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Salahaddin University-Erbil

Matrix Eigenvalue problem

Research project

Submitted to the department of **(**mathematic**)** in partial
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Certification of the supervisor

I certify that this work was prepared under my supervision at the Department of Mathematics/ College of Education /Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics

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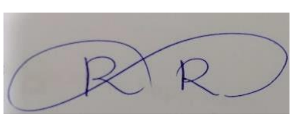
Date: 5 / 4 /2023

In view of the available recommendations, I forward this work for debate by the examining committee.

Signature: 

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Abstract

Solving linear system of equation is a common situation in many scientific and technological problems. Many methods either analytical or numerical, have been developed to solve them so, in this paper I declared some methods to solve linear system of equations for this we need to define some concepts. Generally talked about triangular matrices and non-singular we have defined it, we have taken examples. Matrix Like a general method most used in linear algebra is the Gauss Elimination, Gauss Jordan; in this paper I will explain these by taking an example. Also in this paper I will explain the LU decomposition method to construct triangular matrix. Finally, I will enquire the power method for finding numerical eigenvalues.

Contents

Introduction

Numerical linear algebra is a branch of mathematics that deals with the development and analysis of numerical algorithms for solving problems in linear algebra, such as solving systems of linear equations, finding eigenvalues and eigenvectors, and performing matrix factorizations. Linear systems arise in many applications in science, engineering, and mathematics, and they can often be represented in matrix form. Numerical methods for solving linear systems play a crucial role in many scientific and engineering applications, such as signal processing, image processing, data analysis, and optimization. In this context, a numerical linear system is a system of linear equations that is solved using numerical methods, such as Gaussian elimination, LU factorization, and iterative methods like Jacobi, and conjugate gradient. These methods are used to obtain approximate solutions that are accurate enough for the desired application, while taking into account the computational resources available. Overall, numerical linear algebra is a fundamental tool in scientific and engineering computation, and its applications continue to grow with the increasing complexity of real-world problems.

Chapter one
Basic Concepts

1.1 Triangular Matrices

Definition 1.1.1: A square matrix is said to be **triangular** if the elements above (or below of the main diagonal are zero. For example the matrices

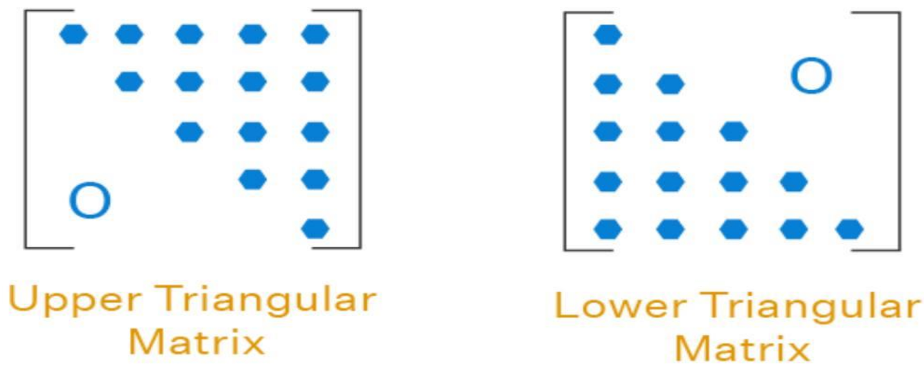
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Definition 1.1.2: An $n \times n$ square matrix $A = [aij]$ is said to be an upper triangular matrix if and only if $aij = 0$, for all $i > j$. This implies that all elements below the main diagonal of a square matrix are zero in an upper triangular matrix. A general notation of an upper triangular matrix is.

$$U = [u\, for\ i \leq j, 0 \text{ for } i > j].$$

Definition 1.1.3: An $n \times n$ square matrix $A = [aij]$ is said to be a lower triangular matrix if and only if $aij = 0$, for all $i < j$. This implies that all elements above the main diagonal of a square matrix are zero in a lower triangular matrix. A general notation of a lower triangular matrix is.

$$L = [lij\, for\ i \geq j, 0 \text{ for } i < j].$$



1.2 Non-Singular Matrix

Definition 1.2.1: A **non-singular matrix** is a square matrix whose determinant is not equal to zero. The non-singular matrix is an invertible matrix, and its inverse can be computed as it has a determinant value. For a square matrix $A = [abcd][abcd]$, the condition of it being a non singular matrix is the determinant of this matrix A is a non zero value. $|A| = |ad - bc| \neq 0$.

Properties of Non-Singular Matrix

The following are some of the important properties of a non-singular matrix.

- The determinant of a non-singular matrix is a non-zero value.
- The non-singular matrix is also called an **invertible** matrix because its determinant can be computed.
- The non-singular matrix is a square matrix because determinants can be calculated only for non-singular matrices.
- The product of two non-singular matrices is a non-singular matrix.

- If A is a non-singular matrix, k is a constant, and then kA is also a non-singular matrix.

Example: Find the determinant value of the matrix $\begin{bmatrix} 1 & -4 \\ 3 & 5 \end{bmatrix}$, and prove if it is a singular or a non-singular matrix.

Solution: The given matrix is A = $\begin{bmatrix} 1 & -4 \\ 3 & 5 \end{bmatrix}$

For a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant of the matrix is $|A| = |ad - bc|$.

$$|A| = 1(5) - (3)(-4) = 5 - (-12) = 5 + 12 = 17$$

Therefore, the determinant of the matrix $|A| = 17$, and it is a nonsingular matrix.

Definition 1.2.2: The **transpose of a matrix** is found by interchanging its rows into columns or columns into rows. The transpose of the matrix is denoted by using the letter "T" in the superscript of the given matrix. For example, if "A" is the given matrix, then the transpose of the matrix is represented by A' or A^T.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3} \qquad A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}_{3 \times 2}$$

Example: Find the transpose of the given matrix

$$M = \begin{bmatrix} 2 & -9 & 3 \\ 13 & 11 & -17 \\ 3 & 6 & 15 \\ 4 & 13 & 1 \end{bmatrix}$$

Solution: Given a matrix of the order 4×3.

The transpose of a matrix is given by interchanging rows and columns.

$$M^T = \begin{bmatrix} 2 & 13 & 3 & 4 \\ -9 & 11 & 6 & 13 \\ 3 & -17 & 15 & 1 \end{bmatrix}$$

Some properties of triangular matrix:

Since we have understood the meaning of a triangular matrix, let us go through some of its important properties. Given below is a list of the properties of a triangular matrix:

1. The transpose of a lower triangular matrix is an upper triangular matrix and vice-versa.
2. If $A1$ and $A2$ two upper(lower) triangular matrices of the same order then $A1 + A2$ are also upper(lower) triangular matrices of the same order.
3. A triangular matrix is invertible if and only if all entries of the main diagonal are non-zero.
4. The product of two lower(upper) triangular matrices is a lower(upper) triangular matrix.
5. The inverse of a triangular matrix is triangular.
6. The determinant of a triangular matrix is the product of the elements of the main diagonal.

7. The inverse of a nonsingular lower(upper) triangular matrix is also a lower(upper) triangular matrix.

Proof(1): Let U=$[a]_{mn}$ be an upper triangular matrix. By definition:

$\forall a_{ij} \in U: i > j \Longrightarrow a_{ij} = 0$

Let U^T=$[b]_{nm}$ be the transpose of U.

That is:

$U^T = [b]_{nm}$: $\forall i \in [1..n]$,j$\in [1..n]$: $b_{ij} = a_{ji}$

Thus:

$\forall b_{ji} \in U^T: i > j \Longrightarrow b_{ji} = 0$

By exchanging i and j in the notation of the above:

$\forall b_{ij} \in U^T: i < j \Longrightarrow b_{ij} = 0$

Thus by definition it is seen that U^T is a lower triangular matrix.

Proof(4): we will prove the result for lower triangular matrices; the proof for upper triangular matrices is similar. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be lower triangular $n \times n$ matrices, and let $C = [c_{ij}]$ be the product $C = AB$. We can prove that C is lower triangular by showing that $c_{ij} = 0$ for $i < j$. But from the definition of matrix multiplication,

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

If we assume that $i < j$, then the terms in this expression can be grouped as follows:

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{i(j-1)}b_{(j-1)j} \quad + a_{ij}b_{jj} + \cdots + a_{in}b_{nj}$$

In the first grouping all of the b factors are zero since B is lower triangular, and in the second grouping all of the a factors are zero since A is lower triangular. Thus, $c_{ij} = 0$, which is what we wanted to prove.

Example: Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ Let

$$A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

Since $AA^{-1} = I$ $$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying the matrices on the left side and equating corresponding elements On both sides, we obtain

$$a_{11} = 1 \qquad a_{22} = 1$$

$$2a_{11} + a_{12} = 0 \qquad 2a_{22} + a_{23} = 0$$

$$a_{12} = -2 \qquad a_{23} = -2$$

$$3a_{11} + 2a_{12} + a_{13} = 0 \qquad a_{33} = 1$$

$$a_{13} = 1$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

1.3 Lu decomposition of a Matrix

Let $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$

be a nonsingular square matrix. Then can be factorized into the form LU. Where

$$L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ l_{21} & 1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ l_{n1} & l_{n2} & \cdots & \cdots & 1 \end{bmatrix} \text{ and } \quad U = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ 0 & u_{22} & \cdots & u_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \text{ if}$$

$a_{11} \neq 0, \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0, \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$, and so on. It is a standard result of linear algebra that such a factorization, when it exists, is unique. Similarly, the factorization LU where

$$L = \begin{bmatrix} l_{11} & 0 & \cdots & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ l_{n1} & l_{n2} & l_{n3} & \cdots & l_{nn} \end{bmatrix} \text{and} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & u_{2n} \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Is also a unique factorization. We outline below the procedure for finding L and U with a square matrix of order 3. Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Multiplying the matrices on the right side. And equating the corresponding elements of both sides, we get

$u_{11} = a_{11}, \qquad u_{12} = a_{12}, \qquad u_{13} = a_{13},$

$l_{21}u_{11} = a_{21}, \qquad l_{21}u_{12} + u_{22} = a_{22}, \qquad l_{21}u_{13} + u_{23} = a_{23},$

$l_{31}u_{11} = a_{31}, \qquad l_{31}u_{12} + l_{32}u_{22} = a_{32}, \quad l_{31}u_{13} + l_{32}u_{23} + u_{33} = a_{33}$

From the above equations, we obtain

$l_{21} = \frac{a_{21}}{a_{11}}, \qquad l_{31} = \frac{a_{31}}{a_{11}}, \quad u_{22} = a_{22} - \frac{a_{21}}{a_{11}}a_{12}, \qquad u_{23} = a_{23} - \frac{a_{21}}{a_{11}}a_{13}$

$l_{32} = \frac{a_{32} - \frac{a_{31}}{a_{11}}a_{12}}{u_{22}}$

from which u_{33} can be computed. The given procedure is a systematic one to evaluate the elements of L and U (where L is unit lower triangular and U upper triangular). First, we determine the first row of U and the first column of L, then we determine the second row of U and the second column of l, and finally, we compute the third row of U. It is obvious that this procedure can be generalized. When the factorization is complete. The inverse of a can be computed.

Example: Solve the following system of equation by LU Decomposition method

$X + 5y + z = 14$

$2x + y + 3z = 13 \qquad \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ 13 \\ 17 \end{pmatrix}$

$3x + y + 4z = 17$

Solution: $A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = LU$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 0 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$u_{11} = 1$, $u_{12} = 5$, $u_{13} = 1$, $l_{21}u_{11} = 2$, $l_{31}u_{11} = 3$

$l_{21}u_{12} + u_{22} = 1$ $l_{21} = 2$, $l_{31} = 3$, $u_{22} = -9$

$l_{21}u_{13} + u_{23} = 3$, $l_{31}u_{12} + l_{32}u_{22} = 1, u_{23} = 1$

$l_{32} = \frac{14}{9}$,$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 4$, $u_{33} = \frac{-5}{9}$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{14}{9} & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & \frac{-5}{9} \end{bmatrix}$$

Chapter Two

2.1 Solution of linear systems-Direct methods

The solution of a linear system of equation can be accomplished by a Numrical method which falls in one of two categories:direct or iterative Methods amongst the direct methods.we will describe the elimination Method by gauss as also its modification and the LU decomposition method.About the iterative types,we will describe only the Jacobi and Gauss-seidel methods

2.1.1 Gauss Elimination

This is the elementary elimination method and it reduces the system of Equations to an equivalent upper-triangular system, which can be solved By back substitution Let the system of n linear equations in n unknowns be given by

$$a_{11}\, x_1 \;+\; a_{12}x_2 \;+ a_{13}x_3 + \cdots + a_{1n}\, x_n = b_1$$

$$a_{21}x_1 \;+\; a_{21}\, x_2 \;+\; a_{23}x_3 + \cdots + \; a_{2n}\, x_n = b_2$$

.

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$$a_{n1}x_1 \;+\; a_{n2} \;\; + a_{n3}x_3 + \cdots + \; a_{2n}\, x_n \; = b_n$$

The elementary row operations used in Gaussian elimination are:

1.interchange two rows.

2.Multiply a row by a nonzero constant.

3. Add a multiple of one row to another row.

Example:

$\mathrm{x} - \mathrm{y} + 2\mathrm{z} = 3$

$\mathrm{x} + \mathrm{y} + 3\mathrm{z} = 5$

$3\mathrm{x} - 4\mathrm{y} - 5\mathrm{z} = -13$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 3 \\ 3 & -4 & -5 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -13 \end{pmatrix}$$

Solution:

$$= \begin{bmatrix} 1 & -1 & 2: & 3 \\ 1 & 1 & 3: & 5 \\ 3 & -4 & -5: & -13 \end{bmatrix}$$ R2→ $R2 - R1$ R3→R3—3R1

$$= \begin{bmatrix} 1 & -1 & 2: & 3 \\ 0 & 2 & 1: & 2 \\ 0 & -1 & -11: & -22 \end{bmatrix}$$ R3→ $3R3 + R2$

$$= \begin{bmatrix} 1 & -1 & 2: & 3 \\ 0 & 2 & 1: & 2 \\ 0 & 0 & -32: & -64 \end{bmatrix}$$

$-32z = -64 \quad z = 2$

$2y + z = 2$ y=0

$x - y2 + 2z = 3 \quad x = -1$

This an example of how Gaussian elimination can be used to solve a system of linear equations.

2.1.2 Pivoting

The pivot or pivot element is the element of a matrix, or an array, which is selected first by an algorithm (e.g. Gaussian elimination, simplex algorithm, etc.), to do certain calculations. In the case of matrix algorithms, a pivot entry is usually required to be at least distinct from zero, and often distant from it; in this case finding this element is called pivoting. Pivoting may be followed by an interchange of rows or columns to bring the pivot to a fixed position and allow the algorithm to proceed successfully and possibly to reduce round-off error. It is often used for verifying row echelon form. Pivoting might be thought of as swapping or sorting rows or columns in a matrix, and thus it can be represented as multiplication by permutation matrices. However, algorithms rarely move the matrix elements because this would cost too much time; instead, they just keep track of the permutations. Overall, pivoting adds more operations to the computational cost of an algorithm. These additional operations are sometimes necessary for the algorithm to work at all. Other times these additional operations are worthwhile because they add numerical stability to the final result.

Example: use Gauss elimination to solve the system

$$2x + y + z = 10$$

$$3x + 2y + 3z = 18$$

$$x + 4y + 9z = 16$$

For this we multiply the first equation by $(-3/2)$ and add to the second to get

$$y + 3z = 6$$

Similarly. we multiply the first equation by $(-1/2)$ and add it to the third to get

$$7y + 17z = 22$$

Next, we have to eliminate y from (i) and (ii). For this we multiply (i) by -7 and add to 9ii0.this gives

$$-4z = -20 \; or \; z = 5$$

The upper triangular form is therefore given by

$$2x + y + z = 10$$

$$y + 3z = 6$$

$$z = 5$$

It follows that the required solution is $x = 7, y = -9$ and $z = 5$ The next example demonstrates the necessity or pivoting in the elimination method.

2.1.3 Gauss-Jordan Method

The following row operations on the augmented matrix of a system produce the augmented matrix of an equivalent system, i.e., a system with the same solution as the original one.

• Interchange any two rows.

• Multiply each element of a row by a nonzero constant.

• Replace a row by the sum of itself and a constant multiple of another row of the matrix.

For these row operations, we will use the following notations.

• $R_i \leftrightarrow R_j$ means: Interchange row i and row j.

• αR_i means: Replace row i with α times row i.

• $R_i + \alpha R_j$ means: Replace row i with the sum of row i and α times row j.

The Gauss-Jordan elimination method to solve a system of linear equations is described in the following steps.

1. Write the augmented matrix of the system.

2. Use row operations to transform the augmented matrix in the form described below, which is called the reduced row echelon form (RREF).

(a) The rows (if any) consisting entirely of zeros are grouped together at the bottom of the matrix.

(b) In each row that does not consist entirely of zeros, the leftmost nonzero element is a 1 (called a leading 1 or a pivot).

(c) Each column that contains a leading 1 has zeros in all other entries.

(d) The leading 1 in any row is to the left of any leading 1's in the rows below it.

3. Stop process in step 2 if you obtain a row whose elements are all zeros except the last one on the right. In that case, the system is inconsistent and has no solutions. Otherwise, finish step 2 and read the solutions of the system from the final matrix.

Note: When doing step 2, row operations can be performed in any order. Try to choose row operations so that as few fractions as possible are carried through the computation. This makes calculation easier when working by hand.

Example: solve the system

$$2x+y+z=10$$

$$3x+2y+3z=18$$

$$X+4y+9z=16$$

Solution:

x+4y+9z=16

2x+y+z=10

3x+2y+3z+18

$$\begin{bmatrix} 1 & 4 & 9 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 16 \\ 10 \\ 18 \end{pmatrix}$$

(A:B)= $\begin{bmatrix} 1 & 4 & 9 & :16 \\ 2 & 1 & 1 & :10 \\ 3 & 2 & 3 & :18 \end{bmatrix}$ R2→R2-2R1 R3→R3-3R1

$=\begin{bmatrix} 1 & 4 & 9 & :16 \\ 0 & -7 & -17 & :-22 \\ 0 & -10 & -24 & :-30 \end{bmatrix}$ R3→ $7R3 - 10R2$ $R1 \rightarrow 7R1 + 4R2$

$$=\begin{bmatrix} 7 & 0 & -5 & :24 \\ 0 & -7 & -17 & :-22 \\ 0 & 0 & 2 & :10 \end{bmatrix}$$

$$=\begin{bmatrix} 14 & 0 & 0 & :98 \\ 0 & -14 & 0 & :126 \\ 0 & 0 & 2 & :10 \end{bmatrix}$$

14x=98 → x=7

-14y=126 → y=-9

2z=10 → z=5

2.1.4 Modification of the Gauss Method to compute the Inverse

We know that X will be the inverse of A if $Ax = I$ where I is the unit matrix of the same order as A. It is required to determine the elements of X such that For example, for third-order matrices may be written as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The reader can easily see that this equation is equivalent to the three equations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We can therefore apply the Gaussian elimination method to each of these systems and the result in each case will be the corresponding column of . Since the matrix of coefficients is the same in each case , we can solve all the three systems simultaneously. Starting with the augmented system

$$\begin{bmatrix} a_{11} & a_{12} & a_{13}:1 & 0 & 0 \\ a_{21} & a_{22} & a_{23}:0 & 1 & 0 \\ a_{31} & a_{32} & a_{33}:0 & 0 & 1 \end{bmatrix}$$

We obtain at the end of the first and second stage, respectively

$$\begin{bmatrix} a_{11} & a_{12} & a'_{13}: & 1 & 0 & 0 \\ 0 & a'_{22} & a'_{23}: & -a_{21}/a_{11} & 1 & 0 \\ 0 & a'_{32} & a'_{33}: & -a_{31}/a_{11} & 0 & 1 \end{bmatrix} \textbf{ and } \begin{bmatrix} a_{11} & a_{12} & a_{13}: & 1 & 0 & 0 \\ 0 & a'_{22} & a'_{23}: & a_{21} & 1 & 0 \\ 0 & 0 & a'_{33}: & a_{31} & a_{32} & 1 \end{bmatrix}$$

Where $a_{21} = -a_{21}/a_{11}$,

$a_{31} = (a_{21}/a_{11})(a'_{32}/a'_{22}) - a_{31}/a_{11}, a_{32} = -a'_{32}/a'_{22}$

The inverse can now be obtained easily, since the back-substitution process with each column of the matrix I will yield the corresponding column of A^{-1} where I is given by.

Example: We shall consider again the system given we have here

$A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ the augmented system is

$$\begin{bmatrix} 2 & 1 & 1{:}1 & 0 & 0 \\ 3 & 2 & 3{:}0 & 1 & 0 \\ 1 & 4 & 9{:}0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 1: & 1 & 0 & 0 \\ 0 & 1/2 & 3/2: & -3/2 & 1 & 0 \\ 0 & 0 & 17/2: & -1/2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1: & 1 & 0 & 0 \\ 0 & 1/2 & 3/2: & -3/2 & 1 & 0 \\ 0 & 0 & -2: & 10 & -7 & 1 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & 3/2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ -3/2 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & 3/2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -7 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & 3/2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \qquad x = \begin{bmatrix} -3 & 5/2 & 1/2 \\ 12 & -15/2 & 3/2 \\ -5 & 7/2 & -1/2 \end{bmatrix}$$

2.2 Numerical Eigen value
2.2.1 The Power Method

Like the Jacobi and Gauss-Seidel methods, the power method for approximating eigenvalues is iterative. First we assume that the matrix A has a dominant eigenvalue with corresponding dominant eigenvectors. Then we choose an initial approximation x_0 of one of the dominant eigenvectors of A. This initial approximation must be a nonzero vector in R^n. Finally we form the sequence give by

$$x_1 = Ax_0$$

$$x_2 = Ax_1 = A(Ax_0) = A^2x_0$$

$$x_3 = Ax_2 = A(A^2x_0) = A^3x_0$$

$$\vdots$$

$$x_k = Ax_{k-1} = A(A^{k-1}x_0) = A^kx_0$$

For large powers of k, and by properly scaling this sequence, we will see that we obtain a good approximation of the dominant eigenvector of A. This procedure is illustrated in the following example.

Example

Complete six iterations of the power method to approximate a dominant eigenvector of $A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$

Solution: We begin with an initial nonzero approximation of $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

We then obtain the following approximations.

$$x_1 = Ax_0 = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ -4 \end{bmatrix} \rightarrow -4\begin{bmatrix} 2.50 \\ 1 \end{bmatrix}$$

$$x_2 = Ax_1 = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} -10 \\ 4 \end{bmatrix} = \begin{bmatrix} 28 \\ 10 \end{bmatrix} \rightarrow 10\begin{bmatrix} 2.80 \\ 1 \end{bmatrix}$$

$$x_3 = Ax_2 = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 28 \\ 10 \end{bmatrix} = \begin{bmatrix} -64 \\ -22 \end{bmatrix} \rightarrow -22\begin{bmatrix} 2.91 \\ 1 \end{bmatrix}$$

$$x_4 = Ax_3 = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} -64 \\ -22 \end{bmatrix} = \begin{bmatrix} 136 \\ 46 \end{bmatrix} \rightarrow 46\begin{bmatrix} 2.96 \\ 1 \end{bmatrix}$$

$$x_5 = Ax_4 = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 136 \\ 46 \end{bmatrix} = \begin{bmatrix} -280 \\ -94 \end{bmatrix} \rightarrow -94\begin{bmatrix} 2.98 \\ 1 \end{bmatrix}$$

$$x_5 = Ax_4 = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} -280 \\ 94 \end{bmatrix} = \begin{bmatrix} 568 \\ 190 \end{bmatrix} \rightarrow 190\begin{bmatrix} 2.99 \\ 1 \end{bmatrix}$$

Note that the approximations in Example appear to be approaching scalar multiples of $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ which we know from Example is a dominant eigenvector of the matrix

$A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$ In Example the power method was used to approximate a dominant eigenvector of the matrix A. In that example we already knew that the dominant eigenvalue of A was For the sake of demonstration, however, let us assume that we do not know the dominant eigenvalue of A. The following theorem provides a formula for determining the eigenvalue corresponding to a given eigenvector.

References

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پوختە

چارەسەرکردنی سیستەمی هێڵی هاوکێشە بارودۆخێکی باوە لە چەندین کێشەی زانستی و تەکنۆلۆژیدا. چەندین ڕێگای شیکاری یان ژمارەیی پێشکەوتوو هەن بۆ چارەسەرکردنیان، بۆیە لەم راپۆرتەدا چەند ڕێگایەکم بۆ شیکارکردنی سیستەمی هێڵی هاوکێشەکان باسکرد.

بۆ ئەمە پێویستە هەندێک چەمک پێناسە بکەین. بە شێوەیەکی گشتی باسی ماتریکسی سێگۆشەیی و ناتاک دەکەین، ئێمە پێناسەمان کردووە، ئێمە نموونەمان وەرگرتووە. ماتریکس وەک ڕێگایەکی گشتی کە زۆر بەکاردێت لە جەبری هێڵییدا بریتییە لە لابردنی گاویس، گاوس جۆردن. لە م بابە تە دا ئە مانە بە نموونە یە ک دە گە یە نم. هەروەها لەم راپۆرتەدا شێوازی LU ڕوون دەکەمەوە بۆ دروستکردنی ماتریکسی سێگۆشەیی. لە کۆتاییدا، من باس لە شێوازی هێز دەکەم بۆ دۆزینەوەی بەهای ژمارەیی.

خلاصه

حل النظام الخطي للمعادلة هو موقف شائع في العديد من المشاكل العلمية والتكنولوجية. تم تطوير العديد من الطرق سواء التحليلية أو العددية لحلها ، لذلك ، في هذه التقرير أعلنت بعض الطرق لحل نظام المعادلات الخطي لهذا نحتاج إلى تعريف بعض المفاهيم.

تحدثنا بشكل عام عن المصفوفات المثلثة وغير المفردة التي حددناها ، لقد أخذنا أمثلة. مصفوفة مثل الطريقة العامة الأكثر استخداما في الجبر الخطي هي إزالة غاوس ، غاوس الأردن. في هذه الورقة سأشرح هذه من خلال أخذ مثال. أيضا في هذه الورقة سوف أشرح طريقة تحلل LU لبناء مصفوفة مثلثة. أخيرا ، سوف أستفسر عن طريقة الطاقة لإيجاد القيم الذاتية العددية.