Chapter2

Particle properties of waves

Electromagnetic waves

Coupled electric and magnetic oscillations that move with the speed of light and exhibit typical wave behavior

The electric and magnetic fields in electromagnetic waves vary together; the fields are perpendicular to each other and to the direction of propagation of the waves.

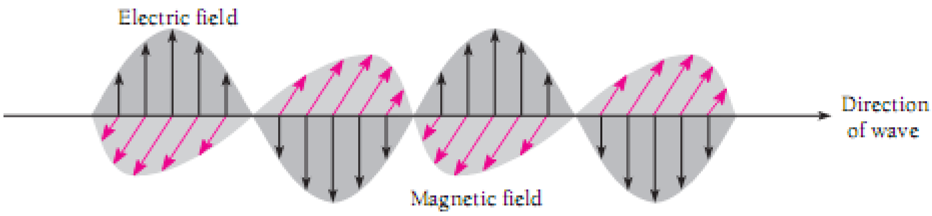

Fig.1: Depiction of electromagnetic wave.

We know

1. $c = 1/\sqrt{\mu_0 \varepsilon_0}$, where μ_0 and ε_0 were determined by magnetic induction and static Gauss law experiments. ⇒ $c = 3 \times 10^8$ m/s, which was known to be speed of light from MM experiment.

⇒ Light = E&M wave.

2. Wave nature of light verified by interference phenomena.

3. Hertz verified interference and velocity c for radio waves. ⇒ Confirmation.

Blackbody radiation

The thermal radiation emitted by a hot body, in general, depends on the composition and the temperature of the body. However, there is a class of bodies, called black bodies, which emit thermal radiation whose quantity and quality depend only on their temperature. For this reason the radiation emitted by such bodies is called the thermal radiation. Such bodies are named black bodies because they absorb all the radiation that falls on it. Lamp black and platinum black are nearest to ideal black bodies. When a black body is maintained at constant high temperature, the emitted radiation is called the black body radiation. For experimental purposes a cavity having a small hole can be regarded as a perfect black body because of its identical behavior with that of the perfectly black body. If radiation is allowed to enter such a cavity, it is reflected back and forth at the inner walls of the cavity and at each reflection some portion of energy is absorbed. After suffering a large number of reflections at the walls it is completely absorbed in the cavity. Therefore at lower temperature the hole appears black. When the cavity is maintained at higher temperature the radiation that comes out of the hole is similar to that emitted by a black body at the same temperature. Thus, a cavity with a small hole acts like a black body.

SPECTRAL DISTRIBUTION OF ENERGY IN THERMAL RADIATION

The spectral energy density u(v) or u(λ) was measured at different temperatures with special spectrograph. The energy density was plotted against the wavelength at different temperatures. The results obtained may be summarized as follows:

1. At a particular temperature the spectral energy density increases with wavelength and attains a maximum value and then falls to zero for longer wavelengths.

2. As the temperature is increased, the wavelength λ_{max} corresponding to maximum energy density shifts towards the shorter wavelength region. It was experimentally found by Wein that

$$\lambda_{max}T = b = 2.898 \times 10^{-3} m.k \qquad (2-1)$$

This relation is known as Wein's displacement law.

3. As the temperature is increased the total energy density u for all wavelengths increases. It was found that the total energy density, which is equal to the area under the curve is proportional to the fourth power of the temperature i.e,

$$R = \frac{c}{4}u = \frac{c}{4}\int_0^\infty u_\lambda d\lambda = \sigma T^4 \qquad \sigma = constant = 5.7 \times 10^{-8} W/m^2K^4 \qquad (2-2)$$

which is the Stefan's law.

In order to explain the dependence of spectral energy density on wavelength and temperature it was realized that certain assumptions regarding the structure of black body on atomic level and its interaction with radiation were necessary.

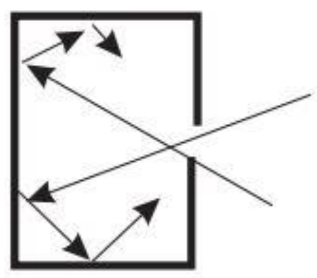

Fig.2 black body

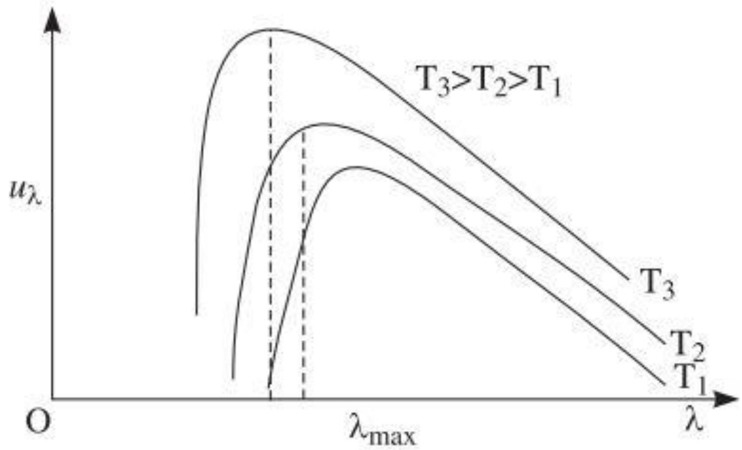

Fig.3 spectral distribution of energy

CLASSICAL THEORIES OF BLACK BODY RADIATION

Wein's Law : Wein, in 1893, from thermodynamic reasoning alone showed that energy density in black body radiation is given by

$$u(\lambda,T)d\lambda = \frac{c_1}{\lambda^5} e^{\frac{-c_2}{\lambda T}} d\lambda \qquad (2-3)$$

where c_1 and c_2 are empirical constants. By proper choice of these constants Wein's law can be made to fit the experimental curve in the shorter wavelength region alone but fails in the longer wavelength region.

Rayleigh and Jeans Law

Rayleigh and Jeans made an attempt to derive a better radiation law on the basis of the following assumptions.

1. The radiation in a cavity is electromagnetic in nature. In a metallic cavity whose walls are perfectly reflecting, the superposition of incident and reflected waves of each frequency results in the formation of standing waves with nodes at the walls. The number of standing waves (or modes) per unit volume in the frequency range ν and ν + dν is given by

$$N(\mathrm{v})d\mathrm{v} = \frac{8\pi \mathrm{v}^2 d\mathrm{v}}{c^3} \qquad (2-4)$$

In terms of wavelength

$$N(\lambda)d\lambda = \frac{8\pi \, d\lambda}{\lambda^4} \qquad (2-5)$$

2. The theorem of equipartition of energy is also valid for electromagnetic waves. According to this theorem the average contribution of each degree of freedom to the total energy of a system is ½ kT where k is Boltzmann constant and T is the temperature of the system. A standing wave is a system of two degrees of freedom, one corresponding to the electric field and the other to the magnetic field. Hence the average energy of each standing wave (or mode) is kT.

The energy density of radiation in the frequency range v and v + dv in a cavity maintained at temperature T is

$$u(\mathrm{v})d\mathrm{v} = \frac{8\pi kT \ \mathrm{v}^2 d\mathrm{v}}{c^3} \qquad (2-6)$$

In terms of wavelength this relation is expressed as

$$u(\lambda)d\lambda = \frac{8\pi kT \ d\lambda}{\lambda^4} \qquad (2-7)$$

Eqns. (2-6) and (2-7) represent the Rayleigh-Jeans formula for black body radiation. A glance at the Rayleigh-Jeans formula, which is a rigorous consequence of classical physics, reveals that it fails to explain the experimental results in the higher frequency (lower wavelength) region. Instead of finite energy density, it predicts infinite energy density at extremely short wavelengths ultraviolet, X-rays and gamma rays. This discrepancy between the theory and the experiment was dramatically called ultraviolet catastrophe.

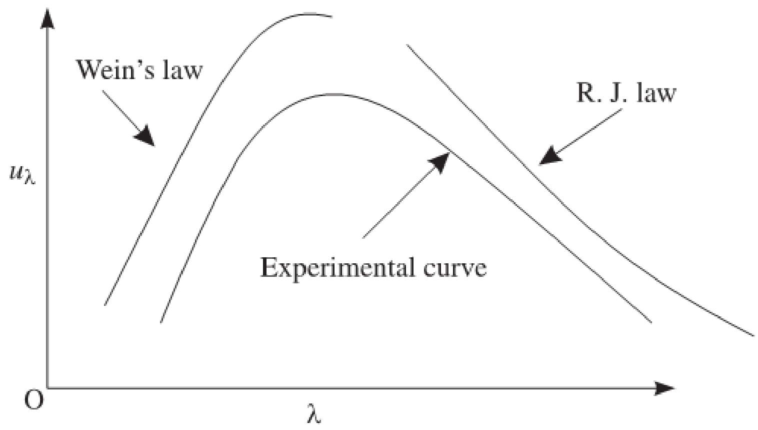

Fig. 4 Comparison of theoretical radiation laws with experimental curve

The failure of Rayleigh-Jeans law was taken more seriously because it was the only possible law that could be derived on the basis of the then known laws of classical physics. The failure of Rayleigh-Jeans law means the failure of the basic assumptions derived from the well-established laws of classical physics. This situation compelled the German physicist Max Planck to look beyond the framework of classical physics. He proposed a revolutionary hypothesis, according to which the emission and absorption of electromagnetic energy takes place in the form of packets (bundles), called quanta. This concept of quantization of energy is foreign to the classical physics.

Planks radiation law

The failure of Rayleigh-Jeans law led Planck to think that it was not possible to obtain a correct radiation law within the framework of classical physics. Planck assumed that the walls of the cavity consist of microscopic oscillators. In thermal equilibrium the absorption and emission of radiation by these oscillators take place at equal rate. According to Planck's hypothesis the emission and absorption of radiation by an oscillator take place in the form of discrete packets of energy called photons, whose energy is proportional to the frequency of radiation.

$$\varepsilon_n = nh\nu \qquad (2-8)$$

n=0, 1, 2,……..

The spectral energy density of blackbody radiation is

$$planck\ radiation\ formula \qquad u(\mathrm{v})d\mathrm{v} = \frac{8\pi h}{c^3}\ \frac{\mathrm{v}^3 d\mathrm{v}}{e^{\frac{h\mathrm{v}}{kT}} - 1} \qquad (2-9)$$

Here h is constant whose value is 6.626×10^{-34} J.s.

Actual average energy per standing wave is

$$\varepsilon = \frac{h\mathrm{v}}{e^{h\mathrm{v}/_{KT}} - 1} \qquad (2-10)$$

Ex. 1. Calculate the average energy of Planck's oscillator for (hv/kT) = 0.01, 0.1, 1.0, 10.

Solution

The average energy of plank oscillator is

$$\varepsilon = \frac{h\mathrm{v}}{e^{h\mathrm{v}/_{KT}} - 1}$$

Putting the values of (hv/kT) given in the problem we find

ε = kT, 0.95 kT, 0.58 kT, 0.00045 kT.

Example

DEDUCTION OF STEFAN'S LAW FROM PLANCK'S LAW

The radiant emitted (energy emitted per unit area per unit time for all wavelengths) of a black body is given by

$$R = \frac{c}{4}u = \frac{c}{4}\int_0^\infty u_\lambda(\mathrm{v}, T)d\lambda$$

$$= \frac{c}{4}\int_0^\infty u(\mathrm{v})d\mathrm{v} = \frac{c}{4}\int_0^\infty \frac{8\pi h}{c^3}\ \frac{\mathrm{v}^3 d\mathrm{v}}{e^{\frac{h\mathrm{v}}{kt}} - 1}$$

$$\int_0^\infty u(\mathrm{v})d\mathrm{v} = \int_0^\infty \frac{8\pi h}{c^3}\ \frac{\mathrm{v}^3 d\mathrm{v}}{e^{\frac{h\mathrm{v}}{kt}} - 1}$$

$$= \frac{8\pi h}{c^3}\left(\frac{KT}{h}\right)^4 \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

Mathematically the $\int_0^\infty \frac{x^3 dx}{e^x-1} = \frac{\pi^4}{15}$

$$R = \frac{2\pi^5 K^4}{15c^2h^3} T^4 = \sigma T^4$$

$$\sigma = 5.57 \times 10^{-8} watt/m^2K^4$$

PHOTOELECTRIC EFFECT

The emission of electrons by a substance under the action of light is called the photoelectric effect. The phenomenon of photoelectric effect can be studied with the help of an apparatus schematically shown in the Fig. (5). within an evacuated glass jacket two electrodes A and B are enclosed and the light radiation is allowed to enter the jacket through a quartz window. The radiation falls on the electrode A, called cathode. The electrode B can be kept at positive or negative potential with respect to the cathode. A sensitive ammeter is put in the circuit to record the current resulting from the photoelectrons. The potential difference between the cathode and the anode can be measured by voltmeter. The experimental observations of photoelectric effect may be summarized as follows:

1. For a constant potential difference between the cathode and anode, the number of electrons emitted from cathode (and hence the photoelectric current) increases with increasing intensity
2. For a constant intensity and frequency of incident radiation the photoelectric current varies with the potential difference V between the cathode and anode and reaches a constant value beyond which further increase of potential difference does not affect the photoelectric current, on the other hand, if the plate B is made more and more negative with respect of the photocathode surface the current decreases. This negative potential (with respect to cathode) of the plate is called retarding potential. For a particular value of retarding potential, the photoelectric current becomes zero. This potential is called cut-off or stopping potential V_0 and is measure of maximum kinetic energy of photoelectrons and we can write

$$T_{max} = eV_0 \qquad (2-11)$$

where T_{max} is the maximum kinetic energy of the ejected electron.

3. The stopping potential V_0 and hence the maximum kinetic energy T_{max} of photoelectrons is independent of the intensity of incident radiation and depends only on the frequency ν of radiation.

4. For each substance there exists a characteristic frequency v_0 such that for radiation with frequency below v_0 the photoelectrons are not ejected from the surface. This frequency is called the threshold frequency and the corresponding wavelength is called threshold wavelength, λ_0.

5. It has been observed that as soon as the light is incident on the substance, the electrons are emitted i.e., there is no time lag between the incidence of radiation and the ejection of electron.

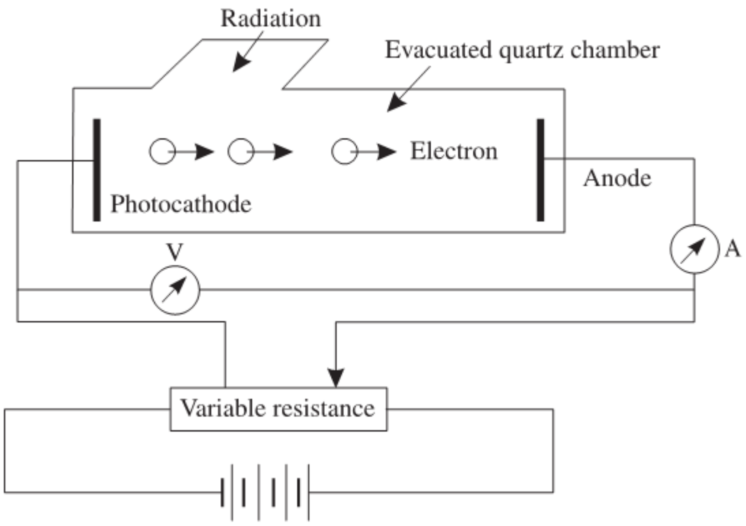

Fig.5 Schematic arrangement of the apparatus used for the study of photoelectric effect

Failure of classical physics:

1- The maximum kinetic energy of the electrons should be proportional to the intensity of the radiation.
2- The photoelectric effect should occur for light of any frequency or wavelength.
3- The first electrons should be emitted in a time interval of the order of seconds after the radiation begins to strike the surface.

Einstein's explanation of photoelectric effect:

According to Einstein, electromagnetic radiation of frequency v consists of small packets, called photons, each of energy hv. When a photon of energy hv (= hc / λ)is incident on the surface of a material, some of its energy is spent in making the electron free and the rest appears as kinetic energy of the electron. The electrons at the surface of the material are most loosely bound and require minimum energy for their liberation. This energy is called the work function φ of the material. The maximum kinetic energy of photoelectrons, ejected from the surface, is given by

$$T_{max} = h\nu - \varphi \qquad (2-12)$$

If ν_0 is the frequency of the incident radiation such that the photon energy $h\nu_0$ is just sufficient to make the electron free from the material the ejected electron has zero kinetic energy. This happens when

$$h\nu_0 = \varphi \qquad (2-13)$$

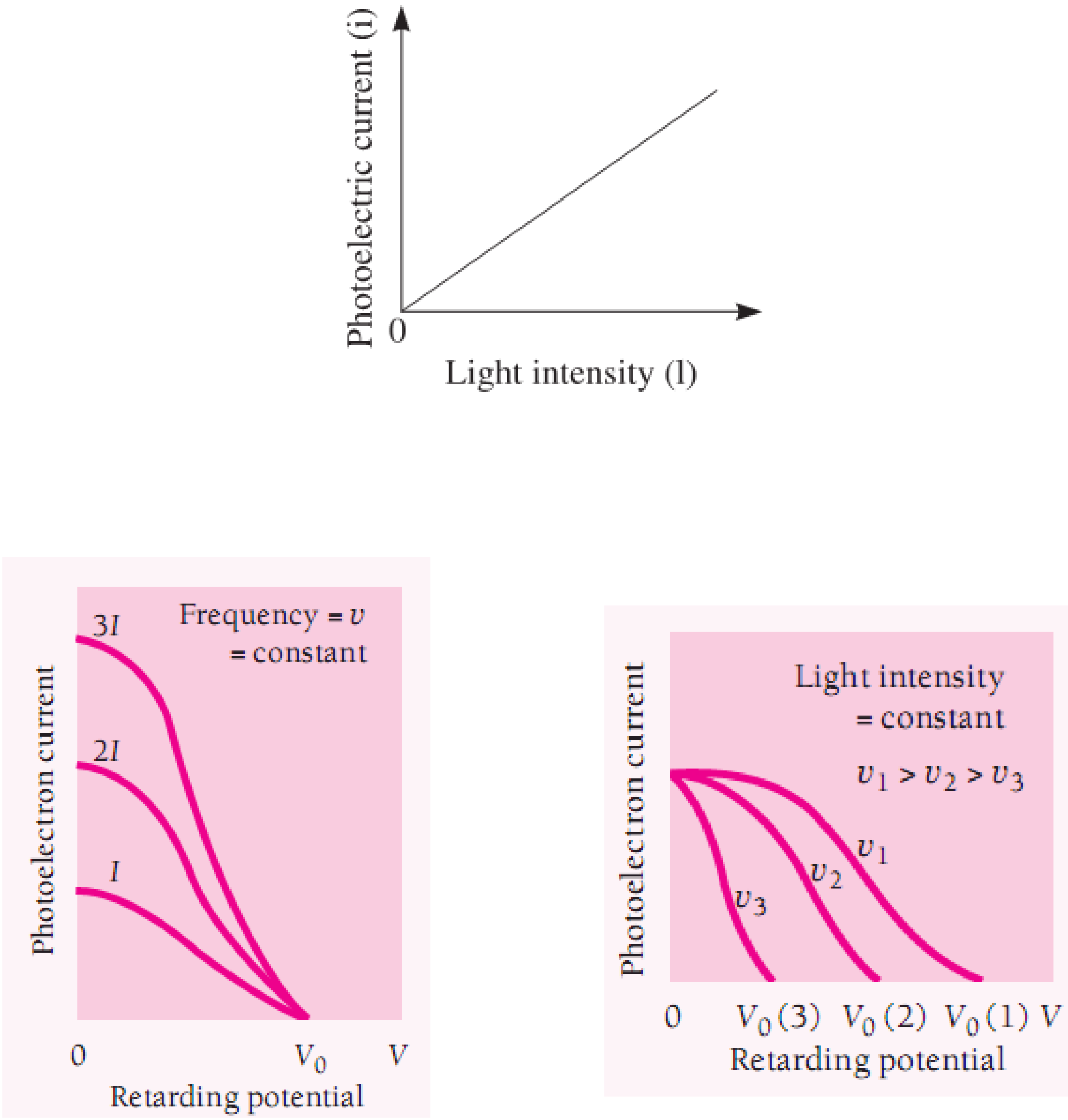

Fig.6 Variation of photoelectric current with intensity and accelerating potential

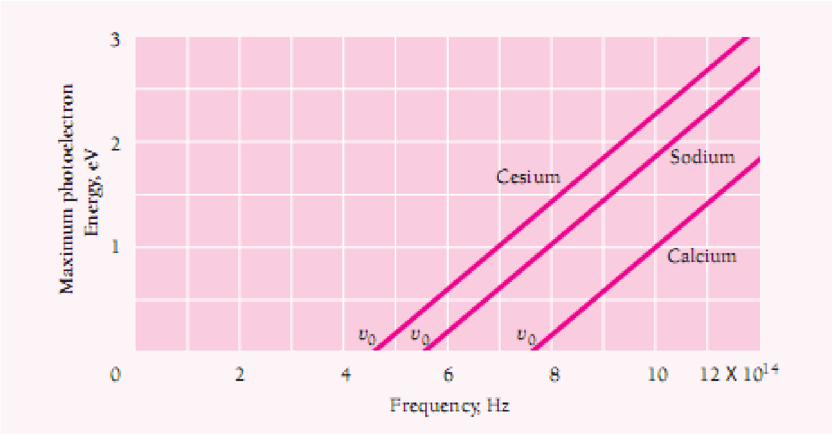

Fig.7 Variation of kinetic energy of photoelectrons with frequency of incident radiation.

We conclude that E-M radiation is not a continuous wave form, but consists of discrete, localized wave packets, called a photon. Either a photon is absorbed entirely, or not at all. Likewise, light is emitted in the form of one or more discrete photons.

Example

Light of wavelength 2000 Å falls on aluminum surface, which has work function of 4.2 eV. Calculate

(a) maximum kinetic energy of photoelectrons.

(b) minimum kinetic energy of photoelectrons.

(c) cut-off wavelength.

(d) stopping potential.

Sol. (a) Maximum kinetic energy of photoelectrons

$$T_{max} = h\nu - \varphi$$

$$= \frac{12400eV\ A^0}{2000A^0} - 4.2eV = 2\ eV$$

B- $T_{min} = 0$

c- Threshold wavelength

$$\lambda_0 = \frac{hc}{\varphi} = \frac{12400eV\ A^0}{2.4eV} = 5950\ A^\circ$$

D- Stopping potential

$$V_0 = \frac{T_{max}}{e} = 2V.$$

X-RAYS

They consist of high-energy photons

In 1895 Wilhelm Roentgen found that a highly penetrating radiation of unknown nature is produced when fast electrons impinge on matter. This radiation is called X-rays in which:-

1- travel in straight lines,
2- unaffected by electric and magnetic fields
3- pass readily through opaque materials
4- cause phosphorescent substances to glow
5- Expose photographic plates.

Not long after this discovery it became clear that x-rays are em waves. Electro- magnetic theory predicts that an accelerated electric charge will radiate em waves, and a rapidly moving electron suddenly brought to rest is certainly accelerated. Radiation produced under these circumstances is given the German name bremsstrahlung ("braking radiation").

Electromagnetic radiation with wavelengths from about 0.01 to about 10 nm falls into the category of x-rays. The boundaries of this category are not sharp: the shorter- wavelength end overlaps gamma rays and the longer-wavelength end overlaps ultraviolet light.

Produced of x-rays

Fig.8 is a diagram of an x-ray tube. A cathode, heated by a filament through which an electric current is passed, supplies electrons by thermionic emission. The high potential difference V maintained between the cathode and a metallic target accelerates the electrons toward the latter. The face of the target is at an angle relative to the electron beam, and the x-rays that leave the target pass through the side of the tube. The tube is evacuated to permit the electrons to get to the target unimpeded.

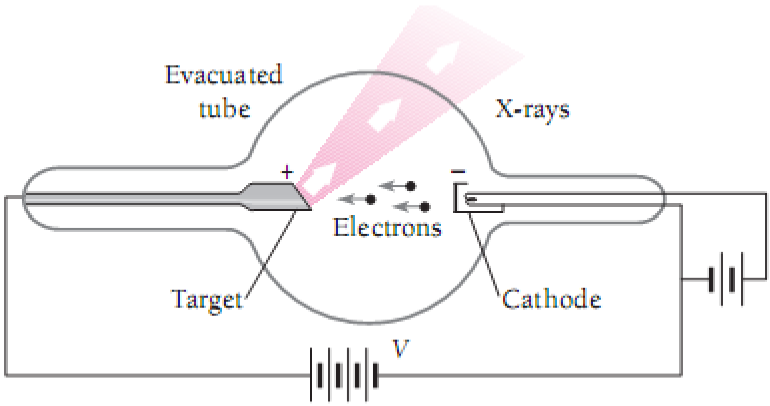

Fig.8 An x-ray tube. The higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the x-rays.

Fig.9 and 10 show the x-ray spectra that result when tungsten and molybdenum targets are bombarded by electrons at several different accelerating potentials. The curves exhibit two features electromagnetic theory cannot explain:

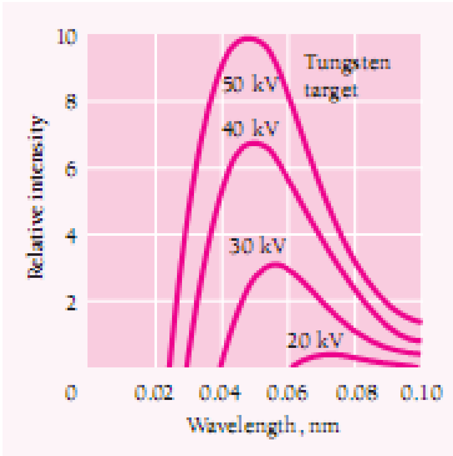

Fig.9 X-ray spectra of tungsten at various accelerating potentials.

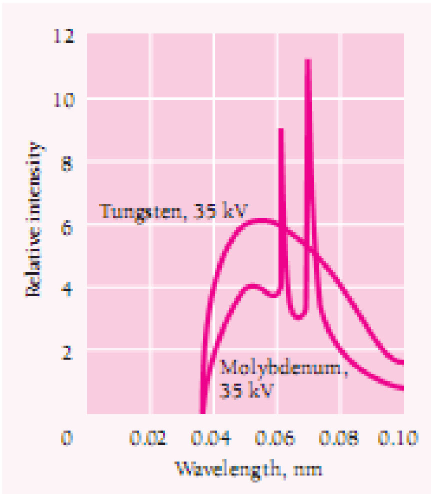

Fig.10 X-ray spectra of tungsten and molybdenum at 35 kV accelerating potential.

1- In the case of molybdenum, intensity peaks occur that indicate the enhanced production of x-rays at certain wavelengths. These peaks occur at specific wavelengths for each target material and originate in rearrangements of the electron structures of the target atoms after having been disturbed by the bombarding electrons. The important thing to note at this point is the presence of x-rays of specific wavelengths, a decidedly no classical effect, in addition to a continuous x-ray spectrum.

2- The x-rays produced at a given accelerating potential V vary in wavelength, but none has a wavelength shorter than a certain value λ_{min}. Increasing V decreases λ_{min}. At a particular V, λ_{min} is the same for both the tungsten and molybdenum targets. Duane and Hunt found experimentally that λ_{min} is inversely proportional to V; their precise relationship is

$$\lambda_{min} = \frac{hc}{eV}$$

$$\lambda_{min} = \frac{1.24 \times 10^{-6} Volt.m}{V\,Volt} \qquad (2-14)$$

Example

Find the shortest wavelength present in the radiation from an x-ray machine whose accelerating potential is 50,000 V.

Solution

$$\lambda_{min} = \frac{1.24 \times 10^{-6} Volt.m}{50000V} = 2.48 \times 10^{-11} m$$

The wavelength corresponding to the frequency

$$\mathrm{v}_{max} = \frac{c}{\lambda_{min}} = \frac{3 \times 10^{8} m/s}{2.48 \times 10^{-11} m} = 1.21 \times 10^{19}\, Hz$$

X-RAY DIFFRACTION

How x-ray wavelengths can be determined

A monochromatic beam of x-rays that falls upon a crystal will be scattered in all directions inside it. However, owing to the regular arrangement of the atoms, in certain directions the scattered waves will constructively interfere with one another while in others they will destructively interfere. The atoms in a crystal may be thought of as defining families of parallel planes, as in Fig. 11, with each family having a characteristic separation between its component planes. This analysis was suggested in 1913by W. L Bragg, in honor of whom the above planes are called Bragg planes.

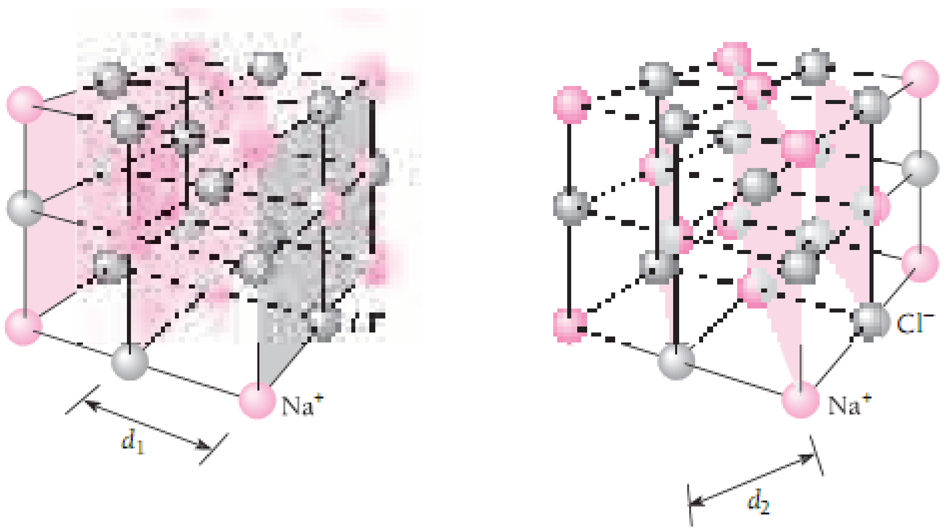

Fig.11 Two sets of Bragg planes in a NaCl crystal.

Constructive interference takes place only between those scattered rays that are parallel and whose paths differ by exactly λ, λ_2, λ_3, and so on. That is, the path difference must be nλ, where n is an integer. The only rays scattered by A and B for which this is true are those labeled I and II in Fig. 12.

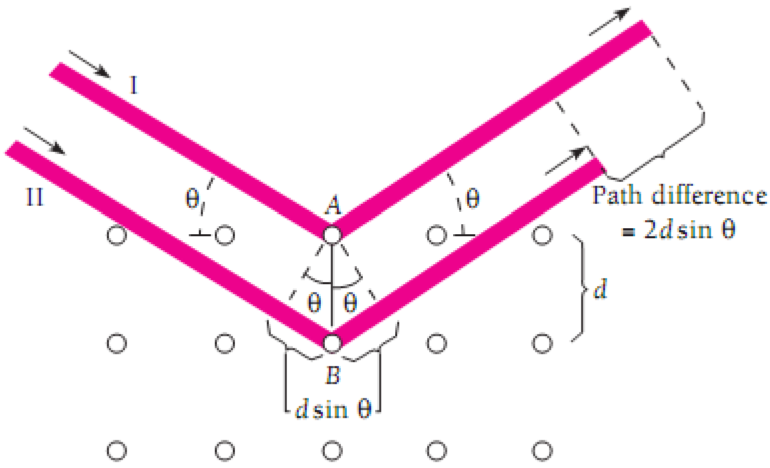

Fig.12. X-ray scattering from a cubic crystal.

The first condition on I and II is that their common scattering angle be equal to the angle of incidence θ of the original beam.

The second condition is that

$$2dsin\theta = n\lambda \qquad (2-15)$$

since ray II must travel the distance 2d sin θ farther than ray I. The integer n is the order of the scattered beam.

The schematic design of an x-ray spectrometer based upon Bragg' s analysis is shown in Fig. 13. A narrow beam of x-rays falls upon a crystal at an angle θ, and a detector is placed so that it records those rays whose scattering angle is also θ. Any x-rays reaching the detector therefore obey the first Bragg condition. As θ is varied, the detector will record intensity peaks corresponding to the orders predicted by Eq. (2-15). If the spacing d between adjacent Bragg planes in the crystal is known, the x-ray wavelength λ may be calculated.

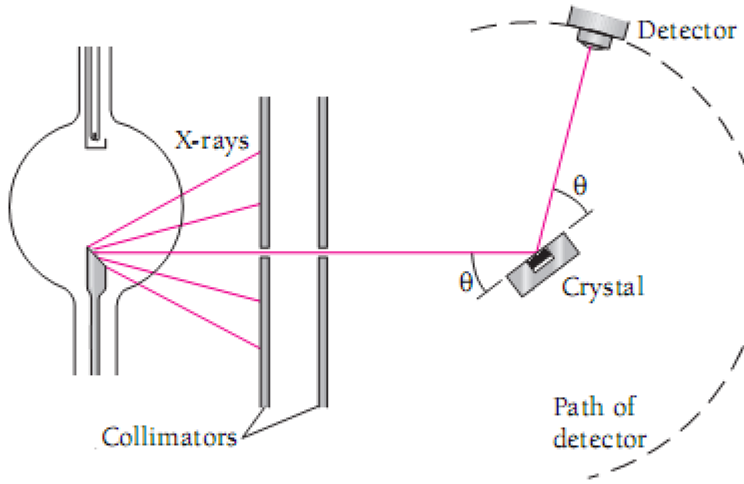

Fig. 13. X-ray spectrometer.

Compton Effect

In 1922, the American physicist Arthur Compton, investigating the scattering of X-rays by different substances, observed that the scattered rays, in addition to radiation of the initial wavelength λ, contain also rays of a greater wavelength λ'. This phenomenon is known as Compton Effect.

Fig.14 shows such a collision: an x-ray photon strikes an electron. If the initial photon has the frequency ν associated with it, the scattered photon has the lower frequency ν', where

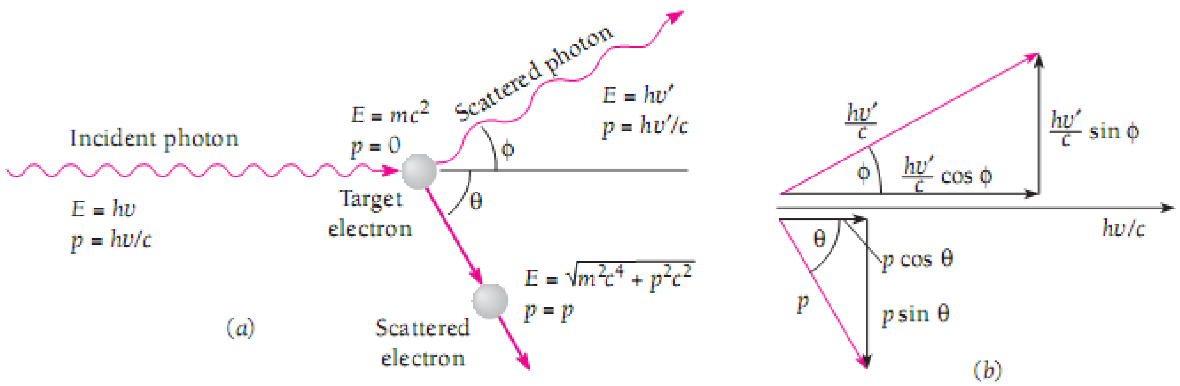

Fig.14(a) The scattering of a photon by an electron is called the Compton effect. (b) Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

Loss in photon energy = gain in electron energy

$$h\text{v} - h\text{v}' = KE \qquad (2-16)$$

for massless particle (photon) the energy and linear momentum given by

$$E = pc \qquad (2-17) \quad and \qquad p = \frac{E}{c} = \frac{h\text{v}}{c} \qquad (2-18)$$

The initial photon momentum is hv/c, the scattered photon momentum is hv'/c, and the initial and final electron momenta are respectively 0 and p. In the original photon direction

Initial momentum = final momentum

$$\frac{h\text{v}}{c} + 0 = \frac{h\text{v}'}{c} cos\phi + pcos\theta \qquad (2-19)$$

and perpendicular to this direction

Initial momentum = final momentum

$$0 = \frac{h\text{v}'}{c} sin\phi - psin\theta \qquad (2-20)$$

The angle ϕ is that between the directions of the initial and scattered photons, and θ is that between the directions of the initial photon and the recoil electron.

The first step is to multiply Eqs. (2.19) and (2.20) by c and rewrite them as

$$pc\ cos\theta = h\text{v} - h\text{v}'cos\phi$$

$$pc\ sin\theta = h\text{v}'sin\phi$$

By squaring each of these equations and adding the new ones together

$$p^2c^2 = (h\text{v})^2 - 2(h\text{v})(h\text{v}')cos\phi + (h\text{v}')^2 \qquad (2-21)$$

Next we equate the two expressions for the total energy of a particle

$$E = KE + m_\circ c^2 \qquad (2-22)$$

$$E = \sqrt{m_\circ{}^2 c^4 + p^2c^2} \qquad (2-23)$$

$$(KE + m_\circ c^2)^2 = m_\circ{}^2 c^4 + p^2 c^2$$

$$p^2 c^2 = KE^2 + 2m_\circ c^2 KE$$

Since

$$KE = h\mathrm{v} - h\mathrm{v}'$$

We have

$$p^2 c^2 = (h\mathrm{v})^2 - 2(h\mathrm{v})(h\mathrm{v}') + (h\mathrm{v}')^2 \ + 2m_\circ c^2 (\ h\mathrm{v} - h\mathrm{v}') \qquad (2-24)$$

Substituting this value in eq.(2-21) we finally obtain

$$2m_\circ c^2 (h\mathrm{v} - h\mathrm{v}') = 2(h\mathrm{v})(h\mathrm{v}')(1 - cos\phi) \qquad (2-25)$$

This relationship is simpler when expressed in terms of wavelength λ. Dividing eq.(2.20) by $2h^2c^2$

$$\frac{m_\circ c}{h}\left(\frac{\mathrm{v}}{c} - \frac{\mathrm{v}'}{c}\right) = \frac{\mathrm{v}}{c}\frac{\mathrm{v}'}{c}(1 - cos\phi)$$

And so since v/c =1/λ and v'/c =1/λ'

$$\frac{m_\circ c}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = (1 - cos\phi)/\lambda\lambda'$$

$$\lambda' - \lambda = \frac{h}{m_\circ c} = (1 - cos\phi) \qquad (2-26) \qquad Compton\ Effect$$

$\lambda c = \frac{h}{m_\circ c}$ is called Compton wavelength of the scattering particle

for electron

λc = 0.02426 A°

The greatest wavelength change possible corresponds to ϕ = 180°, when the wavelength change will be twice the Compton wavelength λc. The Compton Effect is the chief means by which x-rays lose energy when they pass through matter.

Example

X-rays of wavelength 10.0 pm are scattered from a target. (a) Find the wavelength of the x-rays scattered through 45°. (b) Find the maximum wavelength present in the scattered x-rays. (c) Find the maximum kinetic energy of the recoil electrons.

Solution

$$\lambda' - \lambda = \frac{h}{m_{\circ}c} = (1 - cos\phi)$$

$$\lambda' = \lambda + \frac{h}{m_{\circ}c} = (1 - cos\phi)$$

$$= 10\ pm + 0.293\lambda_c$$

$$= 10.7\ pm$$

b- λ - λ' is a maximum when (1 - cos ϕ) = 2, in which case

$$\lambda' = \lambda + 2\lambda_c = 10pm + 4.9pm = 14.9\ pm$$

(c) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$$KE_{max} = h(\mathrm{v} - \mathrm{v}') = hc(\frac{1}{\lambda} - \frac{1}{\lambda'})$$

where λ' is given in (b). Hence

$$KE_{max} = \frac{(6.626 \times 10^{-34} J.s)(3 \times 10^{8} ms^{-1})}{10^{-12} pm} \ (\frac{1}{10\ pm} - \frac{1}{14.9\ pm})$$

$$= 6.54 \times 10^{-15} J$$

Which is equivalent to 40.8 KeV.

Pair production and annihilation

Energy into matter

Another process that can occur when photons encounter atoms is pair production, in which the photon loses all its energy and in the process two particles are created: an electron and a positron.

The photon energy hv is converted into the relativistic total energies E_+ and E_- of the positron and electron:

$$hv = E_+ + E_- = (m_e c^2 + K_+) + (m_e c^2 + K_-) \qquad (2-27)$$

Because K+ and K− are always positive, the photon must have an energy of at least $2m_e c^2$ = 1.02 MeV in order for this process to occur; such high-energy photons are in the region of nuclear gamma rays. Symbolically,

photon → electron + positron

This process, like bremsstrahlung, will not occur unless there is an atom nearby to supply the necessary recoil momentum. The reverse process,

electron + positron → photon

also occurs; this process is known as electron- positron annihilation and can occur for free electrons and positrons as long as at least two photons are created. In this process the electron and positron disappear and are replaced by two photons. Conservation of energy requires that

$$(m_e c^2 + K_+) + (m_e c^2 + K_-) = E_+ + E_- \qquad (2-28)$$

where E1 and E2 are the photon energies. Usually the kinetic energies K+ and K− are negligibly small, so we can assume the positron and electron to be essentially at rest. Momentum conservation then requires the two photons to have equal and opposite momenta and thus equal energies. The two annihilation photons have equal energies of 0.511MeV (= $m_e c^2$) and move in exactly opposite directions.

Example

Show that pair production cannot occur in empty space.

Solution

From conservation of energy,

$$h\nu = 2\gamma mc$$

where hν is the photon energy and γmc^2 is the total energy of each member of the electron-positron pair. Fig.15 is a vector diagram of the linear momenta of the photon, electron, and positron. The angles θ are equal in order that momentum be conserved in the transverse direction. In the direction of motion of the photon, for momentum to be conserved it must be true that

$$\frac{h\nu}{c} = 2pcos\theta$$

$$h\nu = 2pc\ cos\theta$$

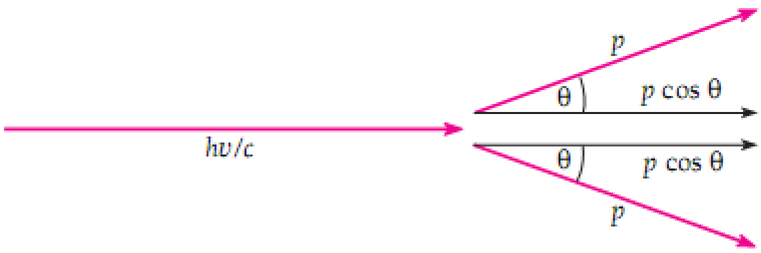

Fig.15 Vector diagram of the momenta involved if a photon were to materialize into an electron- positron pair in empty space.

Since p= γmc for the electron and positron

$$h\nu = 2\gamma mc^2\ \frac{v}{c}cos\theta$$

because υ/c <1 and cosθ <=1

$$h\nu < 2\gamma mc^2$$

But conservation of energy requires that $h\nu=2\gamma mc^2$

Hence it is impossible for pair production to conserve both energy and momentum unless some other object is involved in the process to carry away part of the initial photon momentum.

Photon absorption

The intensity I of an x- or gamma-ray beam is equal to the rate at which it transports energy per unit cross-sectional area of the beam. The fractional energy – dI/I lost by the beam in passing through a thickness dx of a certain absorber is found to be proportional to dx:

$$-\frac{dI}{I} = \mu dx \qquad (2-29)$$

The proportionality constant **μ** is called the linear attenuation coefficient and its value depends on the energy of the photons and on the nature of the absorbing material.

Integrating Eq. (2-29) gives

$$radiation\ intensity \qquad I = I_0 e^{-\mu x} \qquad (2-30)$$

The intensity of the radiation decreases exponentially with absorber thickness x.

$$\frac{I}{I_0} = e^{-\mu x} \quad \Rightarrow \frac{I_0}{I} = e^{-\mu x} \quad \Rightarrow \ln\frac{I_0}{I} = -\mu x$$

$$absorber\ thickness \qquad x = \frac{\ln\frac{I_0}{I}}{\mu} \qquad (2-31)$$

Fig.16 is a graph of the linear attenuation coefficient for photons in lead as a function of photon energy. The contribution to **μ** of the photoelectric effect, Compton scattering, and pair production are shown.

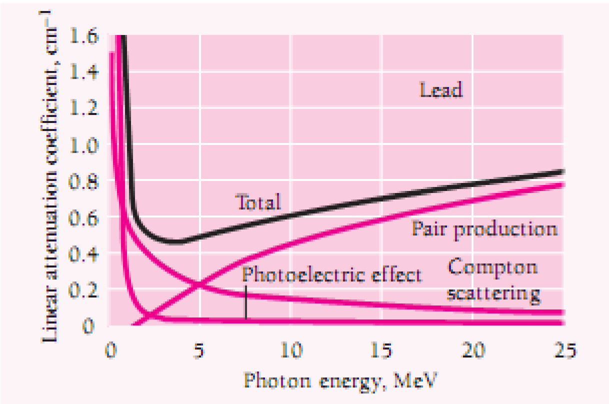

Fig.16 Linear attentuation coefficients for photons in lead.

Example

The linear attenuation coefficient for 2.0-MeV gamma rays in water is 4.9 m^{-1}. (a) Find the relative intensity of a beam of 2.0-MeV gamma rays after it has passed through 10 cm of water. (b)How far must such a beam travel in water before its intensity is reduced to 1 percent of its original value?

Solution

(a) Here $\boldsymbol{\mu}x=(4.9m^{-1})(0.1)=0.49$ and so

$$\frac{I}{I_0} = e^{-\mu x} = e^{-0.49} = 0.61$$

The intensity of the beam is reduced to 61 percent of its original value after passing through 10cm of water.

(b) Since $I_0/I = 100$, we obtain

$$x = \frac{\ln \frac{I_0}{I}}{\mu} = \frac{1n\ 100}{4.9\ m^{-1}} = 0.94\ m$$

Photons and gravity

Although they lack rest mass, photons behave as though they have gravitational mass

A photon that falls in a gravitational field gains energy (increase in frequency from ν toν'). A photon has no rest mass; it nevertheless interacts with electrons as though it has the inertial mass

$$m = \frac{p}{v} = \frac{hv}{c^2} \qquad (2-32)$$

All photons travel with the speed of light and so cannot go any faster. However, a photon that falls through a height H can manifest the increase of mgH in its energy by increase in frequency from ν to ν' (Fig.17).

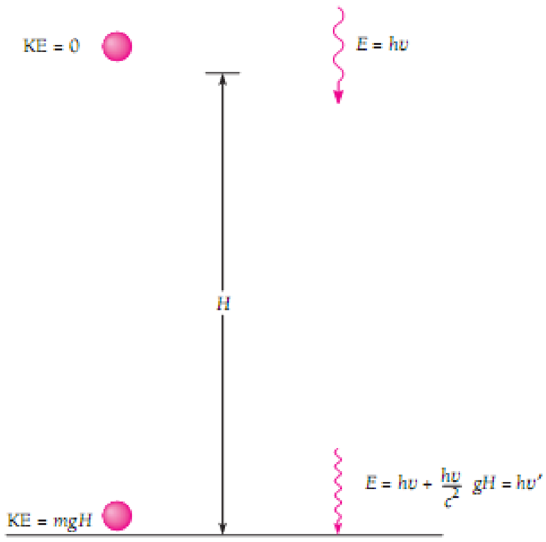

Fig.17 a photon that falls in gravitational field gains energy.

Hence,

Final photon energy = initial photon energy + increase in energy.

$$h\nu' = h\nu + mgH$$

and so

$$h\nu' = h\nu + (\frac{h\nu}{c^2})gH$$

$$h\nu' = h\nu\left(1 + \frac{gH}{c^2}\right) \qquad (2-33)$$

photon energy after falling through height H.

Example

The increase in energy of a fallen photon was first observed in 1960 by Pound and Rebka at Harvard. In their work H was 22.5 m. Find the change in frequency of a photon of red light whose original frequency is 7.3 10^{14}Hz when it falls through 22.5 m.

Solution

$$h\text{v}' = h\text{v}\left(1 + \frac{gH}{c^2}\right)$$

$$\text{v}' - \nu = \left(\frac{gH}{c^2}\right)\text{v} = \frac{\left(\frac{9.8m}{s^2}\right)(22.5m)(7.3\times 10^{14}Hz)}{\left(3\times\frac{10^8 m}{s}\right)^2} = 1.8\,Hz$$

Gravitational Red Shift

If the frequency associated with a photon moving toward the earth increases, then the frequency of a photon moving away from it should decrease.

Suppose a photon of initial frequency v is emitted by a star of mass M and radius R, as in Fig. 18. The potential energy of a mass m on the star's surface is

$$PE = -\frac{GMm}{R}$$

where the minus sign is required because the force between M and m is attractive. The potential energy of a photon of "mass" hv/c^2 on the star's surface is therefore

$$PE = -\frac{GMh\text{v}}{c^2R}$$

and its total energy E, the sum of PE and its quantum energy hv, is

$$E = h\text{v} - \frac{GMh\text{v}}{c^2R} = h\text{v}(1 - \frac{GM}{c^2R})$$

At a larger distance from the star, (at the earth), the photon's energy is now entirely electromagnetic, and

$$E = h\text{v}'$$

where v' is the frequency of the arriving photon. (The potential energy of the photon in the earth's gravitational field is negligible compared with that in the star's field. Hence

$$h\text{v}' = h\text{v}(1 - \frac{GM}{C^2R})$$

$$\frac{\text{v}'}{\text{v}} = 1 - \frac{GM}{C^2R}$$

And the relative frequency change is

$$graviational\ red\ shift \qquad \frac{\Delta\text{v}}{\text{v}} = \frac{\text{v} - \text{v}'}{\text{v}} = 1 - \frac{\text{v}'}{\text{v}} = \frac{GM}{c^2R} \qquad (2-34)$$

The photon has a lower frequency at the earth, corresponding to its loss in energy as it leaves the field of the star. A photon in the visible region of the spectrum is thus shifted toward the red end, and this phenomenon is accordingly known as the gravitational red shift.

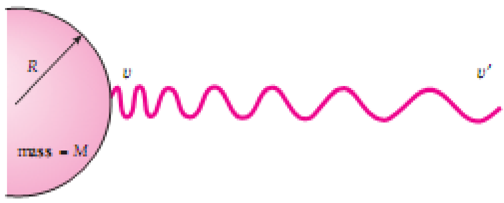

Fig.17 The frequency of a photon emitted from the surface of a star decreases as it moves away from the star.

EXERCISES 2

1-Is it correct to say that the maximum photoelectron energy KE_{max} is proportional to the frequency ν of the incident light? If not, what would a correct statement of the relationship between KE_{max} and ν be?

Solution

No: the relation is given in Equation

$$KE_{max} = h\text{v} - \varphi = h(\text{v} - \text{v}_0)$$

So that while KE_{max} is a linear function of the frequency n of the incident light, KE_{max} is not proportional to the frequency.

2. The maximum wavelength for photoelectric emission in tungsten is 230 nm. What wavelength of light must be used in order for electrons with a maximum energy of 1.5 eV to be ejected?

Solution

$$KE_{max} = h\text{v} - \varphi = h(\text{v} - \text{v}_0)$$

$$\lambda = \frac{hc}{\left(\frac{hc}{\lambda_0}\right) + K_{max}} = \lambda_0[1 + \frac{K_{max}\lambda_0}{hc}]^{-1} = 230nm[1 + \frac{(1.5)(230 \times 10^{-9}m}{1.24 \times 10^{-6}eV.m}]^{-1} = 180\ nm$$

3. What is the maximum wavelength of light that will cause photoelectrons to be emitted from sodium? What will the maximum kinetic energy of the photoelectrons be if 200-nm light falls on a sodium surface?

Solution

The maximum wavelength would correspond to the least energy that would allow an electron to be emitted, so the incident energy would be equal to the work function, and

$$\lambda_{max} = \frac{hc}{\emptyset} = \frac{1.24 \times 10^{-6}eV.m}{2.3\ eV} = 539\ nm$$

where the value of φ for sodium is taken from Table 2.1.

$$KE_{max} = h\text{v} - \varphi = \frac{hc}{\lambda} - \varphi = \frac{1.24 \times 10^{-6}eV.m}{200 \times 10^{-9}m} - 2.3eV = 3.9\ eV.$$

4. A metal surface illuminated by 8.5 x 10^{14} Hz light emits electrons whose maximum energy is 0.52 eV The same surface illuminated by 12.0 x 10^{14} Hz light emits electrons whose maximum energy is 1.97 eV From these data find Planck's constant and the work function of the surface .

Solution

Denoting the two energies and frequencies with subscripts 1 and 2,

$$KE_{max1} = h\nu_1 - \varphi, \qquad KE_{max2} = h\nu_2 - \varphi$$

$$\lambda = \frac{KE_{max2} - KE_{max1}}{\nu_2 - \nu_1} = \frac{1.97eV - 0.52eV}{12 \times 10^{14}Hz - 8.5 \times 10^{14}Hz} = 4.14\ eV.s$$

The work function f may be obtained by substituting the above result into either of the above, yielding φ = 3.0 eV

5. Electrons are accelerated in television tubes through potential differences of about 10 kV. Find the highest frequency of the electromagnetic waves emitted when these electrons strike the screen of the tube. What kind of waves are these?

Solution

For the highest frequency, the electrons will acquire all of their kinetic energy from the accelerating voltage, and this energy will appear as the electromagnetic radiation emitted when these electrons strike the screen. The frequency of this radiation will be

$$\nu = \frac{E}{h} = \frac{eV}{h} = \frac{(1e)(10 \times 10^3 V)}{4.14 \times 10^{-15} eV.s} = 2.4 \times 10^{18} Hz\ , which\ corresponds\ to\ x - rays$$

6. The distance between adjacent atomic planes in calcite (CaCO3) is 0.300 nm. Find the smallest angle of Bragg scattering for 0.030-nm x-rays.

Solution

$$n\lambda = 2d sin\theta, \qquad solving\ for\ \theta\ , when\ n = 1$$

$$\theta = \arcsin\left(\frac{\lambda}{2d}\right) = \arcsin\left(\frac{0.03nm}{2 \times 0.3nm}\right) = 2{,}9°$$

7. What is the frequency of an x-ray photon whose momentum is 1.1 x 10-23 kg m/s?

Solution

$$\nu = \frac{cp}{h} = \frac{(3 \times 10^{8} m/s)(1.1 \times 10^{-23} kg.m/s)}{6.63 \times 10^{-34} j.s} = 5 \times 10^{18} Hz.$$

8. A beam of x-rays is scattered by a target. At 45^o from the beam direction the scattered x-rays have a wavelength of 2.2 pm. What is the wavelength of the x-rays in the direct beam?

Solution

The wavelength of the x-rays in the direct beam,

$$\lambda = \lambda' - \lambda_c(1 - cos\varphi) = 2.2pm - (2.426\, pm)(1 - cos45°) = 1.5\, pm$$

9. An x-ray photon of initial frequency 3.0 x 10^{19} Hz collides with an electron and is scattered through 90^o. Find its new frequency.

Solution

In terms of frequencies , with $\lambda = c/\nu$ and $\lambda' = c/\nu'$, and with cos90°=0,

$$\frac{c}{\nu'} = \frac{c}{\nu} + \lambda_c$$

Solving for ν' gives

$$\nu' = [\frac{1}{\nu} + \frac{\lambda_c}{c}]^{-1} = [\frac{1}{3 \times 10^{19} Hz} + \frac{2.43 \times 10^{-12} m}{3 \times 10^{8} m/s}]^{-1} = 2.4 \times 10^{19} Hz$$

10. At what scattering angle will incident 100-keV x-rays leave a target with an energy of 90 keV?

Solution

$$cos\emptyset = 1 + \frac{\lambda}{\lambda_c} + \frac{\lambda'}{\lambda_c} = 1 + \left(\frac{mc^2}{E} + \frac{mc^2}{E'}\right) = 1 + \left(\frac{511 keV}{100 kEv} + \frac{511 keV}{90 keV}\right)$$
$$= 0.432, \quad from\ which\ \emptyset = 64°$$

11. A photon of frequency ν is scattered by an electron initially at rest. Verify that the maximum kinetic energy of the recoil electron is $KE_{max} = (2h^2 \nu 2/mc^2)/(1 + 2h\nu/mc^2)$.

Solution

For the electron to have the maximum recoil energy, the scattering angle must be 1800, and we obtain $mc^2\ KE_{max} = 2\ (h\nu)\ (h\nu')$, where $KE_{max} = (h\nu - h\nu')$ has been used. To simplify the algebra somewhat, consider

$$\nu' = \nu \frac{\lambda}{\lambda'} = \frac{\nu}{1 + (\frac{\Delta\lambda}{\lambda})} = \frac{\nu}{1 + (2\lambda_c/\lambda)} = \frac{\nu}{1 + (2\nu\lambda_c/c)}$$

Where $\Delta\lambda = 2\ \lambda_c$ for $\varphi = 180°$. With this expression ,

$$KE_{max} = \frac{2(h\nu)(h\nu')}{mc^2} = \frac{2(h\nu)^2/(mc^2)}{1 + (2\nu\lambda_c/c)}$$

Using $\lambda_c = h/(mc)$ gives the desired result.

12. A positron collides head on with an electron and both are annihilated. Each particle had a kinetic energy of 1.00 MeV, Find the wavelength of the resulting photons.

Solution

The energy of each photon will he the sum of one particle's rest and kinetic energies, 1.511 MeV(keeping an extra significant figure). The wavelength of each photon will be

$$\lambda = \frac{hc}{E} = \frac{1.24 \times 10^{-6} eV.m}{1.5 \times 10^6 eV} = 8.21 \times 10^{-13} m = 0.821 pm$$

13.(a) Show that the thickness $x_{1/2}$, of an absorber required to reduce the intensity of a beam of radiation by a factor of 2 is given by $x1/2 = 0.693/\mu$. (b) Find the absorber thickness needed to produce an intensity reduction of a factor of 10.

Solution

$$I = I_0 e^{-\mu x} \Rightarrow x_{1/2} = \frac{ln2}{\mu} = \frac{0.693}{\mu}$$

(b) similarly, with $\frac{I_0}{I} = 10,$

$$x_{1/10} = \ln 10/\mu = \frac{2.3}{\mu}$$

14. The linear absorption coefficient for 1-MeV gamma rays in lead is 78 m^{-1}. find the thickness of lead required to reduce by half the intensity of a beam of such gamma rays.

Solution

$$x_{1/2} = \frac{ln2}{\mu} = \frac{0.693}{78m^{-1}} = 8.9mm$$

15. The linear absorption coefficients for 2.0-MeV gamma rays are 4.9 m^{-1} in water and 52 m^{-1} in lead. What thickness of water would give the same shielding for such gamma rays as 10 mm of lead?

Solution

$$\mu_{H2o}\ x_{H2o} = \mu_{pb}\, x_{pb} \quad or$$

$$x_{H2o} = x_{pb} \quad \frac{\mu_{pb}}{\mu_{H2o}} = (10 \times 10^{-3}m)\frac{52\ m^{-1}}{4.9m^{-1}} = 0.106m$$

16. The sun's mass is 2.0 x 10^{30} kg and its radius is 7.0 x 10^{8} m. Find the approximate gravitational red shift in light of wavelength 500 nm emitted by the sun.

Solution

$$\frac{GM}{c^2R} = \frac{(6.67 \times 10^{-11}N.\frac{m^2}{kg^2})(2 \times 10^{30}kg)}{\left(3 \times 10^8\frac{m}{s}\right)^2(7 \times 10^8m)} = 2.12 \times 10^{-6}$$

$$\Delta\lambda = \lambda\frac{GM}{c^2R} = (500 \times 10^{-9}m)(\ 2.12 \times 10^{-6}) = 1.06 \times 10^{-12}m = 1.06\ pm$$