Columns and Struts

A machine part subjected to an axial compressive force is called a ***strut*.** A strut may be horizontal, inclined, or even vertical. But a vertical strut is known as a ***column, pillar*** or ***stanchion*.**

Failure of a Column or Strut

It has been observed that when a column or a strut is subjected to a compressive load and the load is gradually increased, a stage will reach when the column will be subjected to ultimate load. Beyond this, the column will fail by crushing and the load will be known as the ***crushing load*.**

It has also been experienced, that sometimes, a compression member does not fail entirely by crushing, but also by bending *i.e.* buckling. This happens in the case of long columns. It has also been observed, that all the

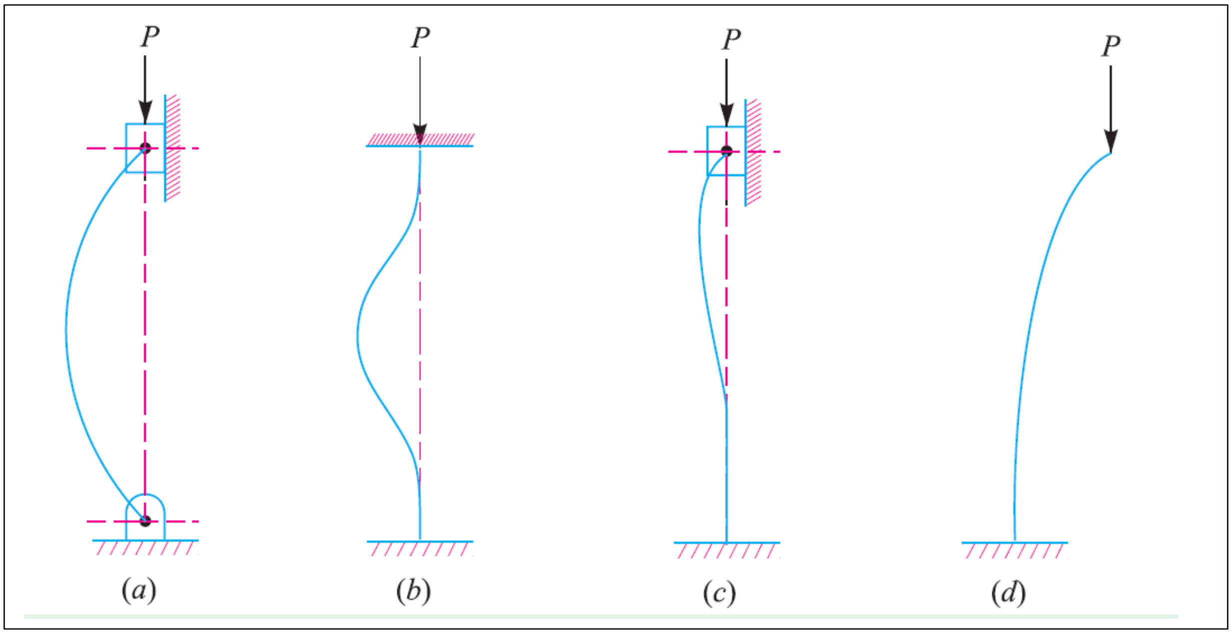
*short columns fail due to their crushing. But, if a

long column is subjected to a compressive load, it is subjected to compressive stress. If the load is gradually increased, the column will reach a stage, when it will start buckling. The load, at which the column tends to have lateral displacement or tends to buckle is called a ***buckling load, critical load,* or ***crippling load*** and the column is said to have developed an elastic instability.

Types of End Conditions of Columns

In actual practice, there are a number of end conditions for columns. But we shall study the Euler's column theory on the following four types of end conditions which are important from the subject's point of view:

1. Both the ends are hinged or pin jointed as shown in Fig. (*a*),
2. Both the ends are fixed as shown in Fig. (*b*),
3. One end is fixed and the other hinged as shown in Fig. (*c*), and
4. One end is fixed and the other free as shown in Fig. (*d*).



Euler's Column Theory

The first rational attempt, to study the stability of long columns, was made by **Mr. Euler.** He **derived an equation, for the buckling load of long columns based on the bending stress.** While deriving this equation, **the effect of direct stress is neglected**. This may be justified by the statement, that the direct stress induced in a long column is negligible as compared to the bending stress. It may be noted that **Euler's formula cannot be used in the case of short columns, because the direct stress is considerable, and hence cannot be neglected.**

* The columns which have **lengths less than 8 times their diameter**, are called ***short columns***.

** The columns which have **lengths more than 30 times their diameter,** are called ***long columns*****.**

Assumptions in Euler's Column Theory

The following simplifying assumptions are made in Euler's column theory:

1. Initially the column is perfectly straight, and the load applied is truly axial.

2. The cross-section of the column is uniform throughout its length.

3. The column material is perfectly elastic, homogeneous, and isotropic, and thus obeys Hooke's law.

4. The length of the column is very large as compared to its cross-sectional dimensions.

5. The shortening of the column, due to direct compression (being very small) is neglected.

6. The failure of the column occurs due to buckling alone.

7. The weight of the column itself is neglected.

Euler's Formula

According to Euler's theory, the crippling or buckling load (*Wcr*) under various end conditions is represented by a general equation,

$$W_{cr} = \frac{C\,\pi^2 E\,I}{l^2} = \frac{C\,\pi^2 E\,A\,k^2}{l^2} \qquad \ldots (\because I = A.k^2)$$

$$= \frac{C\,\pi^2 E\,A}{(l/k)^2}$$

where E = Modulus of elasticity or Young's modulus for the material of the column,

A = Area of cross-section,

k = Least radius of gyration of the cross-section,

l = Length of the column, and

C = Constant, representing the end conditions of the column or end fixity coefficient.

The following table shows the values of end fixity coefficient (C) for various end conditions.

S. No.	*End conditions*	*End fixity coefficient (C)*
1.	Both ends hinged	1
2.	Both ends fixed	4
3.	One end fixed and other hinged	2
4.	One end fixed and other end free	0.25

The vertical column will have two moments of inertia (*viz*. *Ixx* and *Iyy*). Since the column will tend to buckle in the direction of the least moment of inertia, therefore the least value of the two moments of inertia is to be used in the relation.

Slenderness Ratio

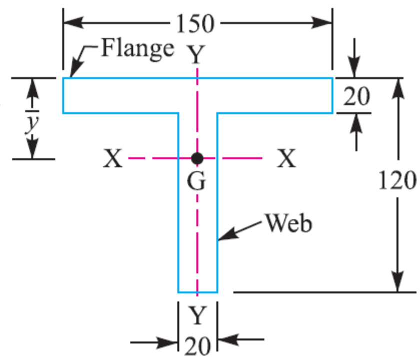
In Euler's formula, the ratio *l* / *k* is known as the ***slenderness ratio***. It may be defined as the ratio of the effective length of the column to the least radius of gyration of the section. It may be noted that the formula for crippling load, in the previous article is based on the assumption that the slenderness ratio *l* / *k* is so large, that the failure of the column occurs only due to bending, the effect of direct stress (*i.e.* *W* / *A*) being negligible.

$$\sigma_{cr} = \frac{W_{cr}}{A} = \frac{C\ \pi^2\ E}{(l / k)^2}$$

Example: A T-section 150 mm × 120 mm × 20 mm is used as a strut of 4 m long hinged at both ends. Calculate the crippling load, if Young's modulus for the material of the section is 200 kN/mm^2.

Solution. Given : $l = 4$ m = 4000 mm ; $E = 200$ kN/mm^2 = 200×10^3 N/mm^2

First of all, let us find the centre of gravity (G) of the T-section as shown in Fig. 16.2.



All dimensions in mm.

Let $\bar{y}$ be the distance between the centre of gravity (G) and top of the flange,

We know that the area of flange,

$$a_1 = 150 \times 20 = 3000 \text{ mm}^2$$

Its distance of centre of gravity from top of the flange,

$$y_1 = 20 / 2 = 10 \text{ mm}$$

Area of web, $a_2 = (120 - 20)\,20 = 2000 \text{ mm}^2$

Its distance of centre of gravity from top of the flange,

$$y_2 = 20 + 100 / 2 = 70 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{3000 \times 10 + 2000 \times 70}{3000 + 2000} = 34 \text{ mm}$$

We know that the moment of inertia of the section about X-X,

$$I_{XX} = \left[\frac{150\,(20)^3}{12} + 3000\,(34 - 10)^2 + \frac{20\,(100)^3}{12} + 2000\,(70 - 34)^2\right]$$

$$= 6.1 \times 10^6 \text{ mm}^4$$

and $$I_{YY} = \frac{20\,(150)^3}{12} + \frac{100\,(20)^3}{12} = 5.7 \times 10^6 \text{ mm}^4$$

Since I_{YY} is less than I_{XX}, therefore the column will tend to buckle in Y-Y direction. Thus we shall take the value of I as $I_{YY} = 5.7 \times 10^6$ mm^4.

Moreover as the column is hinged at its both ends, therefore equivalent length,

$$L = l = 4000 \text{ mm}$$

We know that the crippling load,

$$W_{cr} = \frac{\pi^2 E I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times 5.7 \times 10^6}{(4000)^2} = 703 \times 10^3 \text{ N} = 703 \text{ kN}$$ **Ans.**