmitted by the shafts can be determined.

For a solid shaft …………………………… $T_{max} = \frac{\pi}{□6} * \tau * D^3$

But for a hollow shaft …………………….. $T_{max} = \frac{\pi}{□6} * \tau * (Do^4 – Di^4)/ Do$



Solid Shaft



Hollow Shaft

Power Transmitted by Shafts

Once the expression for torque (T) for a solid or a hollow shaft is obtained, power transmitted by the shafts can be determined.

Let:

N = revolution per minute (r.p.m) of the shaft

T = Mean torque transmitted in N.m

ω = angular speed of the shaft

Then

$$\text{Power} = \frac{□ * \pi * N * T}{6} \text{ watt}$$

$$\text{Power} = \omega * T \qquad \omega = \frac{□ * \pi * N}{6}$$

Problem 1: A solid shaft of 150 mm diameter is used to transmit torque. Find the maximum torque transmitted by the shaft if the maximum shear induced to the shaft is 45 N/mm^2

Solution:

Problem 2: The shearing stress is a solid shaft is not to exceed 40 N/mm^2 when the torque transmitted is 20000 N.m. Determine the minimum diameter of the shaft.

Solution:

Problem 3: Two shafts of the same material and of same lengths are subjected to the same torque, if the first shaft is of a solid circular section and the second shaft is of hollow circular section, whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the shaft.

Solution:

TORQUE IN TERMS OF POLAR MOMENT OF INERTIA

Polar moment of inertia of a plane area is defined as the moment of inertia of the area about an axis perpendicular to the plane of the figure and passing through the center of gravity of the area. Or polar moment of inertia is the resistance of any object for twisting moment and **it is denoted by symbol J.**

Polar moment inertia for circular shaft is

For solid shafts: $$J = \frac{\pi}{32} * D^4$$

For hollow shafts: $$J = \frac{\pi}{32} * (Do^4 - Di^4)$$

$$\theta = \frac{T * L}{G * J}$$

Where

θ : Angle of twist of the shaft in radian
T : torque transmitted by the shaft
L : length of the shaft
G : modulus of rigidity
J : Polar moment of inertia

POLAR MODULUS

Polar modulus is defined as the ratio of the polar moment of inertia to the radius of the shaft. It is also called torsional section modulus. It is denoted by Z_p. Mathematically.

$$Z_p = \frac{J}{R}$$

For a solid shaft: $$Z_p = \frac{\pi}{16} * D^3$$

For Hollow shaft: $$Z_p = \frac{\pi}{16 * Do}$$

*** $(Do^4 - Di^4)$**

STRENGTH OF A SHAFT AND TORSIONAL RIGIDITY

The strength of a shaft means the maximum torque or maximum power the shaft can transmit. Torsional rigidity or stiffness of the shaft is defined as the product of modulus of rigidity (G) and polar moment of inertia of the shaft (J). Hence mathematically, the torsional rigidity is given as,

Torsional rigidity = G * J

Torsional rigidity is also defined as the torque required to produce a twist of one radian per unit length of the shaft.

Let a twisting moment (**T**) produces a twist of (**θ**) radians in a shaft of length (**L**).

$$\theta = \frac{T*L}{G*J} \quad \Longrightarrow \quad \textbf{Torsional rigidity} = \frac{T*L}{\theta}$$

If

L = one meter and

θ = one radian

The torsional rigidity = Torque

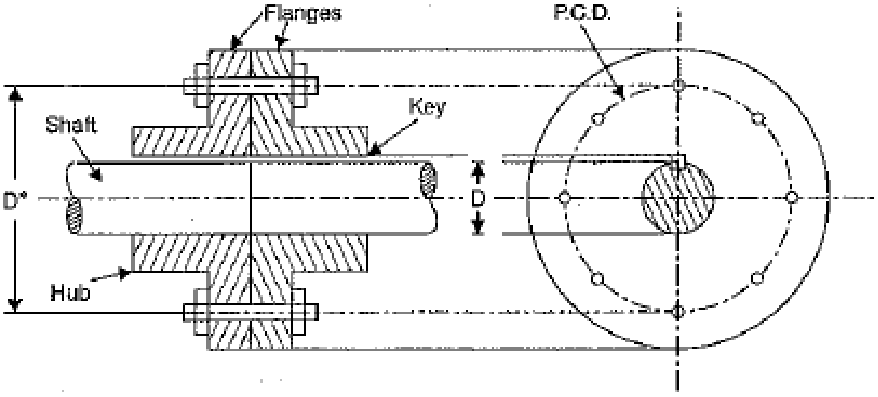
<u>Problem 4: (H.W)</u>

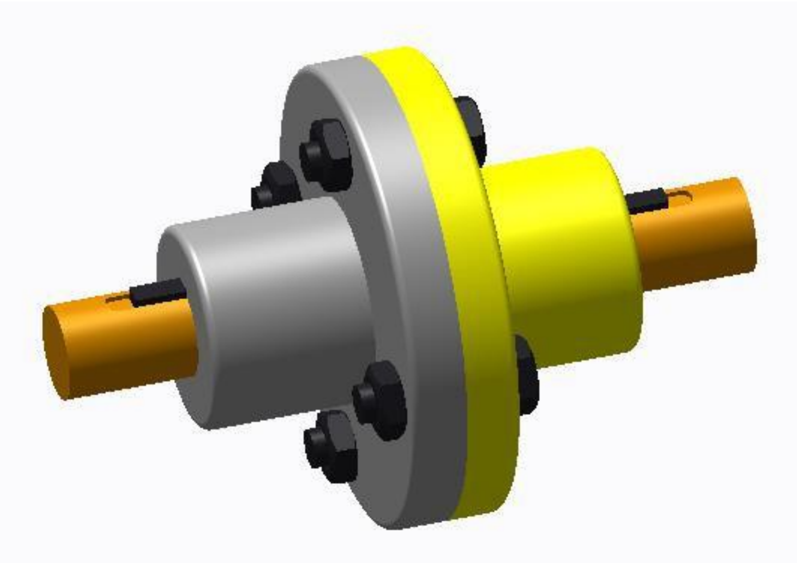
Determine the diameter of a solid steel shaft which will transmit 90 kW at 160 r.p.m. Also determine the length of the shaft if the twist must not exceed 1^o over the entire length. The maximum shear stress is limited to 60 N/mm^2. Take the value of modulus of rigidity = $8 * 10^4$ N/mm^2

Problem 5: A hollow shaft of diameter ratio 3/8 (internal dia. To outer dia.) is to transmit 375 kW power at 100 r.p.m. the maximum torque being 20% greater than the mean. The shear stress is not to exceed 60 N/mm^2 and twist in a length of 4 m not to exceed 2^o. Calculate its external and internal diameters which would satisfy both the above conditions. Assume modulus of rigidity, $G = 0.85 * 10^5$ N/mm^2.

FLANGED COUPLING

A flange coupling is used to connect two shafts as shown in figure.





The flanges of the two shafts are joined together by bolts and nuts (or rivets) and torque is then transferred from one shaft to another through the bolts. Connection between each shaft and coupling is provided by the key. The bolts are arranged along a circle called the pitch circle. The bolts are subjected to shear stress when torque is transmitted from one shaft to another.

Let:

τ = Shear stress in the shaft

q = Shear stress in the bolt

d = diameter of bolt

D = diameter of shaft

D* = Diameter of bolt pitch circle

n = number of bolts

Maximum load that can be resisted by one bolt = Stress in bolt * Area of one bolt

$$= \mathbf{q * \frac{\pi}{4} * d^2}$$

Torque resisted by one bolt = load resisted by one bolt * Radius of pitch circle

$$= \mathbf{q * \frac{\pi}{4} * d^2 * \frac{D}{2}}$$

Total torque resisted by n bolts

$$= \mathbf{n * q * \frac{\pi}{4} * d^2 * \frac{D}{2} \quad = \quad n * q * \frac{\pi * d^2 * D}{8}} \quad \ldots\ldots\ldots\text{(i)}$$

But the torque transmitted by the shaft,

$$\mathbf{T = \frac{\pi}{16} * \tau * D^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Since the torque resisted by the bolts should be equal to the torque transmitted by the shaft, therefore, equating (i) and (ii), we get.

$$\mathbf{n * q * \frac{\pi * d^2 * D}{8} = \frac{\pi}{16} * \tau * D^3}$$

from the above equation the unknown value of any parameter (say number of bolts or diameter of bolt) can be calculated.

Problem 6: Two shafts are connected end to end by means of a flanged coupling in which there are 12 bolts, the pitch circle diameter being 25 cm. the maximum shear stress is limited to 55 N/mm^2 in the shafts and 20 N/mm^2 in the bolts. If one shaft is solid of 5 cm diameter and the other is hollow of 10 cm external diameter. Calculate the internal diameter of the hollow shaft and the bolt diameter so that both shafts and the coupling are all equally strong in torsion.

STRENGTH OF A SHAFT OF VARYING SECTION

When a shaft is made up of different length of different diameters, the torque transmitted by individual sections should be calculated first. The strength of such a shaft is the minimum value of these torque.

Problem 7: A shaft of ABC of 500 mm length and 40 mm external diameter is bored, for a part of its length AB, to a 20 mm diameter and for the remaining length BC to a 30 mm diameter bore. If the shear stress is not to exceed 80 N/mm^2, find the maximum power, the shaft can transmit at a speed of 200 r.p.m. if the angle of twist in the length of 20 mm diameter bore is equal to that in the 30 mm diameter bore, find the length of the shaft that has been bored to 20 mm and 30 mm diameter.