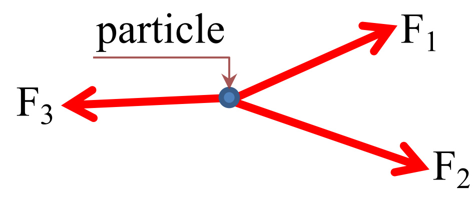
Equilibrium of a Particle

By particle we mean an object whose physical dimensions are of no significance to the analysis of forces acting on it. All the forces in this case would pass through a single point resulting in a concurrent force system.

Condition for the equilibrium of the particle:-
When a particle is at rest or moves with constant velocity, it is in equilibrium (static equilibrium).All the forces acting on the particle form a zero force resultant (Newton's 1st law of motion).

$$\sum \boldsymbol{F} = \mathbf{0}$$



The Free-Body diagram:-
This diagram is simply a sketch which shown the particle "free" from its surroundings with all the forces that act on it.

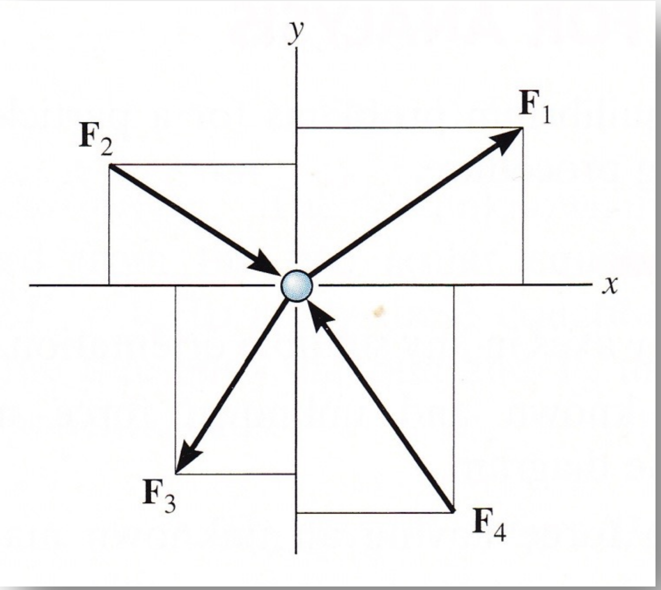
Coplanar Force Systems

If a particle is subjected to a system of coplanar forces that lie in the x–y plane, then each force can be resolved into its x and y components. For equilibrium.

$$\Sigma \mathbf{F} = \mathbf{0}$$

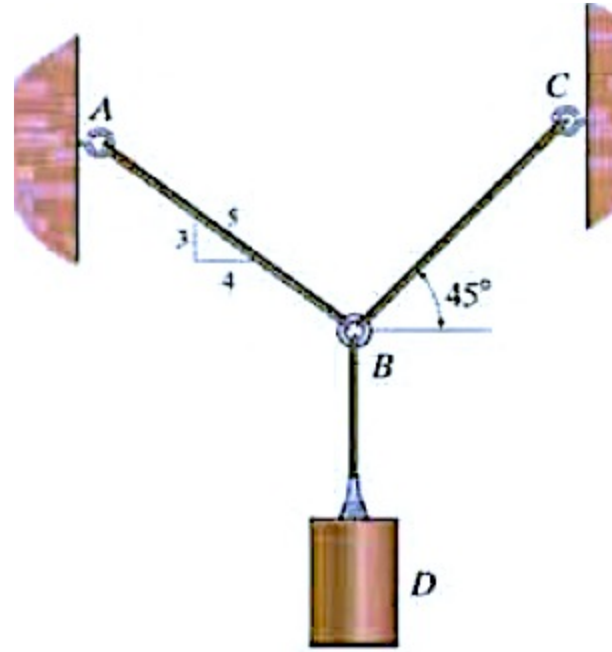
For equilibrium both the x and y components must be equal to zero.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$



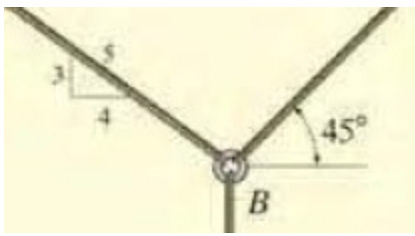
EXAMPLE

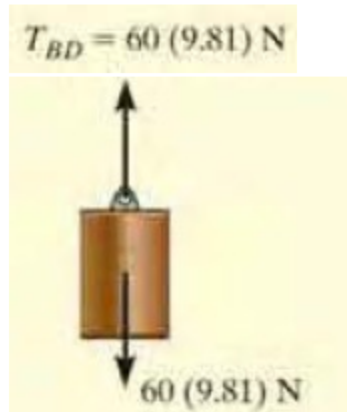
Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig. 3-6a.

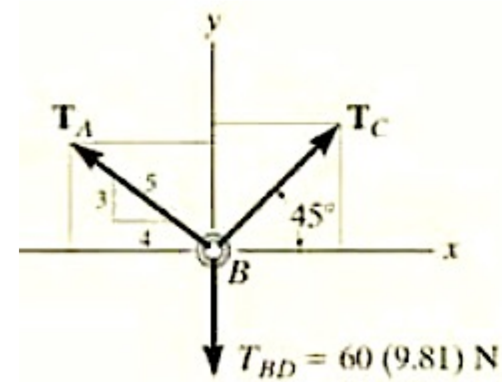


SOLUTION

Free-Body Diagram.







Equations of Equilibrium.

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad T_C \cos 45^\circ - \left(\frac{4}{5}\right)T_A = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0;$ $\qquad T_C \sin 45^\circ + \left(\frac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0 \qquad (2)$

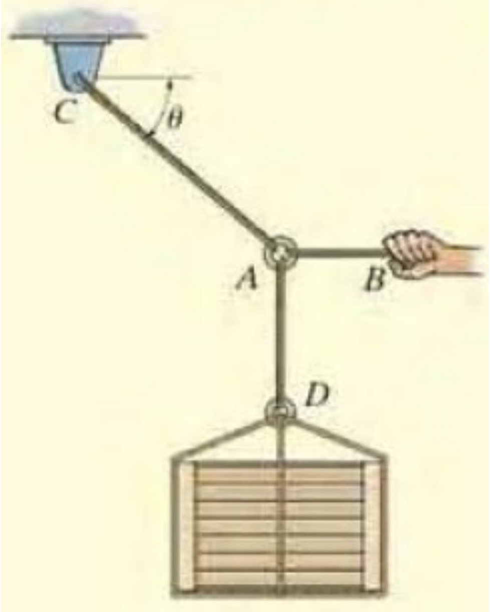
Equation (1) can be written as $T_A = 0.8839 T_C$

Eq. (2) $\qquad T_C \sin 45^\circ + \left(\frac{3}{5}\right)(0.8839 T_C) - 60(9.81)\text{ N} = 0$ $\qquad T_C = 475.66\text{ N} = 476\text{ N}$

Substituting $\qquad T_A = 420\text{ N}$

EXAMPLE

The 200-kg crate in Fig. 3-7*a* is suspended using the ropes *AB* and *AC*. Each rope can withstand a maximum force of 10 kN before it breaks. If *AB* always remains horizontal, determine the smallest angle θ to which the crate can be suspended before one of the ropes breaks.



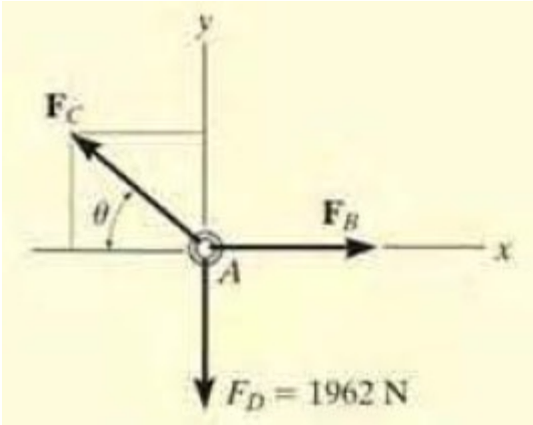
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad -F_C\cos\theta + F_B = 0; \quad F_C = \frac{F_B}{\cos\theta} \qquad (1)$$

$$+\uparrow\Sigma F_y = 0; \qquad F_C\sin\theta - 1962\text{ N} = 0 \qquad (2)$$

F_C is always greater than F_B since $\cos\theta \leq 1$.

Substituting $F_C = 10$ kN into Eq. (2).

$$[10(10^3)\text{ N}]\sin\theta - 1962\text{ N} = 0 \qquad \theta = \sin^{-1}(0.1962) = 11.31^\circ$$



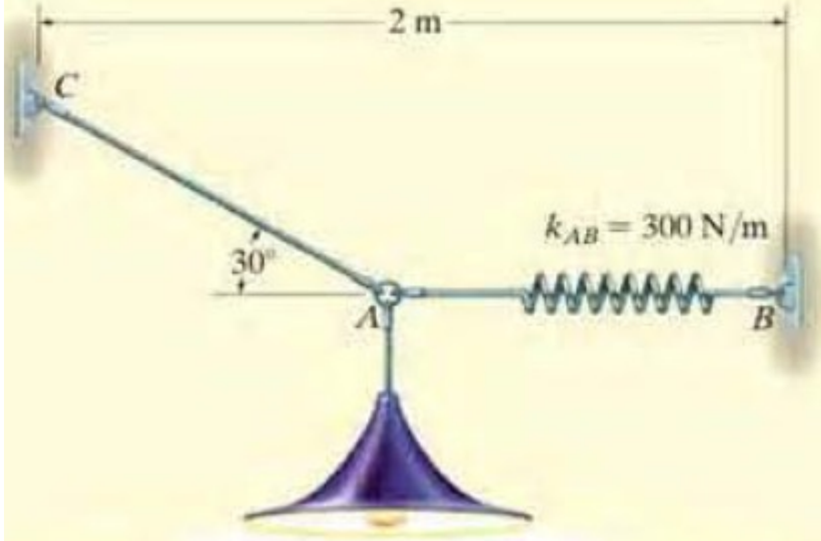
substituting for θ and F_C into Eq. (1).

$$10(10^3)\text{ N} = \frac{F_B}{\cos 11.31^\circ}$$

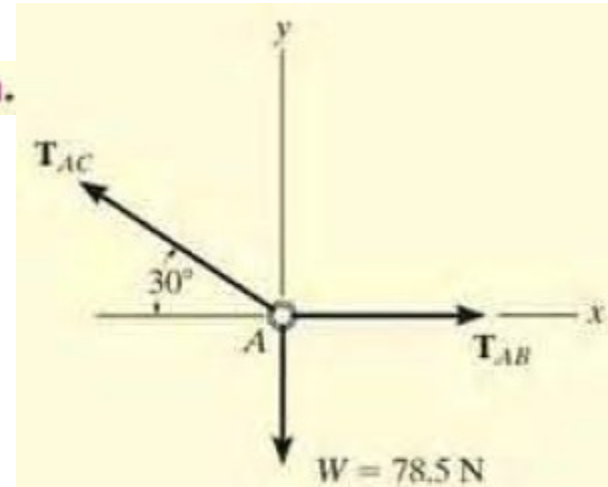
$$F_B = 9.81\text{ kN}$$

EXAMPLE

Determine the required length of cord AC in Fig. 3–8a so that the 8-kg lamp can be suspended in the position shown. The *undeformed* length of spring AB is $l'_{AB} = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m.



Free-Body Diagram.



Equations of Equilibrium.

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad T_{AB} - T_{AC} \cos 30° = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad T_{AC} \sin 30° - 78.5 \text{ N} = 0$

Solving,

$T_{AC} = 157.0 \text{ N}$

$T_{AB} = 135.9 \text{ N}$

The stretch of spring AB is therefore using $F = ks$

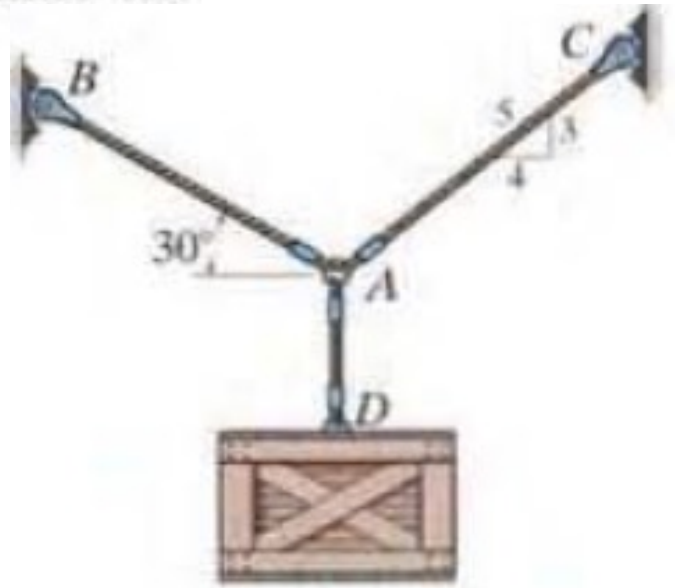
$T_{AB} = k_{AB} s_{AB};$ $\quad 135.9 \text{ N} = 300 \text{ N/m}(s_{AB})$ $\quad s_{AB} = 0.453 \text{ m}$

so the stretched length is $l_{AB} = l'_{AB} + s_{AB}$ $\quad l_{AB} = 0.4 \text{ m} + 0.453 \text{ m} = 0.853 \text{ m}$

The horizontal distance from C to B, Fig. 3–8a, requires

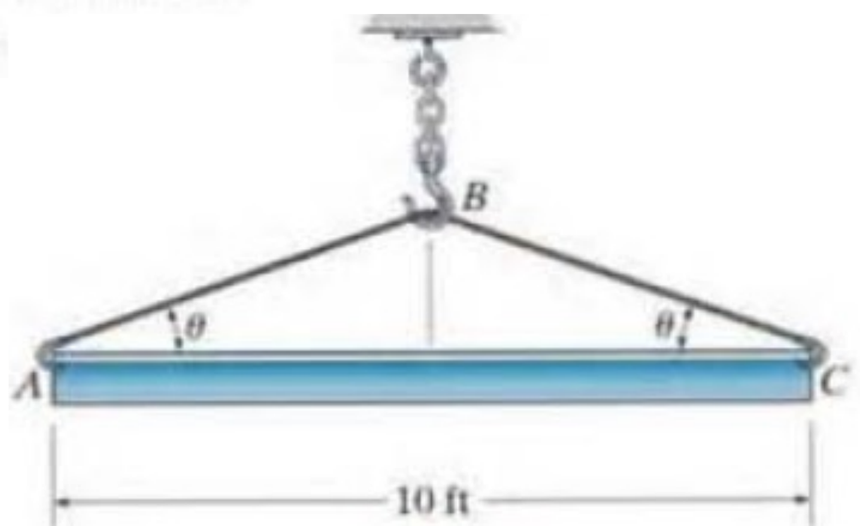
$2 \text{ m} = l_{AC} \cos 30° + 0.853 \text{ m}$ $\quad l_{AC} = 1.32 \text{ m}$

F3–1. The crate has a weight of 550 lb. Determine the force in each supporting cable.

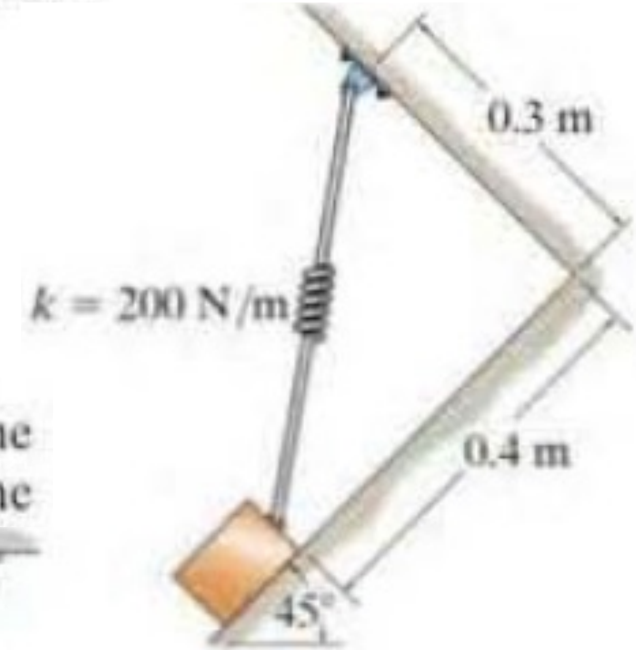


F3–1

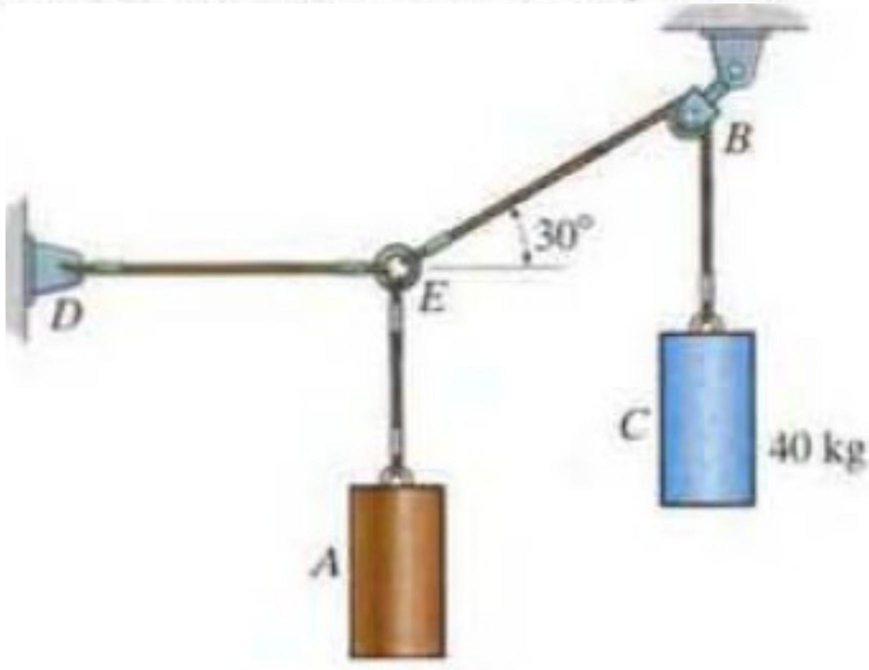
F3–2. The beam has a weight of 700 lb. Determine the shortest cable ABC that can be used to lift it if the maximum force the cable can sustain is 1500 lb.



F3–4. The block has a mass of 5 kg and rests on the smooth plane. Determine the unstretched length of the spring.



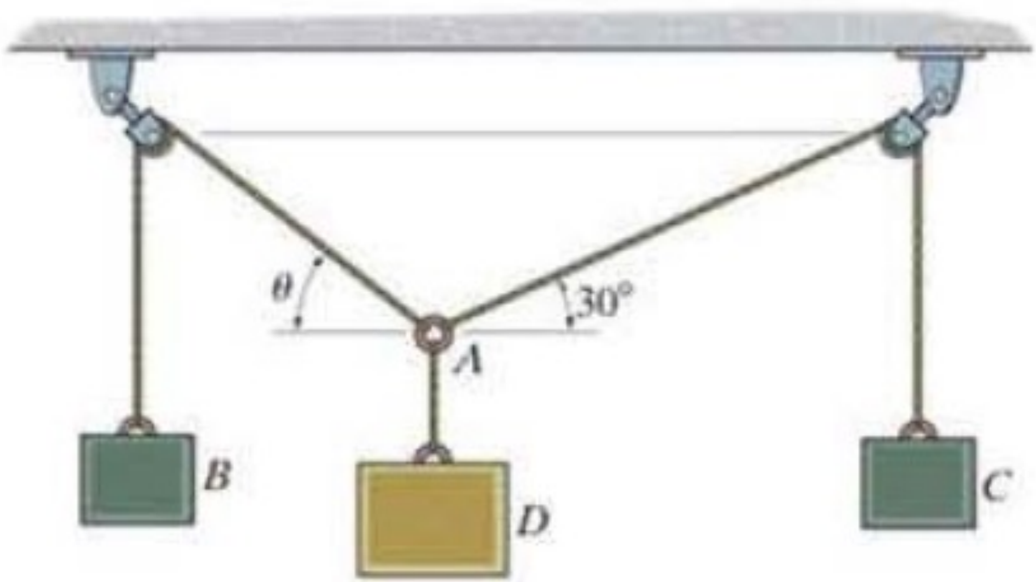
F3–5. If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.



F3–5

*3–12. If block B weighs 200 lb and block C weighs 100 lb, determine the required weight of block D and the angle θ for equilibrium.

•3–13. If block D weighs 300 lb and block B weighs 275 lb, determine the required weight of block C and the angle θ for equilibrium.



3–18. Determine the forces in cables AC and AB needed to hold the 20-kg ball D in equilibrium. Take $F = 300$ N and $d = 1$ m.

3–19. The ball D has a mass of 20 kg. If a force of $F = 100$ N is applied horizontally to the ring at A, determine the dimension d so that the force in cable AC is zero.