Kurdistan Region
Salahaddin University-Erbil
College of Engineering
Mechanic & Mechatronics Engineering Department

Fundamentals of Design

For Second Stage Students
In Mechanic & Mechatronics Dept.

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Simple Bending in Beams

if a length of a beam is subjected to a constant bending moment and no shear force, then the stresses will be set up in that length of the beam due to bending moment. The stresses set up in that length of beam are known as bending stresses.

Beam bending stress equation (flexure formula) is developed under the following assumptions:

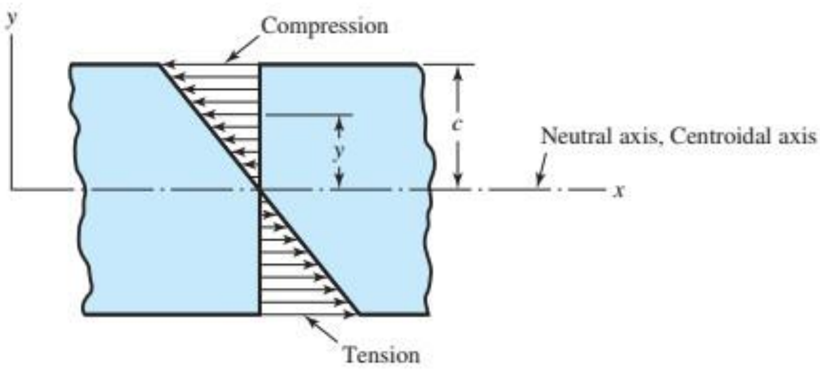
- The beam is straight, long and having a constant cross-section with an axis of symmetry in the plane of bending.
- The material is isotropic, homogeneous, and linearly elastic.
- The beam is subjected to pure bending moment (no axial force, shear or torsion).

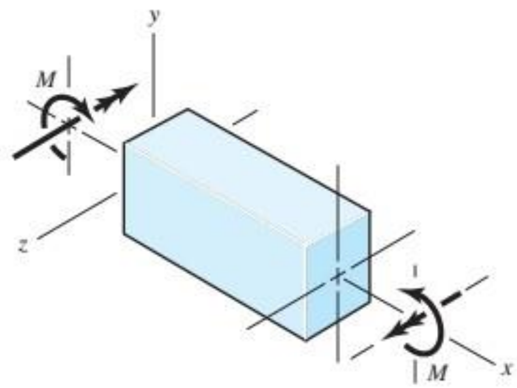
The bending stress in beams subjected to bending moment is found as

$$\sigma = -\frac{My}{I}$$

Where, y: is the height from the neutral axis (centroidal axis)

I: is the moment of inertia about the z axis.
M: is the bending moment





Bending Stresses in symmetrical Section

The neutral axis (N.A.) of a symmetrical section (circular, rectangular or square) lies at a distance of (d/2) from the outermost layer of the section, where (d) is the diameter for a circular section or depth for rectangular or a square section. There is no stress at the neutral axis. The maximum stress takes place at the outermost layer.

The maximum tensile and compressive stresses are at the top and bottom surfaces.
- The maximum bending stress in the beam is usually found using:

$$\sigma = \frac{Mc}{I}$$ where $c = y_{max}$

or sometimes it is written as:

$$\sigma = \frac{M}{Z}$$ where $Z = \frac{I}{c}$ is called the Section Modulus

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis.

$$\frac{I}{y_{max}} = Z$$

where I = M.O.I. about neutral axis
and y_{max} = Distance of the outermost layer from the neutral axis.

From equation (7.4), we have

$$\frac{M}{I} = \frac{\sigma}{y}$$

The stress σ will be maximum, when y is maximum. Hence above equation can be written as

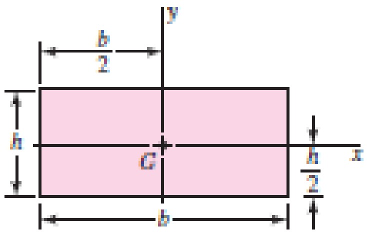
$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$\therefore \quad M = \sigma_{max} \cdot \frac{I}{y_{max}}$$

But $$\frac{I}{y_{max}} = Z$$

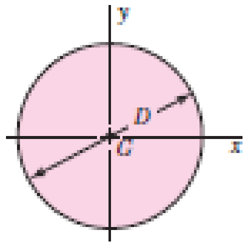
$$\therefore \quad M = \sigma_{max} \cdot Z$$

Rectangle



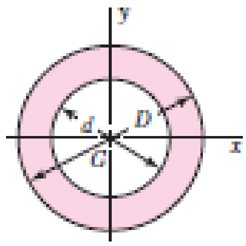
$A = bh \qquad I_x = \frac{bh^3}{12} \qquad I_y = \frac{b^3h}{12} \qquad I_{xy} = 0$

Circle



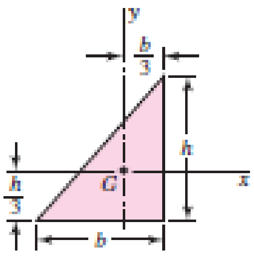
$A = \frac{\pi D^2}{4} \qquad I_x = I_y = \frac{\pi D^4}{64} \qquad I_{xy} = 0 \qquad J_G = \frac{\pi D^4}{32}$

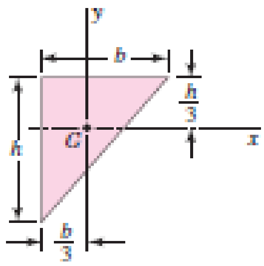
Hollow circle



$A = \frac{\pi}{4}(D^2 - d^2) \qquad I_x = I_y = \frac{\pi}{64}(D^4 - d^4) \qquad I_{xy} = 0 \qquad J_G = \frac{\pi}{32}(D^4 - d^4)$

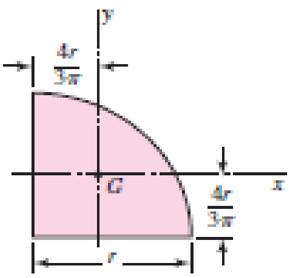
Right triangles

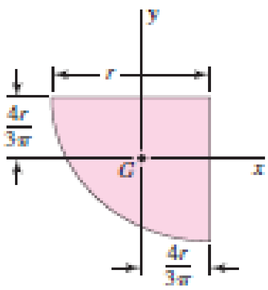




$A = \frac{bh}{2} \qquad I_x = \frac{bh^3}{36} \qquad I_y = \frac{b^3h}{36} \qquad I_{xy} = \frac{b^2h^2}{72}$

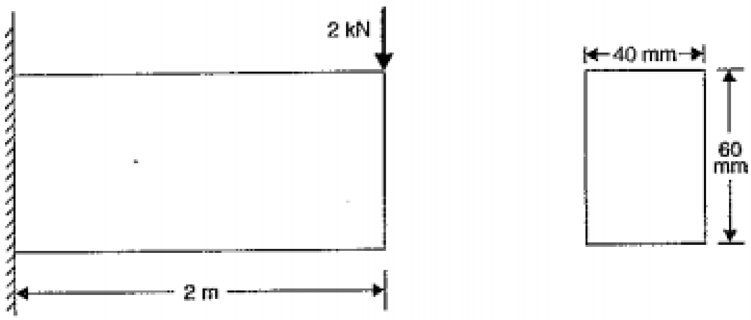
Quarter-circles





$A = \frac{\pi r^2}{4} \qquad I_x = I_y = r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right) \qquad I_{xy} = r^4\left(\frac{1}{8} - \frac{4}{9\pi}\right)$

Example: A cantilever of length 2 meter fails when a load of 2 kN is applied at the free end. If the section of the beam is 40 mm * 60 mm, find the stress at the failure.



Solution

Given:

Length, L = 2 m Load, W = 2 kN = 2000 N section of beam (40*60) mm

Width of beam, b = 40 mm Depth of beam, d = 60 mm

section modulus of a rectangular section is given by

$$Z = \frac{I}{y_{max}} = \frac{bd^3}{12 \times \left(\frac{d}{2}\right)} = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6} \qquad Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6} = 24000 \text{ mm}^3$$

Maximum bending moment for a cantilever is at the fixed end.

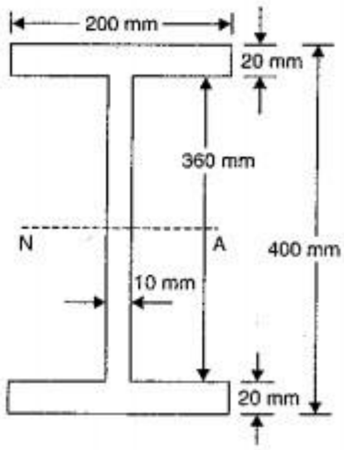
$\therefore$ $M = W \times L = 2000 \times 2 \times 10^3 = 4 \times 10^6$ Nmm

Let σ_{max} = Stress at the failure

$$M = \sigma_{max} . Z$$

$$\sigma_{max} = \frac{M}{Z} = \frac{4 \times 10^6}{24000} = \mathbf{166.67\ N/mm^2}. \quad \textbf{Ans.}$$

Example: A roller steel joist of I section has dimensions as shown in figure. If the maximum bending moment on the section is 5*10^8 N.mm, calculate the maximum stress produced due to bending.



Solution

Moment of inertia about the neutral axis

$$= \frac{200 \times 400^3}{12} - \frac{(200-10) \times 360^3}{12}$$
$$= 1066666666 - 738720000$$
$$= 327946666 \text{ mm}^4$$

Now using the relation,

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\therefore \quad \sigma = \frac{M}{I} \times y$$

$$\sigma_{max} = \frac{M}{I} \times y_{max} = \frac{5 \times 10^8}{327946666} \times 200 \qquad (\because \; y_{max} = 200 \text{ mm})$$

$$= \mathbf{304.92 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

Bending Stress in Unsymmetrical Section

In case of symmetrical section, the neutral axis passes through the geometrical center of the section. But in case of unsymmetrical section such as L, T sections, the neutral axis does not pass through the geometrical center of the section. Hence the value of y for the topmost layer or bottom layer of the section from neutral axis will not be same. For finding bending stress in the beam, the bigger value of y is used. Hence in unsymmetrical sections, first the center of gravity is calculated.

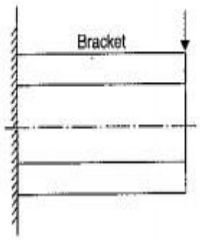
Example: A cast iron bracket subjected to bending has the cross-section of I- form with unequal flanges. The dimensions of the section are shown in figure. Find the position of the neutral axis and moment of inertia of the section about the neutral axis. If the maximum bending moment on the section is 40 MN.mm, determine the maximum bending stress.

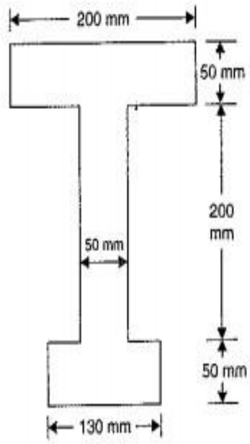
Solution

Max. B.M., $M = 40$ MN mm $= 40 \times 10^6$ Nmm

Let us first calculate the C.G. of the section. Let $\bar{y}$ is the distance of the C.G. from the bottom face. The section is symmetrical about y-axis and hence $\bar{y}$ is only to be calculated. Then,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{(A_1 + A_2 + A_3)}$$





where A_1 = Area of bottom flange = $130 \times 50 = 6500$ mm^2

y_1 = Distance of C.G. of A_1 from bottom face

$= \frac{50}{2} = 25$ mm

A_2 = Area of web = $200 \times 50 = 10000$ mm^2

y_2 = Distance of C.G. of A_2 from bottom face

$= 50 + \frac{200}{2} = 150$ mm

A_3 = Area of top flange = $200 \times 50 = 10000$ mm^2

y_3 = Distance of C.G. of A_3 from bottom face

$= 50 + 200 + \frac{50}{2} = 275$ mm.

$$\therefore \quad \bar{y} = \frac{6500 \times 25 + 10000 \times 150 + 10000 \times 275}{6500 + 10000 + 10000}$$

$$= \frac{162500 + 1500000 + 2750000}{26500}$$

$$= \frac{4412500}{26500} = 166.51 \text{ mm}$$

Hence neutral axis is at a distance of 166.51 mm from the bottom face. **Ans.**

Moment of inertia of the section about the N.A.

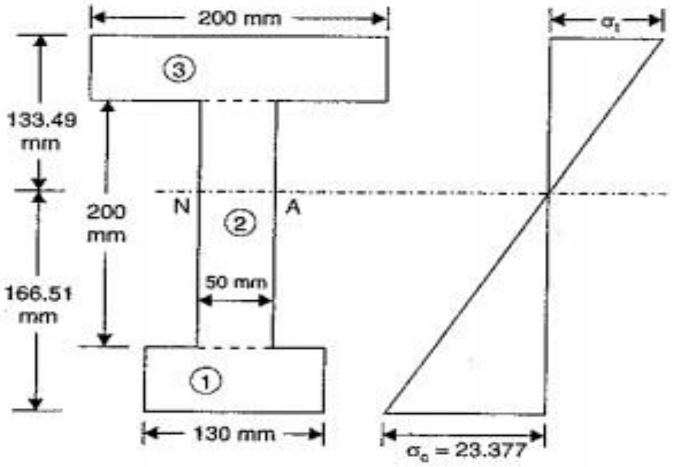
$$I = I_1 + I_2 + I_3$$

where I_1 = M.O.I. of bottom flange about N.A.

= M.O.I. of bottom flange about an axis passing through its C.G. + A_1 × (Distance of its C.G. from N.A.)^2

$$= \frac{130 \times 50^3}{12} + 6500 \times (166.51 - 25)^2$$

$$= 1354166.67 + 130163020 = 131517186.6 \text{ mm}^4$$



Similarly I_2 = M.O.I. of web about N.A.

$$= \frac{50 \times 200^3}{12} + A_2 . (166.51 - y_2)^2$$

$$= \frac{50 \times 200^3}{12} + 10000\,(166.51 - 150)^2$$

$$= 33333333.33 + 272580.1$$

$$= 33605913.43 \text{ mm}^4$$

and I_3 = M.O.I. of top flange about N.A.

$$= \frac{200 \times 50^3}{12} + A_3 . (y_3 - 166.51)^2$$

$$= \frac{200 \times 50^3}{12} + 10000 \times (275 - 166.51)^2$$

$$= 2083333.33 + 117700801 = 119784134.3 \text{ mm}^4$$

$\therefore$ $I = I_1 + I_2 + I_3 = 131517186.6 + 33605913.43 + 119784134.3$

$= \mathbf{284907234.9\ mm^4}$. **Ans.**

Now distance of C.G. from the upper top fibre

$$= 300 - \bar{y} = 300 - 166.51 = 133.49 \text{ mm}$$

and the distance of C.G. from the bottom fibre

$$= \bar{y} = 166.51 \text{ mm}$$

Hence we shall take the value of y = 166.51 mm for maximum bending stress.

Now using the equation

$$\frac{M}{I} = \frac{\sigma}{y}$$

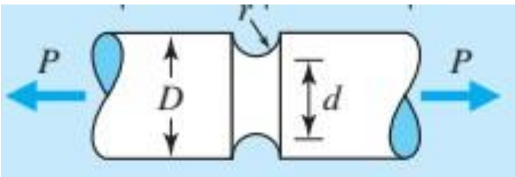
$$\therefore \quad \sigma = \frac{M}{I} \times y = \frac{40 \times 10^6}{284907234.9} \times 166.51 = 23.377 \text{ N/mm}^2$$

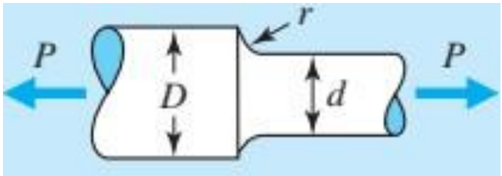
$\therefore$ Maximum bending stress

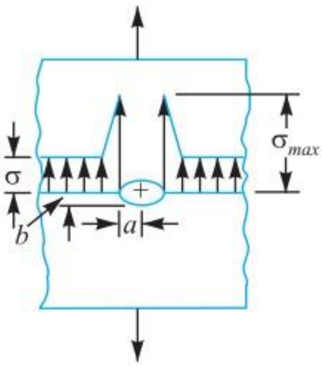
$= \mathbf{23.377\ N/mm^2}$. **Ans.**

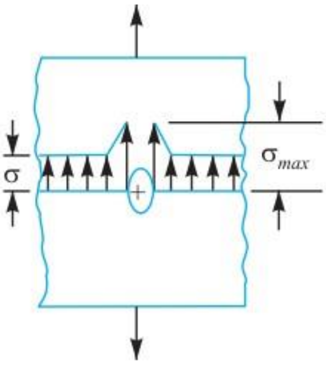
Stress Concentration Factor

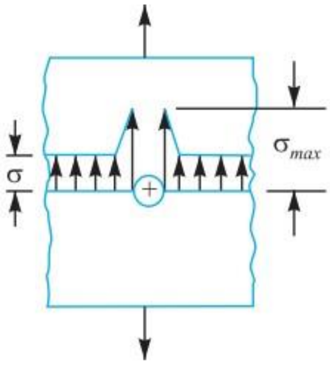
The presence of discontinuities (such as a hole in a plate) alters the stress distribution causing higher stress near the discontinuity. Any type of discontinuity (*hole, shoulder, notch, inclusion*) serve as a stress raiser where it increases the stress in the vicinity of the discontinuity.
Stress concentration occurs at the region in which stress raisers are present, and a stress concentration factor (K_t or K_{ts}) is used to relate the actual maximum stress at the discontinuity to the nominal stress without the discontinuity.











$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$

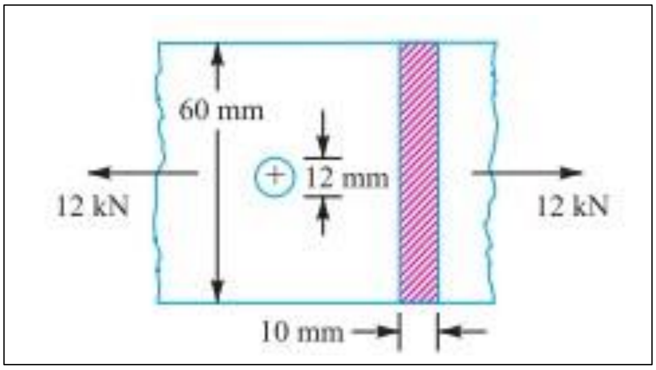
where K_t is used for normal stresses and K_{ts} for shear stresses.

Stress concentration factors are independent of the material properties (as long as the material is in the linear elastic region). They depend only on the type of discontinuity and the geometry.

Tables A-15 & A-16 in the text give the stress concentration factors for some geometric and loading configurations.

Example: Find the maximum stress induced in the following case taking stress concentration into account:

A rectangular plate (60 × 10) mm with a hole 12 mm diameter as shown in fig. and subjected to a tensile load of 12 kN.



Solution. Case 1. Given : $b = 60$ mm ; $t = 10$ mm ; $d = 12$ mm ; $W = 12$ kN $= 12 \times 10^3$ N

We know that cross-sectional area of the plate,

$$A = (b - d)\, t = (60 - 12)\, 10 = 480 \text{ mm}^2$$

$$\therefore \quad \text{Nominal stress} = \frac{W}{A} = \frac{12 \times 10^3}{480} = 25 \text{ N/mm}^2 = 25 \text{ MPa}$$

Ratio of diameter of hole to width of plate,

$$\frac{d}{b} = \frac{12}{60} = 0.2$$

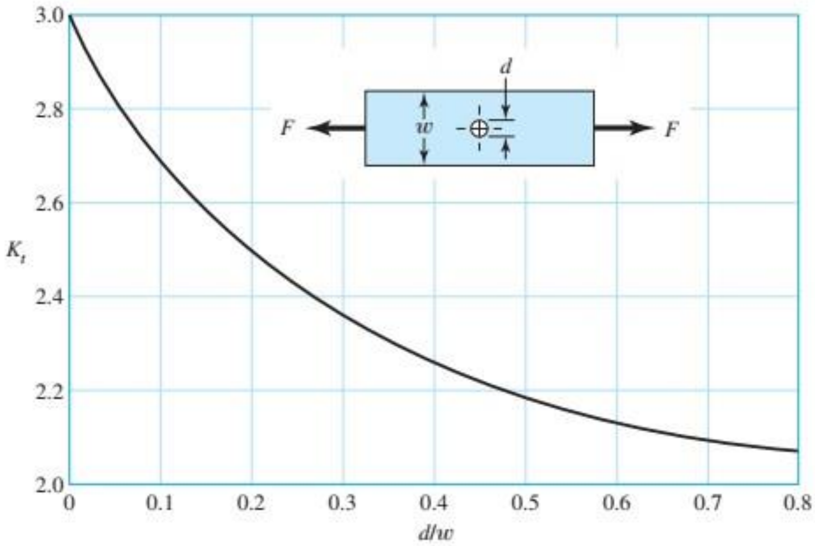
From table A 15 we find that for $d / b = 0.2$, theoretical stress concentration factor,

$$K_t = 2.5$$

$$\therefore \quad \text{Maximum stress} = K_t \times \text{Nominal stress} = 2.5 \times 25 = 62.5 \text{ MPa}$$ **Ans.**

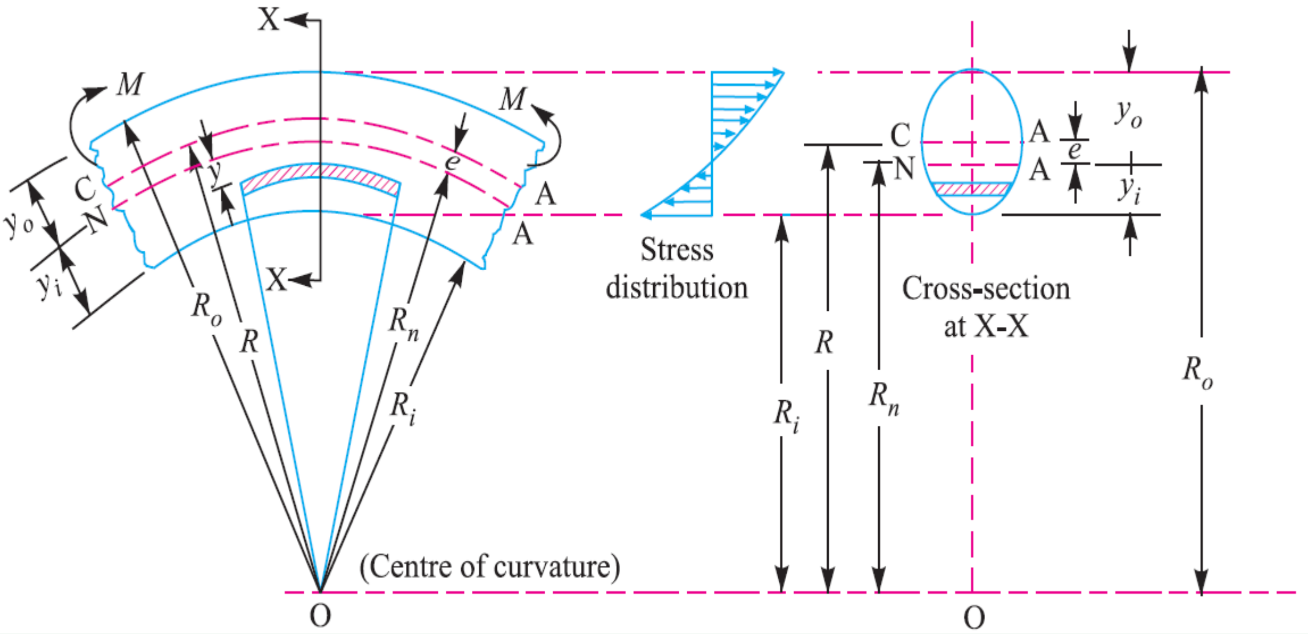
Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.



Bending Stress in Curved Beams

We have seen in the previous article that for the straight beams, the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in case of curved beams, the neutral axis of the cross-section is shifted towards the center of curvature of the beam causing a non-linear (hyperbolic) distribution of stress, as shown in Fig. 5.8. It may be noted that the neutral axis lies between the centroidal axis and the centre of curvature and always occurs within the curved beams. The application of curved beam principle is used in crane hooks, chain links and etc.



Consider a curved beam subjected to a bending moment *M*, as shown in Fig. In finding the bending stress in curved beams, the same assumptions are used as for straight beams. The general expression for the bending stress (σ_b) in a curved beam at any fiber at a distance *y* from the neutral axis, is given by

$$\sigma_b = \frac{M}{A \cdot e}\left(\frac{y}{R_n - y}\right)$$

where M = Bending moment acting at the given section about the centroidal axis,

A = Area of cross-section,

e = Distance from the centroidal axis to the neutral axis = $R - R_n$,

R = Radius of curvature of the centroidal axis,

R_n = Radius of curvature of the neutral axis, and

y = Distance from the neutral axis to the fibre under consideration. It is positive for the distances towards the centre of curvature and negative for the distances away from the centre of curvature.

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i}$$

where y_i = Distance from the neutral axis to the inside fibre = $R_n - R_i$, and

R_i = Radius of curvature of the inside fibre.

The maximum bending stress at the outside fibre is given by

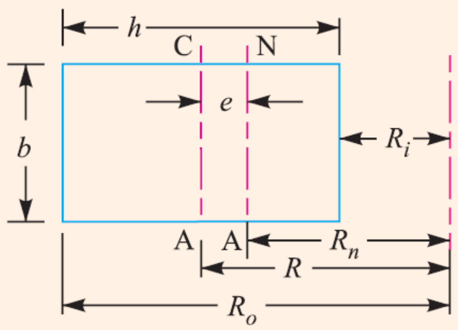
$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o}$$

where y_o = Distance from the neutral axis to the outside fibre = $R_o - R_n$, and

R_o = Radius of curvature of the outside fibre.

It may be noted that the bending stress at the inside fibre is ***tensile*** while the bending stress at the outside fibre is ***compressive.***

Resultant stress, $\sigma = \sigma_d \pm \sigma_b$

Section	*Values of R_n and R*
	$R_n = \frac{h}{\log_e\left(\frac{R_o}{R_i}\right)}$ $R = R_i + \frac{h}{2}$

<table>
<tr><th>Section</th><th>Values of R_n and R</th></tr>
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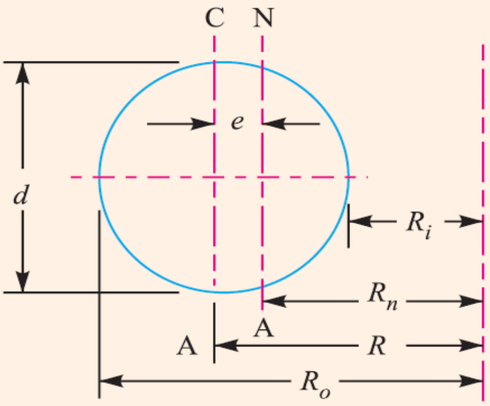

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$$R_n = \frac{\left[\sqrt{R_o} + \sqrt{R_i}\right]^2}{4}$$

$$R = R_i + \frac{d}{2}$$

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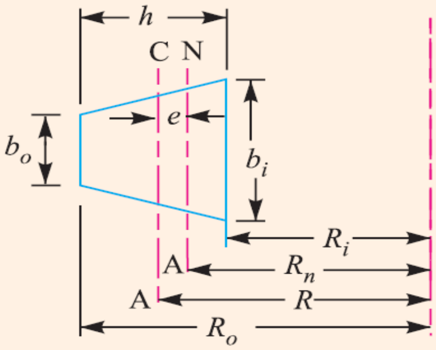

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$$R_n = \frac{\left(\frac{b_i + b_o}{2}\right) h}{\left(\frac{b_i R_o - b_o R_i}{h}\right) \log_e \left(\frac{R_o}{R_i}\right) - (b_i - b_o)}$$

$$R = R_i + \frac{h\,(b_i + 2b_o)}{3\,(b_i + b_o)}$$

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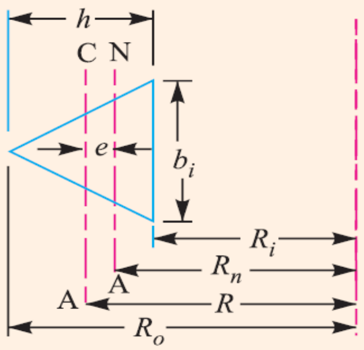
<tr><td>


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$$R_n = \frac{\frac{1}{2} b_i \times h}{\frac{b_i R_o}{h} \log_e \left(\frac{R_o}{R_i}\right) - b_i}$$

$$R = R_i + \frac{h}{3}$$

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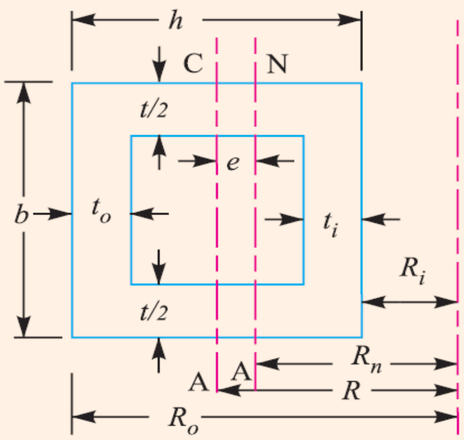

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$$R_n = \frac{(b - t)(t_i + t_o) + t.h}{b\left[\log_e\left(\frac{R_i + t_i}{R_i}\right) + \log_e\left(\frac{R_o}{R_o - t_o}\right)\right] + t.\log_e\left(\frac{R_o - t_o}{R_i + t_i}\right)}$$

$$R = R_i + \frac{\frac{1}{2}h^2.t + \frac{1}{2}t_i^2(b - t) + (b - t)\,t_o\,(h - \frac{1}{2}t_o)}{h.t + (b - t)(t_i + t_o)}$$

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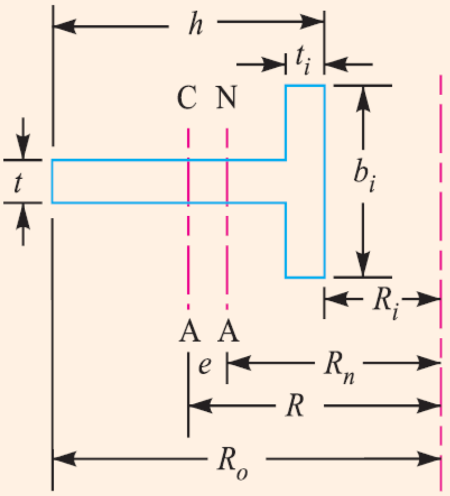
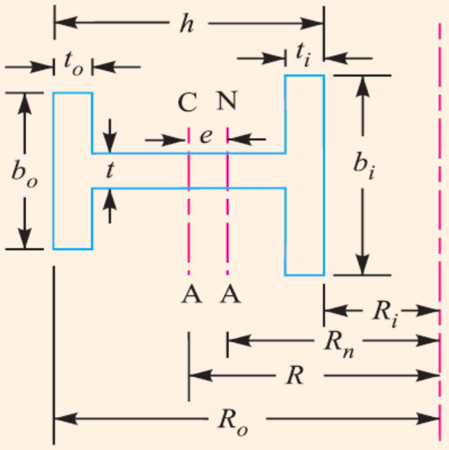
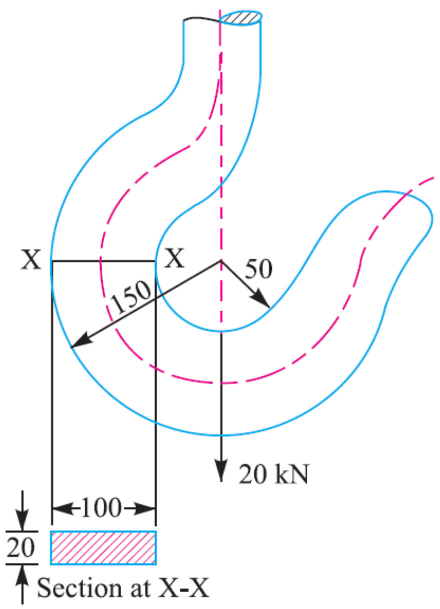
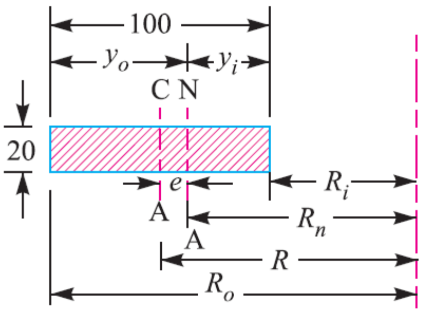
Section	*Values of R_n and R*
	$R_n = \dfrac{t_i(b_i - t) + t.h}{(b_i - t)\log_e\left(\dfrac{R_i + t_i}{R_i}\right) + t.\log_e\left(\dfrac{R_o}{R_i}\right)}$ $R = R_i + \dfrac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t)}{h.t + t_i(b_i - t)}$
	$R_n = \dfrac{t_i(b_i - t) + t_o(b_o - t) + t.h}{b_i\log_e\left(\dfrac{R_i + t_i}{R_i}\right) + t\log_e\left(\dfrac{R_o - t_o}{R_i + t_i}\right) + b_o\log_e\left(\dfrac{R_o}{R_o - t_o}\right)}$ $R = R_i + \dfrac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t) + (b_o - t)\,t_o\,(h - \frac{1}{2}t_o)}{t_i(b_i - t) + t_o(b_o - t) + t.h}$

Example. The crane hook carries a load of 20 kN as shown in Fig. The section at X-X is rectangular whose horizontal side is 100 mm. Find the stresses in the inner and outer fibers at the given section.



All dimensions in mm.



All dimensions in mm.

Solution. Given : $W = 20$ kN $= 20 \times 10^3$ N ; $R_i = 50$ mm ; $R_o = 150$ mm ; $h = 100$ mm ; $b = 20$ mm

We know that area of section at *X-X*,

$$A = b.h = 20 \times 100 = 2000 \text{ mm}^2$$

The various distances are shown in Fig. 5.12.

We know that radius of curvature of the neutral axis,

$$R_n = \frac{h}{\log_e \left(\frac{R_o}{R_i}\right)} = \frac{100}{\log_e \left(\frac{150}{50}\right)} = \frac{100}{1.098} = 91.07 \text{ mm}$$

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h}{2} = 50 + \frac{100}{2} = 100 \text{ mm}$$

∴ Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 100 - 91.07 = 8.93 \text{ mm}$$

and distance between the load and the centroidal axis,

$$x = R = 100 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W \times x = 20 \times 10^3 \times 100 = 2 \times 10^6 \text{ N-mm}$$

The section at *X-X* is subjected to a direct tensile load of $W = 20 \times 10^3$ N and a bending moment of $M = 2 \times 10^6$ N-mm. We know that direct tensile stress at section *X-X*,

$$\sigma_t = \frac{W}{A} = \frac{20 \times 10^3}{2000} = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$

We know that the distance from the neutral axis to the inside fibre,

$$y_i = R_n - R_i = 91.07 - 50 = 41.07 \text{ mm}$$

and distance from the neutral axis to outside fibre,

$$y_o = R_o - R_n = 150 - 91.07 = 58.93 \text{ mm}$$

∴ Maximum bending stress at the inside fibre,

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i} = \frac{2 \times 10^6 \times 41.07}{2000 \times 8.93 \times 50} = 92 \text{ N/mm}^2 = 92 \text{ MPa (tensile)}$$

and maximum bending stress at the outside fibre,

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o} = \frac{2 \times 10^6 \times 58.93}{2000 \times 8.93 \times 150} = 44 \text{ N/mm}^2$$

$$= 44 \text{ MPa (compressive)}$$

∴ Resultant stress at the inside fibre

$$= \sigma_t + \sigma_{bi} = 10 + 92 = 102 \text{ MPa (tensile)}$$ **Ans.**

and resultant stress at the outside fibre

$$= \sigma_t - \sigma_{bo} = 10 - 44 = -34 \text{ MPa} = 34 \text{ MPa (compressive)}$$ **Ans.**