

# **Basic components of a Laser System**

**This chapter is based on:**

- 1- Introduction to Laser Technology-3<sup>rd</sup> Edition**
- 2- principles of laser- 5<sup>th</sup> edition.**
- 3- University of Technology lectures.**
- 4- internet websites.**

**Dr.Abdullah O. Hamza**  
**[abdulla.hamza@su.edu.krd](mailto:abdulla.hamza@su.edu.krd)**  
**2022-2023**

# Laser System

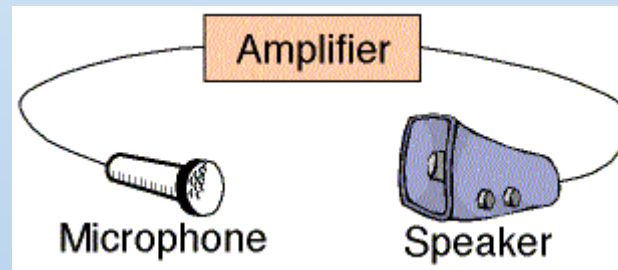
The laser is a system that is similar to an **electronic oscillator**.

An **Oscillator** is a system that produces oscillations without an external driving mechanism. To demonstrate an oscillator, we can use the familiar acoustic analog:

A **sound amplification system** has a **microphone**, **amplifier** and **speaker**.



When the microphone is placed in front of the speaker, a **closed circuit** is formed, and a whistle is heard out of the speaker.

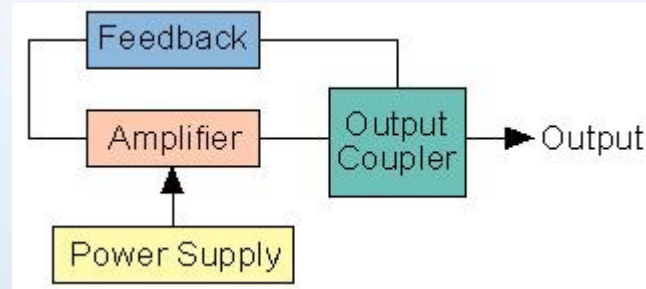


**The whistle is created spontaneously, without any external source.**

**Explanation:** The speaker's internal noise is detected by the microphone, amplified and the amplified signal is again collected by the microphone. This **positive feedback** continues until a loud whistle is heard.

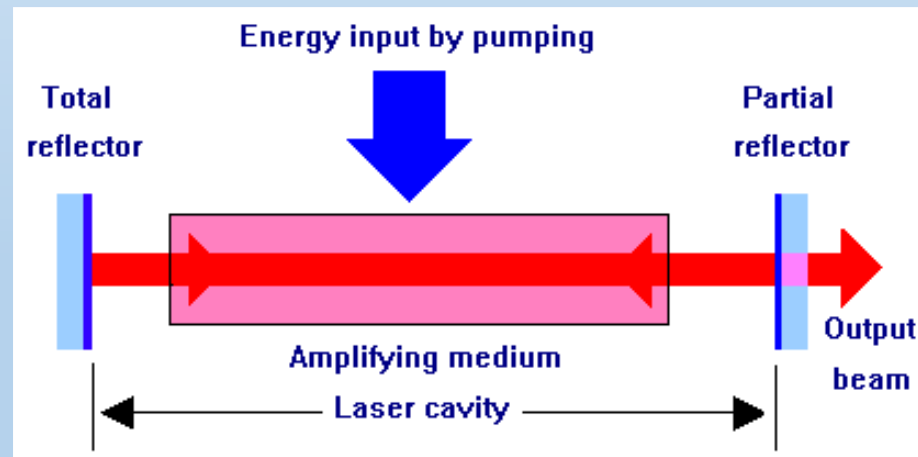
**Every oscillator has 4 main parts** (as seen in figure below):

1. Amplifier.
2. Positive resonance feedback.
3. Output coupler.
4. Power source



In analogy to the electronic amplifier, **the laser** can be described as composed of four structural units (see figure below):

1. **Active medium**, which serves as an optical amplifier.
2. **Excitation mechanism (Pumping)** : to produce population inversion
3. **The Resonator** ( for Optical feedback).
4. **Output coupler**, to allow electromagnetic radiation out of the laser device.



# 1- The laser active medium:

- The **active medium** is a collection of atoms or molecules, which can be excited into a **population inversion** situation, and can have electromagnetic radiation extracted out of it by stimulated emission.

- The **active medium can be in any state of matter: solid, liquid, gas or plasma.**

**Solid – Crystal ..... (Ex. Ruby, Nd:YAG, ....) \_ Glass ..... (Ex. Nd:glass)**

**Dye \_ Organic material dissolved in certain solvent.**

**Gas \_ Atomic (He-Ne,.....) \_ Molecular (Co<sub>2</sub>, N<sub>2</sub>, ...) \_ Ions (Ar<sup>+</sup>)**

- The **active medium determines the possible wavelengths that can be emitted from the laser.** These wavelengths are determined by the specific transitions between the laser energy levels in this material.

- The list of materials that lase under certain laboratory conditions include hundreds of substances, and the number increases with time.

- The **basic physics of the laser is similar for all types of lasers**, and we will use the term "**Active Medium**" and assume that it is composed of "**atoms**". In reality, the active medium can be atoms, molecules, ions, or semiconductors, according to the laser type.

## 2- Pumping Process

As we mentioned in the previous chapter that the **excitation mechanism** is the source of energy that raises the atoms in the active medium into their excited state, thus creating **population inversion**.

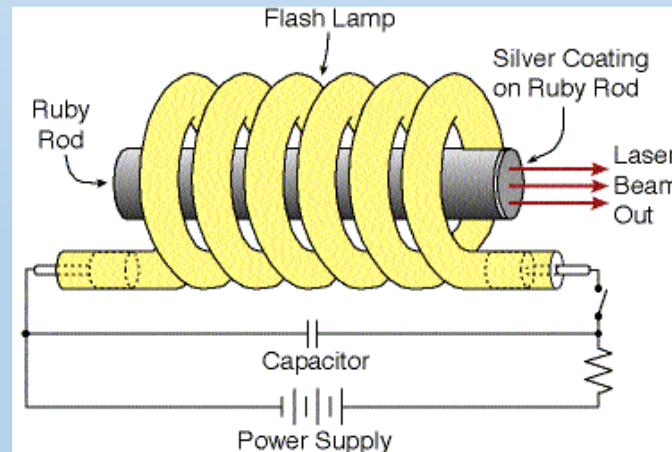
**There are few types of excitation mechanisms:**

### **a- Optical pumping - Excitation by photons:**

In lasers with solid or liquid active medium, it is common to supply the excitation energy in a form of electromagnetic radiation (photons) which are absorbed in the active medium.

The electromagnetic radiation source can be of different kinds:

- **Flash lamps**, which are build from a quartz tube filled with gas at low pressure. Usually **Xenon gas** is used, but sometimes when higher energy is required, other noble gasses with lower atomic weights such as **Krypton** or **Helium** are used.
- **Another laser** or **any other light source** such as the light from the sun.



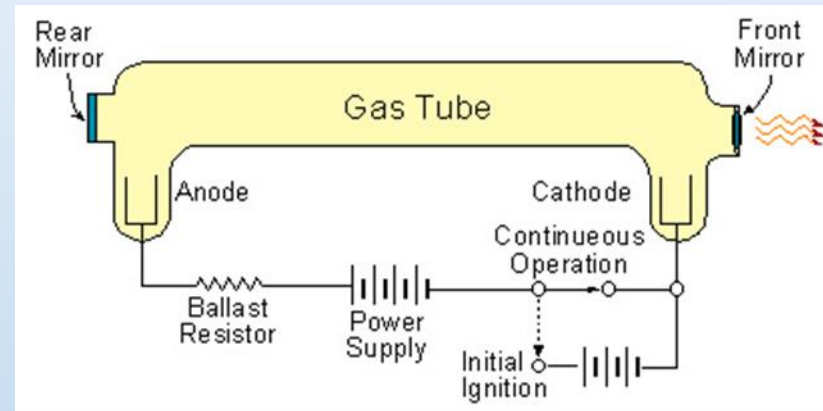
## b- Electrical pumping :

### Used in :

Gas laser – electrical discharge

Semiconductor – potential difference.

- When the active medium is in the gas state, the best excitation is by electrical discharge of the gas (see figure below).



**Electrical Excitation of a Gas Laser**

- The gas in the tube is electrically neutral, and as long as no external energy is applied, most of the molecules are in the ground state.
- When the **high electrical voltage** is applied, electrons are released from the cathode and accelerated toward the anode. On their way, these **electrons collide with the gas molecules and transfer energy to them**. Thus, the gas molecules are raised to excited state.
- **Higher voltage is required to start the electrical discharge in the tube than to keep the discharge**. Thus, a preliminary high voltage pulse is applied for initial discharge, and then the voltage is lowered to its operating value. The **ballast resistor** is used to limit the current in the tube after discharge is achieved.

## Collisions with atoms

This is the standard excitation mechanism in the commercial gas lasers such as: Helium-Neon laser, or Carbon-Dioxide laser. In this method at least two gasses are inside the laser tube.

**One gas** receives the energy from the collision with the accelerated free electrons. The **second gas** receives energy from collisions with the excited molecules of the first gas

### Example: Helium-Neon Laser

□ Figure shows the energy level diagram of Helium-Neon laser, with the possible transitions.

The mass of the Helium atom is about one-fifth of the mass of the Neon atom.

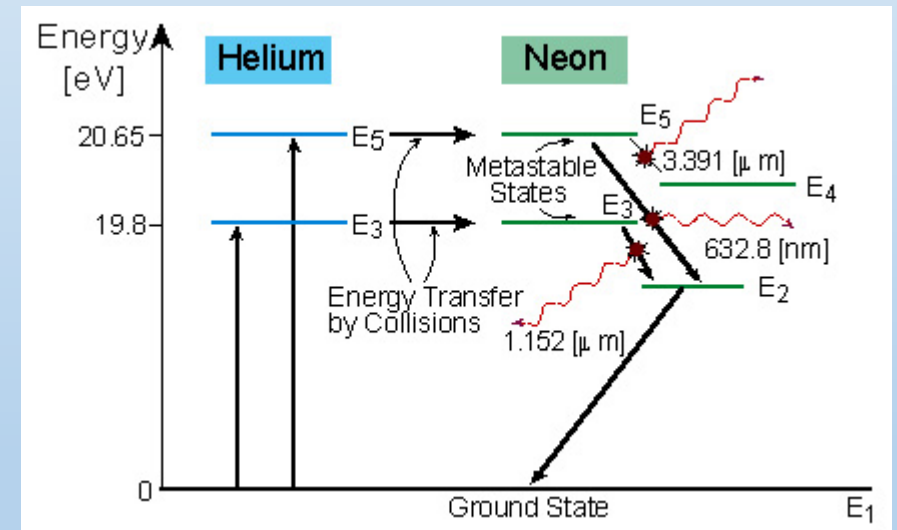
The amount of Helium in the tube is about 6 times the amount of Neon. Thus Helium atoms have more chance to receive energy from the accelerated electrons, and transfer into the excited energy levels  $E_3$  and  $E_5$ .

□ Neon atom have two excited energy levels ( $E_3$  and  $E_5$ ) which are very close to the excited energy levels of the Helium atom. The excited Helium atoms transfer their excitation energy to the Neon atoms by collisions - Resonance excitation. Energy from the He-Ne laser is emitted at wavelengths which correspond to the energy difference between the levels:

$$E_5 - E_4 \Rightarrow \lambda_1 = 3.391 \text{ } [\mu\text{m}]$$

$$E_5 - E_2 \Rightarrow \lambda_2 = 0.632 \text{ } [\mu\text{m}]$$

$$E_3 - E_2 \Rightarrow \lambda_3 = 1.152 \text{ } [\mu\text{m}]$$



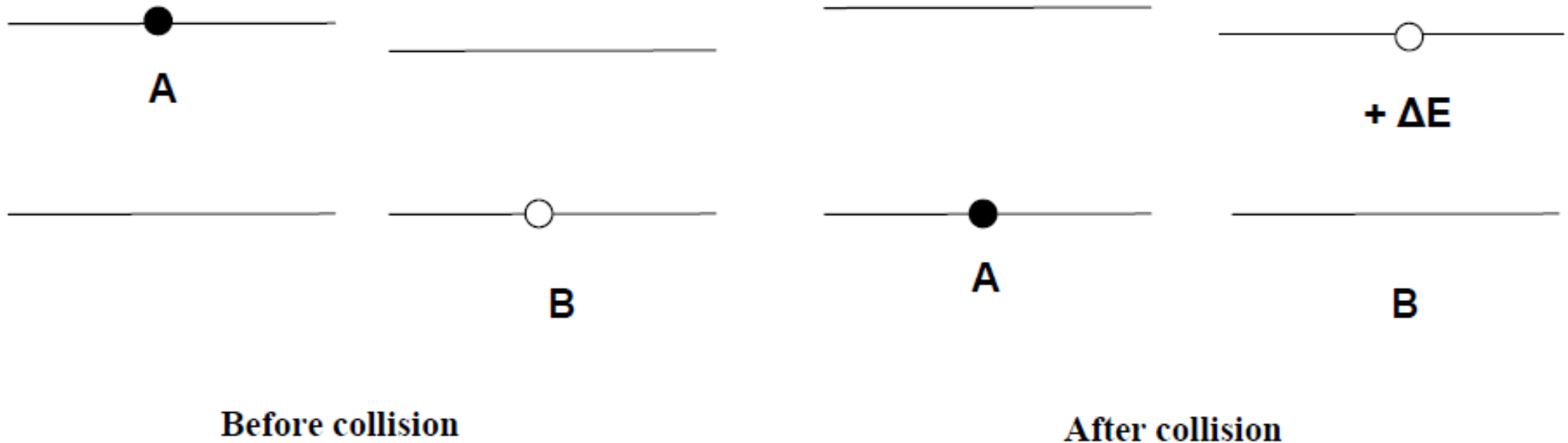
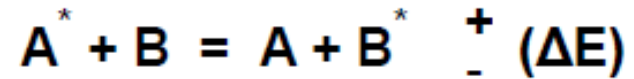
Energy Level Diagram of Helium-Neon Laser

Later we shall understand how to choose the specific required wavelength at the output of the laser.

In summary Electrical pumping can be achieved by one of the following methods:

A)  $e$  (fast) + X (atom in the ground state) =  $X^*$  (atom in the excited state) +  $e$  (Slow)

B) A gas comprising of TWO species (A & B)



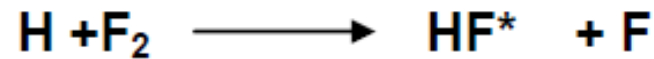
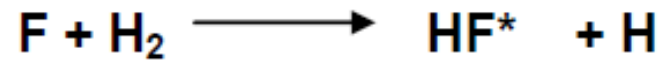


## C- Chemical Pumping

In this excitation, the **energy is supplied by the chemical reaction between two atoms or molecules.**

So , it Does not need external source of energy, where the out put of the chemical reaction represents the active medium and the reaction generated energy can be used to excite the active medium and getting the population inversion.

### Ex: Hydrogen fluoride laser



↓  
Active Medium

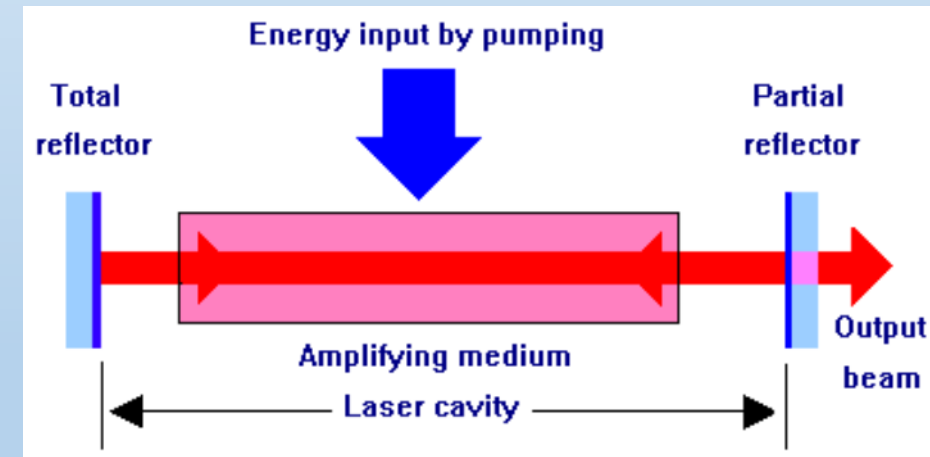
### 3- The Laser Resonator

The optical resonant cavity, plays a crucial role in the operation of laser. In the majority of cases the gain of a pumped active medium is quite small, so that the amplification of an optical beam passing once through the medium is minimal. Amplification is increased by placing highly reflecting mirrors at each end of the medium. **(Usually one mirror is 100% reflecting, so all the radiation coming toward the mirror is reflected back to the active medium. The other mirror is partially reflecting (10%-99%), according to the laser type).**

Usually the **feedback mechanism** is done by **using mirrors at both ends of the active medium**. These mirrors are aligned so that the radiation is moving back and forth between them. In this way an **optical cavity** is created.

The mirrors from an optical resonant cavity ( often **called as Fabry-Perot resonator**), and together with the active medium constitutes an optical oscillator rather than amplifier.

- ❑ The part of the radiation which is not reflected back into the optical cavity, is transmitted out, and it is the **laser output**.
- ❑ **The feedback allows each photon to pass many times through the active medium**, so enough **amplification** will result.
- ❑ Because of the feedback mechanism, only photons which move between the mirrors remain in the active medium, which give the **directionality of the output beam**.



**Laser Resonant Cavity**

# Resonator Configurations

There are several additional resonant cavity configurations that are more practical than is the original plane-parallel (Fabry-perot) setup (shown in the previous slide).

- It can be seen that each optical cavity has 2 end mirrors with radiuses of curvature  $R_1$  and  $R_2$ .

**Two parameters determine the structure of the optical cavity:**

- 1.The volume of the laser mode inside the active medium.
- 2.The stability of the optical cavity.

In the following pages, each type of optical cavity is described:

- 1.Parallel Plane Cavity.
- 2.Concentric Circular Cavity.
- 3.Confocal Cavity.
- 4.Cavity with Radius of Curvature of the mirrors Longer than Cavity length.
- 5.Hemispherical Cavity.
- 6.Half Curve with longer than cavity radius of curvature.
- 7.Unstable resonator.

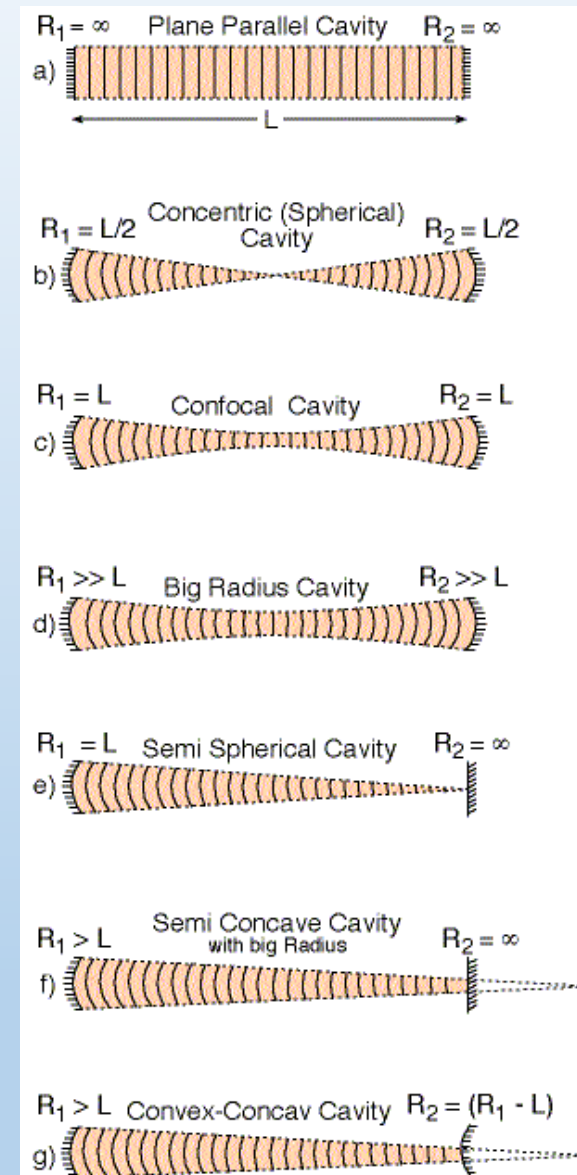


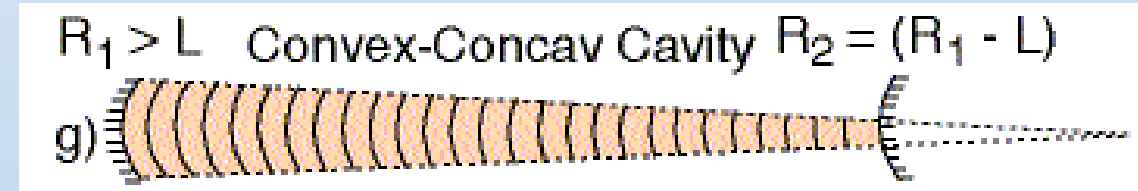
Figure describes the most common optical cavities.

- ❖ **Laser cavities are said to be either stable or unstable to the degree that the beam tends to retrace itself and so remain relatively close to the optical axis.**

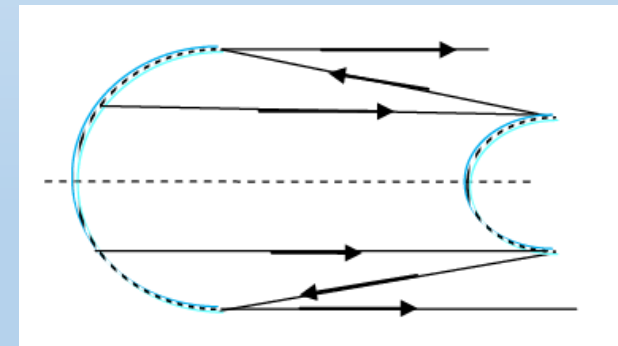
## 1- Unstable resonator.

A beam in an unstable cavity will walk out, going farther from the axis on each reflection until it quickly leaves the cavity altogether. (i.e. **Light ray keep on moving away from resonator axis**)

- ❑ **An example for such cavity is created by convex concave arrangement of spherical mirrors.**



- ❑ The **concave mirror** is big and its radius of curvature is **longer** than the length of the cavity.
- ❑ The **convex mirror** is small and its radius of curvature is small.
- ❑ In such cavity no **standing wave pattern** is created inside the cavity.



- ❑ Unstable resonators are commonly used in **high-power lasers**, where the fact that the beam traces across a wide region of the active medium enhances the **amplification and allows for more energy to be extracted**.
- ❑ This approach will be especially useful for media (like carbo dioxide, CO<sub>2</sub> or Argon, Ar) wherein the beam gains more energy on each trip in the cavity.
- ❑ The disadvantage of this kind of resonator is the beam shape has a **hole in the middle**.

## 2- Stable resonator.

Most lasers, in which the curvature of the mirrors keeps the light concentric near the resonator axis.

In a **stable cavity** is a cavity in which the radiation is captured inside the cavity, creating **standing waves** while the beam move between the mirrors.

### Note

**The geometry of the cavity determines if the cavity is stable or not.**

# The stability condition of a resonator

For an arbitrary mirror separation ( $L$ ) and arbitrary curvatures ( $R_1$ ) & ( $R_2$ ), stability condition is satisfied by a low-loss resonator configuration, the **geometric- parameters** is defined for each of the mirror by:

$$g_1 = 1 - \frac{L}{R_1} \quad , g_2 = 1 - \frac{L}{R_2}$$

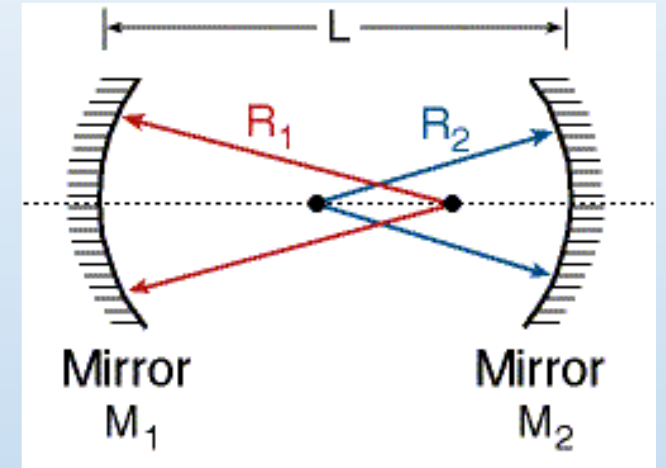
$g_1$  &  $g_2$  : dimensionless parameters.

∴ Stability condition is:

$$0 < g_1 g_2 < 1$$

If  $g_1 g_2 < 0$  or  $g_1 g_2 > 1$   $\Rightarrow$  **unstable resonator**

When  $g_1 g_2 = 0,1$  then the laser is on the boundary between stability & instability the called **marginally stable**.

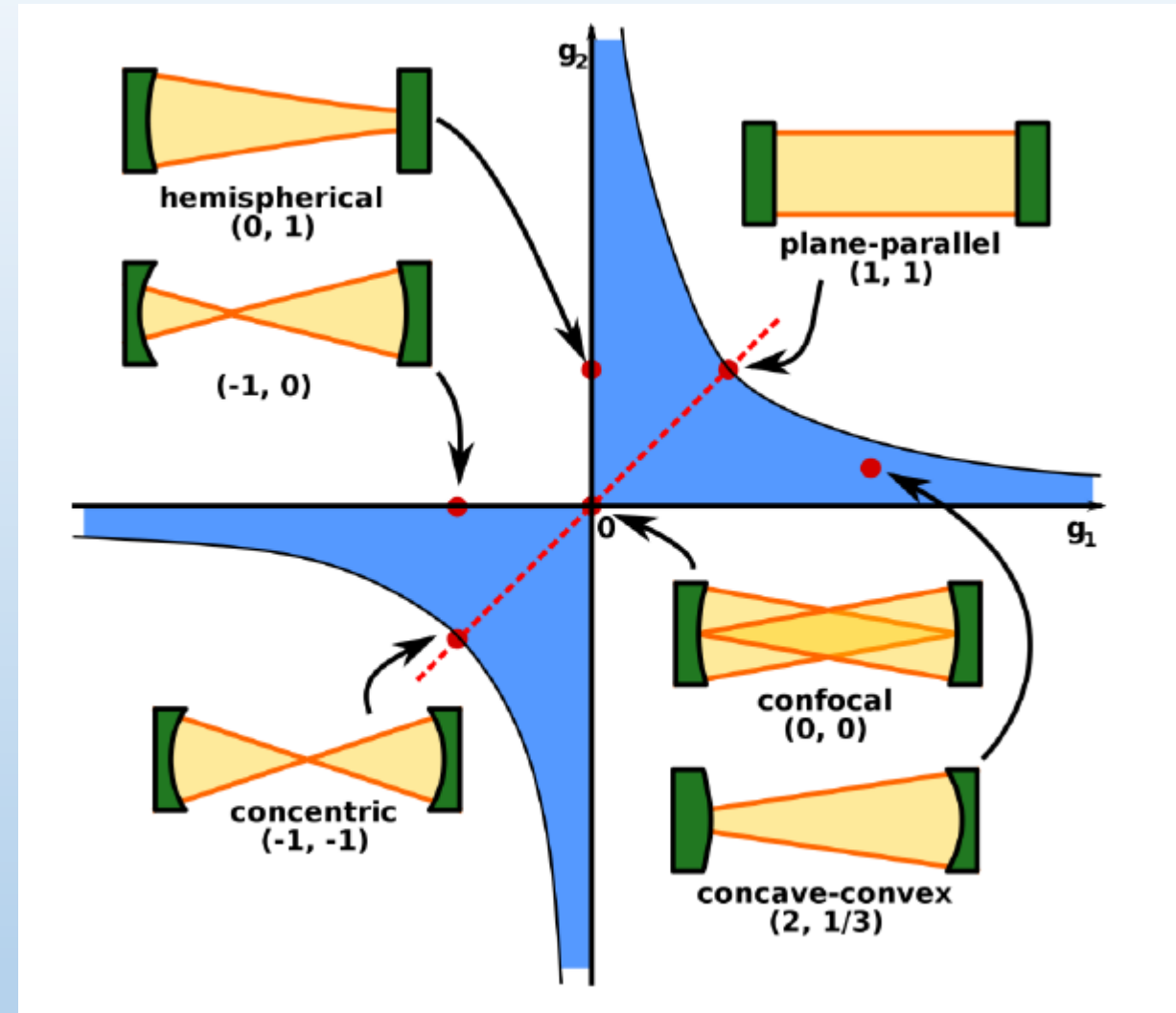


The condition of stability can be expressed in the form of the stability diagram as shown in figure below

Figure shows the stability diagram of all laser cavities.

In the stability diagram,

- ❑ The blue region marks the area of stability.
- ❑ When  $g_1 g_2 = 0, 1$  then the laser is on the boundary between stability and instability is called **marginally stable**.
- ❑ The stability region is surrounded by two hyperbolas defined by the stability criterion.
- ❑ **Red dotted line** making an angle of  $45^\circ$  with axes corresponds to resonators having mirrors of the same radius of curvature  $R$  (symmetric resonator).
- ❑ A few common cavities are marked on the stability diagram.
- ❑ **A cavity is stable** if the center of curvature of one of the mirrors, or the position of the mirror itself, but not both, are between the second mirror and its center of curvature.
- ❑ Pay special attention for cavities on the edges of the stability region !
- ❑ So the stability diagram depends on the cavity parameters.



**stability diagram** of all laser cavities.

**Example:**

The laser cavity length is 1 m. At one end a concave mirror with radius of curvature of 1.5 m. At the other end a convex mirror with radius of curvature of 10 cm. Find if this cavity is stable.

**Solution:**

$$R_1 = 1.5 \text{ m.}$$

As common in optics, a convex mirror is marked with minus sign:

$$R_2 = -0.1 \text{ m}$$

$$g_1 = 1 - L/R_1 = 1 - 1/1.5 = 0.333.$$

$$g_2 = 1 - L/R_2 = 1 + 1/0.1 = 11$$

The product:

$$g_1 * g_2 = 11 * 0.333 > 1$$

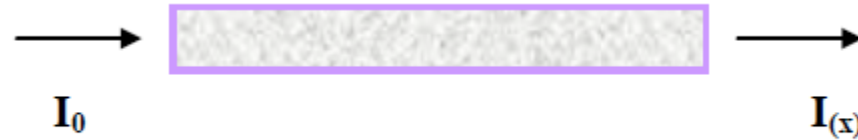
The product is greater than 1, so the cavity is unstable.



# The Gain coefficient

Assume a monochromatic light of intensity ( $I_0$ ), passing through a transparent medium, so the intensity out of the medium is ( $I_{(x)}$ ):

$$I_{(x)} = I_0 e^{-\alpha x}$$



$\alpha$  = Absorption coefficient

$\alpha$  – depends on the no. of atoms ( $N_1$ ) in ( $E_1$ ) and ( $N_2$ ) in ( $E_2$ )

Now, in thermal equilibrium,  $N_1 \gg N_2$ , So  $I_{(x)}$  decreases exponentially.

But if  $N_2 > N_1$  (Population inversion), so  $\alpha$  is negative, hence  $-\alpha$  is positive, So the intensity will increase exponentially as follows:

$$I_{(x)} = I_0 e^{kx}$$

Where  $k$  is called the Gain coefficient.

A relation between  $k$  and the population inversion and some other laser medium parameters can be established:

$$k = (N_2 - N_1) \frac{nh\nu B_{21}}{c\Delta\nu}$$

## Threshold Gain coefficient $k_{th}$

To sustain laser oscillation the gain coefficient must be at least large enough to overcome the losses in the laser system. the sources of loss are due to the following:

1. **Transmission at mirrors.(out put).**
2. **Absorption and scattering by mirrors.**
3. **Diffraction around the mirror edges.**
4. **Absorption in laser active medium (due to unwanted transitions).**
5. **Scattering at optical inhomogeneities in laser medium.**

The minimum **or threshold gain coefficient**  $k_{th}$  required from the condition that the **round trip gain, G**, in the irradiance of the beam must be least unity.

**Losses are represented by ( $\gamma$ ) the loss coefficient**, which reduces the gain coefficient to be **( $k - \gamma$ )**.

**The round trip gain ( $G$ ) = 1** (This is **threshold gain condition**)

**If  $G < 1$  oscillation will decay** (Or oscillations would die out)

**If  $G > 1$  oscillation will grow**

Assume active medium of length (L) between two mirrors of reflectivities ( $R_1$  &  $R_2$ ) (figure below), In traveling from  $M_1$  to  $M_2$  in the laser resonator cavity, the intensity (beam irradiance) will increase from  $I_0$  to  $I_1$ , where ,

$$I_1 = I_0 e^{(k-\gamma) L}$$

After reflection at  $M_2$  the beam irradiance will be

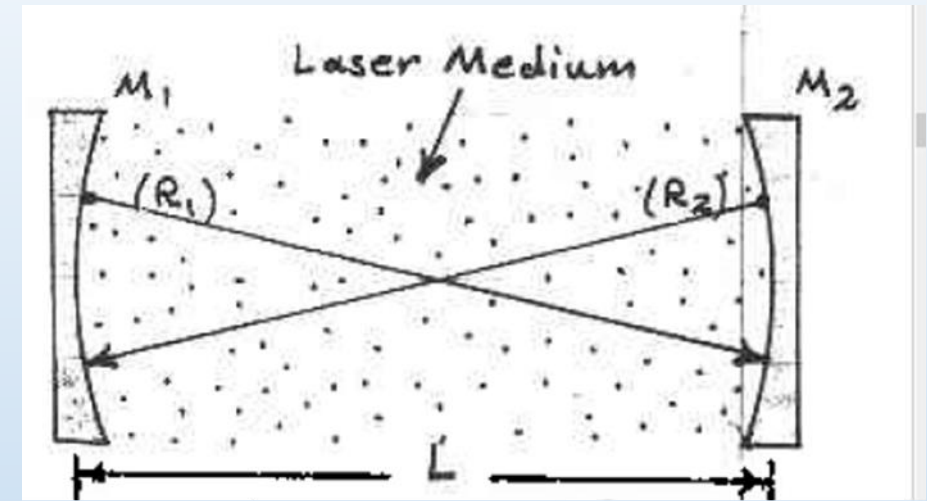
$$I_1 = I_0 R_2 e^{(k-\gamma) L}$$

And after complete round trip the irradiance beam will be

$$I_1 = I_0 R_1 R_2 e^{2(k-\gamma) L}$$

So , the round trip gain (G):

$$G = \frac{\text{Final intensity}}{\text{initial intensity}} = \frac{I_0 R_1 R_2 e^{2(k-\gamma) L}}{I_0} = R_1 R_2 e^{2(k-\gamma) L}$$



$R_1$  : Reflectance at  $M_1$   
 $R_2$  : Reflectance at  $M_2$

**Threshold condition for laser oscillation :**

$$R_1 R_2 e^{2(k-\gamma)L} = 1$$

**Hence:**

**The threshold gain coefficient  $k_{th}$  , is given by**

$$k_{th} = \gamma + (1/2L) \ln(1/R_1 R_2)$$

- ❑ **The first term in  $K_{th}$  equation** , represents the volume loss while the second is the loss in the form of the useful output; thus the condition for steady-state laser operation is that the **gain equals the sum of the losses**.
- ❑ In lasers designed for continuous output (CW) the gain becomes constant at the threshold value. This is because only round trip gain  **$G = 1$** , the cavity energy (and hence the laser output) settles down to a steady-state value. This phenomenon is referred to as gain saturation.
- ❑ **The actual value of the gain depends** on the **population inversion** and **on the physical properties of the medium**. If  **$k$  is high** then it is relatively easy to achieve laser action and mirror alignment and cleanliness are not too critical. With low gain media, the mirrors must be accurately aligned, have high reflectance's and clean

**Example/** In a ruby laser ( $\lambda=694.3$  nm), the ruby crystal is 0.1m long and the mirror reflectances are 97% and 90%. Given that the losses are 10% per round trip. Calculate the threshold gain coefficient?

Sol.

$$k_{th} = \gamma + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$$

$$k_{th} = 0.1 + \frac{1}{2 \times 0.1} \ln \left( \frac{1}{(0.95)(0.90)} \right)$$

$$k_{th} = 0.88 \text{ cm}^{-1}$$

**Example:**

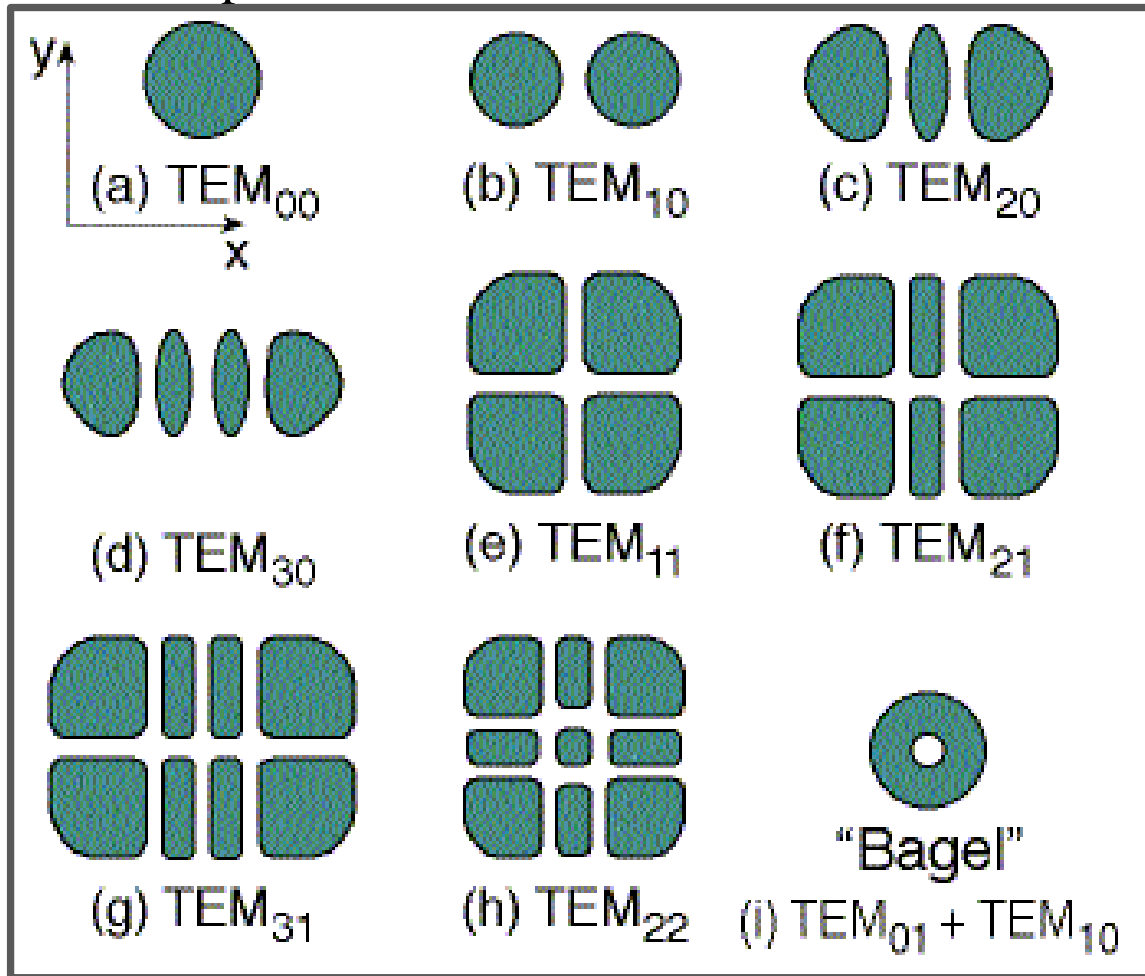
If the intensity of transmitted light  $I$  is increased by a factor of two from the incident light intensity  $I_o$  when the light passes through laser active medium of 0.5 m length find the gain coefficient  $k$  assume there are not any losses in the system.

Hint:

$$\begin{aligned} I &= 2I_o \\ \frac{2I_o}{I_o} &= e^{k \times 0.5 \text{ m}} \end{aligned}$$

$$k = 1.38 \text{ m}^{-1}$$

Figure below shows the energy distribution of the first few transverse electromagnetic modes. **The dark areas** mark places where laser radiation hit.



## Comment

□ From figure each transverse mode (TEM) is marked with two indexes: **TEM<sub>mn</sub>**.

$m, n$ , are integer numbers. Assuming the beam advance in  $z$  direction:

**m** = Number of points of zero illumination (between illuminated regions) along  $x$  axis.

**n** = Number of points of zero illumination (between illuminated regions) along  $y$  axis.

□ **The proffered transverse mode that oscillates within the cavity depends on:** (a) On the aperture of the gain medium and (b) the radial dependence of the gain.

□ A small misalignment of the laser mirrors causes different path length for different “rays” inside the cavity. Thus, the distribution of intensity is not the perfect **Gaussian distribution**.

There is one transverse mode which does not fit this classification, and it has a special name (according to its shape) due to its importance: **“Bagel”**. It is composed of  $TEM_{01}$  and  $TEM_{10}$  oscillating together

## Question

Which TEM<sub>mn</sub> mode is widely used? Why?

## Answer

The TEM<sub>00</sub> transverse mode is the most widely used, and this for several reasons: the flux density is ideally Gaussian over the beam cross section; it is completely spatially coherent; the beam's angular divergence is the smallest; and it can be focused down to the smallest – sized spot.

## Question

*Draw the shapes of the following transverse laser modes?*

TEM<sub>00</sub> TEM<sub>10</sub> TEM<sub>20</sub> TEM<sub>30</sub> TEM<sub>01</sub> TEM<sub>11</sub> TEM<sub>21</sub>  
TEM<sub>31</sub> TEM<sub>02</sub> TEM<sub>12</sub> TEM<sub>22</sub> TEM<sub>33</sub> TEM<sub>34</sub> TEM<sub>60</sub>



# The properties and propagation of a Gaussian laser beam

The TEM<sub>00</sub> mode is so important that there are several names for it in laser technology, all meaning the same thing. The TEM<sub>00</sub> mode is called the *Gaussian mode*, the *fundamental mode*, or even the *diffraction-limited mode*. No matter what it's called, it's a very important mode, and this section describes how the light produced by a Gaussian-mode laser propagates through space.

But before you can understand how a Gaussian beam propagates, you must understand two important properties of the Gaussian laser beam

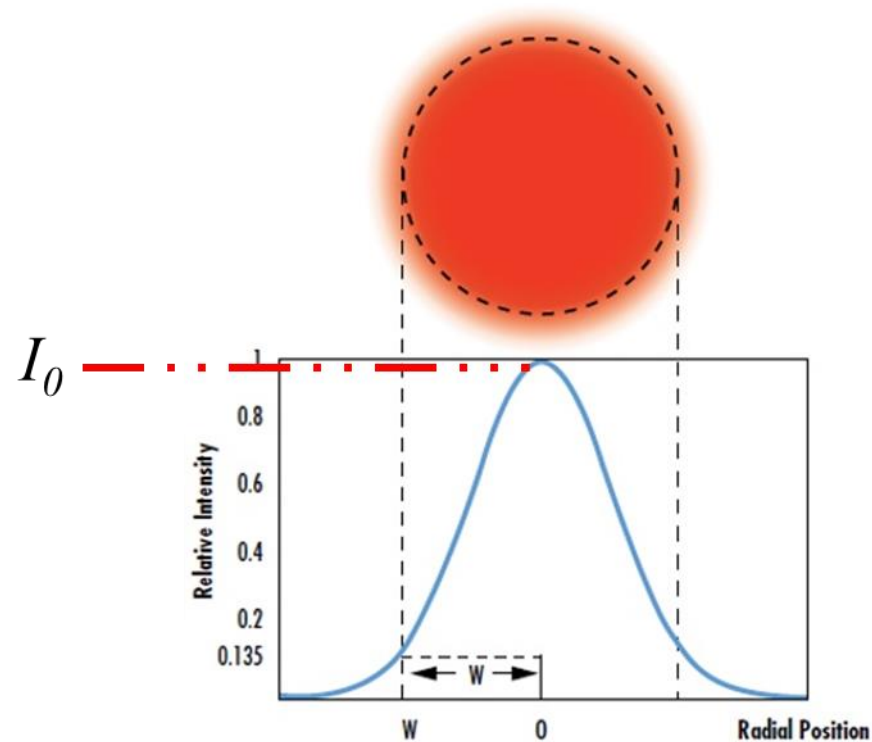
1- The beam has a Gaussian transverse profile at **all locations**. Such a Gaussian beam can be characterized completely at any spatial location by defining both its „beam waist“ and its „wavefront curvature“ at a specific location of the beam. **(The spatial distribution of a Gaussian beam remains Gaussian while the beam propagate through space).**

2- A Gaussian beam always has a minimum beam waist ( $w_0$ ) at one location in space. **(Have the maximum Spatial Coherence compared to other transverse modes).**

**The transverse distribution of the intensity of a Gaussian beam is given by the following equation:**

$$I = I_0 e^{\frac{-2x^2}{w^2}} \quad (1)$$

in which  $I_0$  is the intensity at the center,  $x$  is the distance from the center, and  $w$  is the radius of the laser beam where the intensity is  $1/e^2$  (13.5%) of  $I_0$ . The intensity profile is showed in Fig. below.



*Fig (1) the waist of a Gaussian beam is defined as the location where the irradiance is  $1/e^2$  (13.5%) of its maximum value*

However, this irradiance profile does not stay constant as the beam propagates through space, hence the dependence of  $w(z)$  on  $z$ .

The Gaussian beam minimum waist  $w_0$  for a typical laser resonator mode occurs in the region between the two mirrors of an optical resonator. For example, the minimum beam waist  $w_0$  in a confocal optical resonator ( $R_1=R_2=L$ ) occur halfway between the two mirrors figure (2).

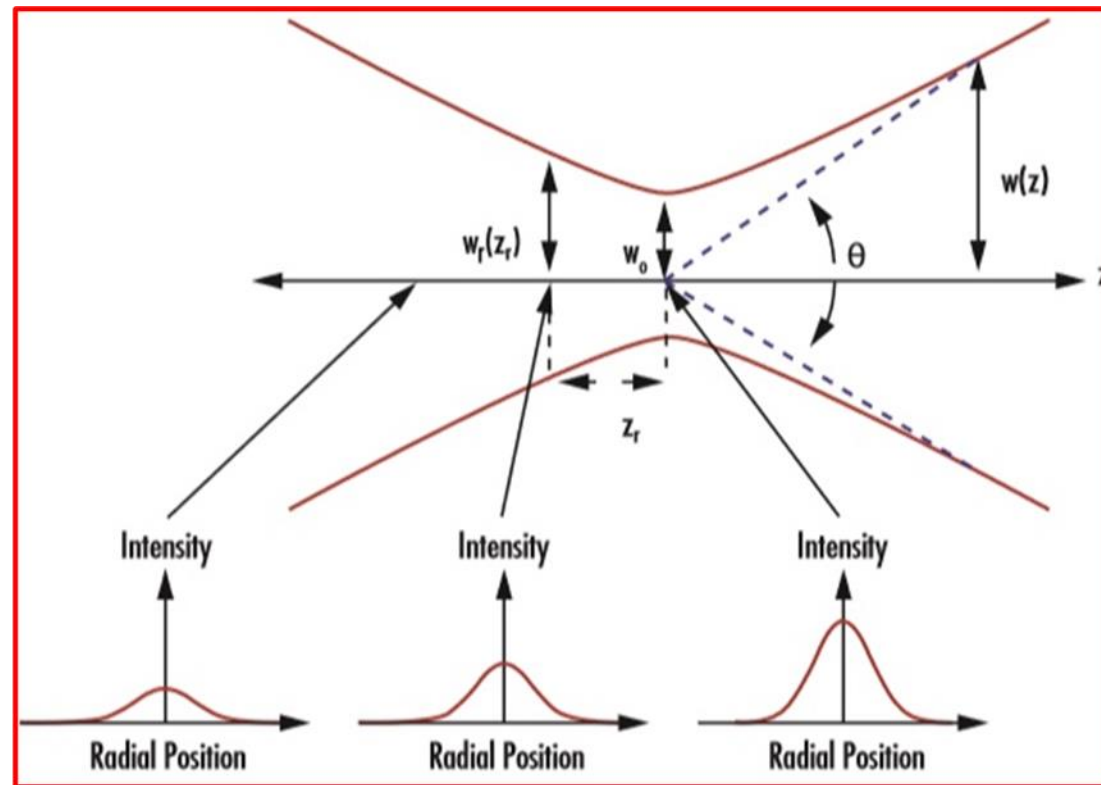


Fig. (2) Gaussian beams are defined by their beam waist ( $w_0$ ), Rayleigh range ( $z_R$ ), and divergence angle ( $\theta$ )

As shown in fig. 2, The beam converges and diverges equally on both sides of the beam waist by the divergence angle  $\theta$ . The beam waist and divergence angle are both measured from the axis and their relationship can be seen in *Equation 2* and *Equation 3*:

$$w_0 = \frac{\lambda}{\pi\theta} \quad (2)$$

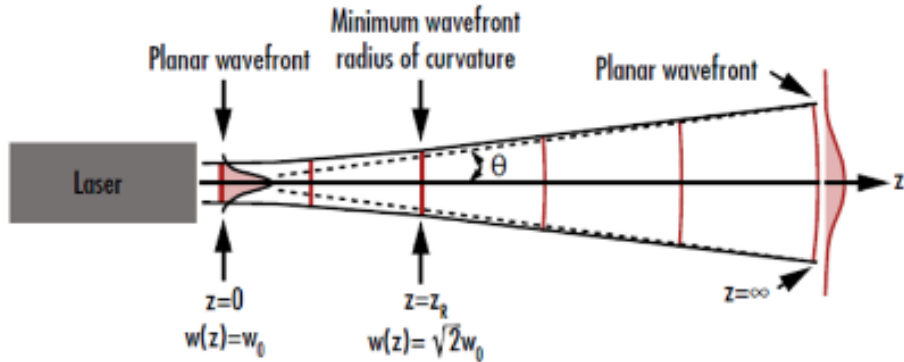
$$\theta = \frac{\lambda}{\pi w_0} \quad (3)$$

- As seen in Equation 3, a **small beam waist results in a larger divergence angle**, while a **large beam waist results in a smaller divergence angle** (or a more collimated beam). This explains why beam **expanders** can reduce beam divergence by increasing beam diameter.
- As the Gaussian beam propagate it expand and diverges from that location, such that the beam waist at a distance of  $\pm z$  from the minimum beam waist  $w_0$  can be described as

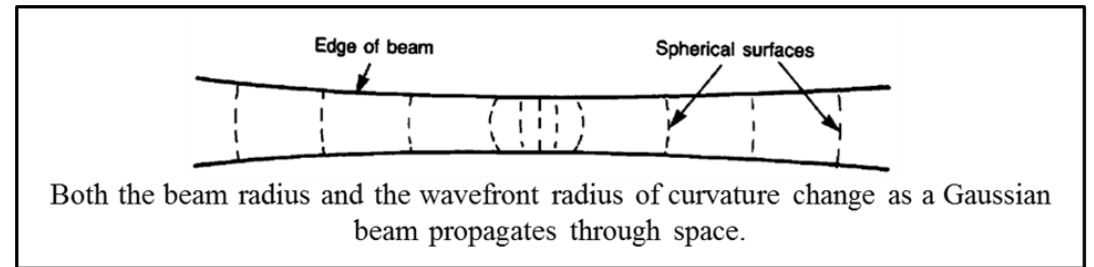
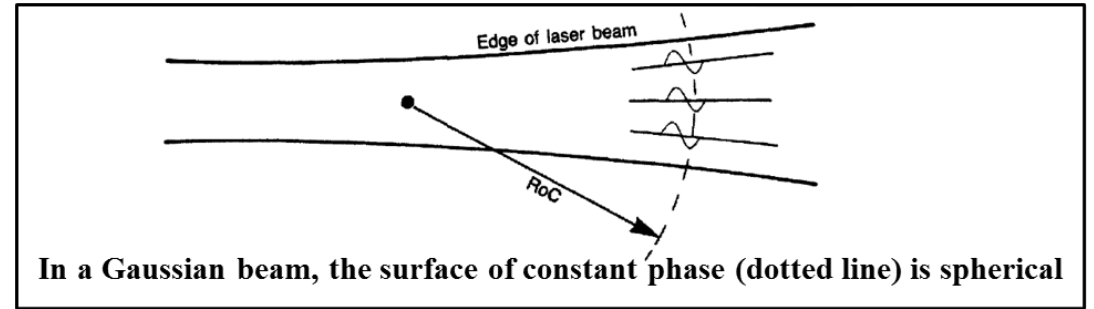
$$w(z) = w_0 \sqrt{1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2} \quad (4)$$

**This equation gives the beam radius**

The wavefront of the laser is planar at the beam waist and approaches that shape again as the distance from the beam waist region increases. This occurs because the radius of curvature of the wavefront begins to approach infinity. The radius of curvature of the wavefront decreases from infinity at the beam waist to a minimum value at the Rayleigh range, and then returns to infinity when it is far away from the laser (*Figure 3*); this is true for both sides of the beam waist.



**Figure 3:** The curvature of the wavefront of a Gaussian beam is near-zero when it is both very close and very far away from the beam waist



The beam wavefront curvature of a Gaussian beam at a location  $z$ , in term of the minimum beam waist  $w_0$  and the wavelength  $\lambda$ , is given by

$$R_z = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right]$$

**(5)** This Equation gives the wavefront radius of curvature.

### □ Application of Eq. 4

Suppose you wanted to do a laser-ranging experiment to measure the distance from the earth to the moon. You would aim a pulse of laser light at the retroreflector left on the moon by the astronauts. By very carefully measuring how long it took the pulse to travel to the moon and back, you'd be able to figure out the distance with an accuracy of feet or even inches. **But how wide is the beam by the time it gets to the moon?** If it's too wide, only a tiny fraction of the light will be reflected back by the **retroreflector**, and the amount that returns to earth could be too small to detect.

**Example:** Let's suppose that we're using an Nd:YAG laser whose wavelength is  $1.06 \mu\text{m}$  and whose beam waist is  $0.5\text{-mm}$  radius. The approximate earth-moon distance is  $239,000 \text{ mi}$ . We must invoke the equation 4:

$$w(z) = w_0 \sqrt{1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2}$$

Let's write down the known quantities and express them all in the same dimension, meters:

$$\lambda = 1.06 \times 10^{-6} \text{ m}$$

$$w_0 = 5 \times 10^{-4} \text{ m}$$

$$z = 239,000 \text{ mi} = 3.84 \times 10^8 \text{ m}$$

Next, substitute the known values into the following equation:

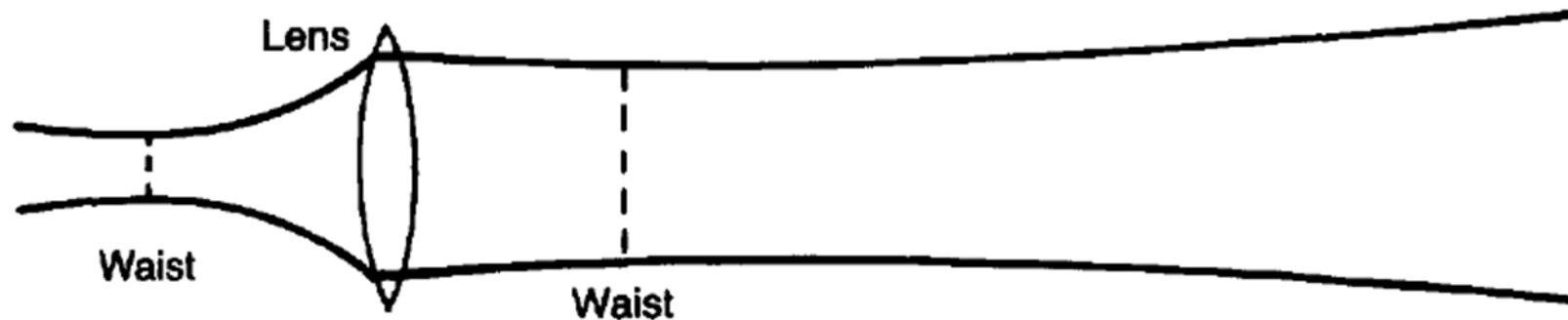
$$w = (5 \times 10^{-4} \text{ m}) \left\{ 1 + \left[ \frac{(1.06 \times 10^{-6} \text{ m})(3.84 \times 10^8 \text{ m})}{\pi (5 \times 10^{-4} \text{ m})^2} \right]^2 \right\}^{1/2}$$

Finally, do the arithmetic:

$$\begin{aligned} w &= (5 \times 10^{-4} \text{ m}) [1 + (5.2 \times 10^8)^2]^{1/2} \\ &\approx (5 \times 10^{-4} \text{ m})(5.2 \times 10^8) = 2.5 \times 10^5 \text{ m} \end{aligned}$$

This beam is too wide. The retroreflector is only about a meter in diameter, so only a minuscule fraction of the light will be reflected back toward earth. **What could you do to decrease the size of the beam on the moon?**

Take a look at the equation again. There are three parameters:  $\lambda$ ,  $z$ , and  $w_0$ . **The laser wavelength is fixed**, and you cannot move the moon any **closer to the earth** to make the problem easier. But you can adjust the radius of the **laser beam**, and that's the solution. The larger the waist of a Gaussian beam, the smaller its divergence will be. If you expand the beam with a telescope before it leaves earth, you can greatly reduce its divergence.



**A lens converts one Gaussian beam into another.**

## □ Application of Eq. 5

**Eq. 5 tells you what mirrors you must use to produce a given Gaussian beam in a resonator.** Let's look at an example. Suppose you wanted to design an argon laser ( $\lambda = 514.5 \text{ nm}$ ) whose beam had a 0.5-mm diameter right at the center of the laser and whose mirrors were 1 m apart. Which mirrors would you use?

You would want to calculate the wavefront radius of curvature at the points where the mirrors are to go, then obtain mirrors with the same curvature.

Begin by writing the equation:

$$R_z = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right]$$

Then write down the known parameters, again putting everything in meters:

$$z = 5 \times 10^{-1} \text{ m}$$

$$\lambda = 5.14 \times 10^{-7} \text{ m}$$

$$w_0 = 2.5 \times 10^{-4} \text{ m}$$

Substitute the values into the equation:

$$R = (5 \times 10^{-1} \text{ m}) \left\{ 1 + \left[ \frac{\pi(2.5 \times 10^{-4} \text{ m})^2}{(5.14 \times 10^{-7} \text{ m})(5 \times 10^{-1} \text{ m})} \right]^2 \right\}$$

Finally, do the arithmetic:

$$R = (5 \times 10^{-1} \text{ m}) [1 + (0.76)^2]$$

$$= (5 \times 10^{-1} \text{ m}) (1.58)$$

$$= 0.79 \text{ m, or } 79 \text{ cm}$$

**Thus, you would want to use mirrors whose radius of curvature was about 80cm.**