State Space Definition

• Steps of control system design.
– Modeling: Equation of motion of the system
– Analysis: test system behavior
– Design: design a controller to achieve the required specification
– Implementation: Build the designed controller – Validation and tuning: test the overall sysetm

• In SS :Modeling, analysis and design in time domain

SS-Definition

- In the classical control theory, the system model is represented by a transfer function
- The analysis and control tool is based on classical methods such as root locus and Bode plot

• It is restricted to single input/single output system

- It depends only the information of input and output and it does not use any knowledge of the interior structure of the plant,
- It allows only limited control of the closed-loop behavior using feedback control is used
- Modern control theory solves many of the limitations by using a much "richer" description of the plant dynamics.
- The so-called state-space description provide the dynamics as a set of coupled first-order differential equations in a set of internal variables known as state variables, together with a set of algebraic equations that combine the state variables into physical output variables.

- The Philosophy of SS based on transforming the equation of motions of order n (highest derivative order) into an n equation of 1storder
- State variable represents storage element in the system which leads to derivative equation between its input and output; it

  could be a physical or mathematical variables

- # of state=#of storage elements=order of the system

- For example if a system is represented by

$$\frac{d^3y}{dt^3} + 7\frac{d^2y}{dt^2} + 19\frac{dy}{dt} + 13y = 13\frac{du}{dt} + 26u$$

- This system of order 3 then it has 3 state and 3 storage elements
- The concept of the state of a dynamic system refers to a minimum set of variables, known as state variables, that fully describe the system and its response to any given set of inputs
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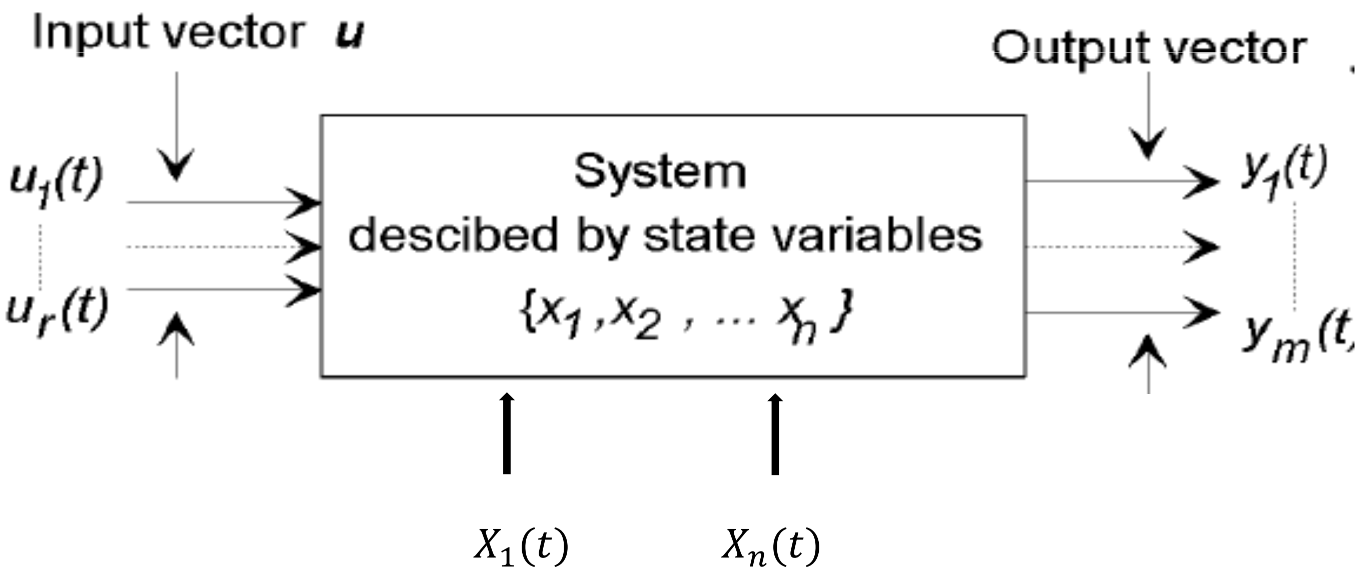

The state variables are an internal description of the system which completely characterize the system state at any time t, and from which any output variables yi(t) may be computed.

The State Equations

A standard form for the state equations is used throughout system dynamics. In the standard form the mathematical description of the system is expressed as a set of n coupled first-order ordinary differential equations, known as the state equations, in which the time derivative of each state variable is expressed in terms of the state variables x1(t), . . . , xn(t) and the system inputs u1(t), . . . , ur(t).

$$
\begin{aligned}
\dot{x}_1 &= f_1(\mathbf{x}, \mathbf{u}, t) \\
\dot{x}_2 &= f_2(\mathbf{x}, \mathbf{u}, t) \\
\vdots &= \vdots \\
\dot{x}_n &= f_n(\mathbf{x}, \mathbf{u}, t)
\end{aligned}
$$

It is common to express the state equations in a vector form, in which the set of n state variables is written as a state vector x(t) = $[x_1(t), x_2(t), \ldots, x_n(t)]^T$, and the set of r inputs is written as an input vector u(t) = $[u_1(t), u_2(t), \ldots, u_r(t)]^T$. Each state variable is a time varying component of the column vector x(t).

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)\ .$$

where $\mathbf{f}(\mathbf{x}, \mathbf{u}, t)$ is a *vector* function with n components $f_i(\mathbf{x}, \mathbf{u}, t)$.

In this note we restrict attention primarily to a description of systems that are linear and time-invariant (LTI), that is systems described by linear differential equations with constant coefficients.

$$
\begin{array}{ccccccccccccccccc}
\dot{x}_1 & = & a_{11}x_1 & + & a_{12}x_2 & + & \ldots & + & a_{1n}x_n & + & b_{11}u_1 & + & \ldots & + & b_{1r}u_r \\
\dot{x}_2 & = & a_{21}x_1 & + & a_{22}x_2 & + & \ldots & + & a_{2n}x_n & + & b_{21}u_1 & + & \ldots & + & b_{2r}u_r \\
\vdots & & \vdots & & & & & & & & & & & & \\
\dot{x}_n & = & a_{n1}x_1 & + & a_{n2}x_2 & + & \ldots & + & a_{nn}x_n & + & b_{n1}u_1 & + & \ldots & + & b_{nr}u_r
\end{array}
$$

$$\begin{array}{ccccccccccccccc} \dot{x}_1 & = & a_{11}x_1 & + & a_{12}x_2 & + & \ldots & + & a_{1n}x_n & + & b_{11}u_1 & + & \ldots & + & b_{1r}u_r \\ \dot{x}_2 & = & a_{21}x_1 & + & a_{22}x_2 & + & \ldots & + & a_{2n}x_n & + & b_{21}u_1 & + & \ldots & + & b_{2r}u_r \\ \vdots & & \vdots & & & & & & & & & & & & \\ \dot{x}_n & = & a_{n1}x_1 & + & a_{n2}x_2 & + & \ldots & + & a_{nn}x_n & + & b_{n1}u_1 & + & \ldots & + & b_{nr}u_r \end{array}$$

where the coefficients aij and bij are constants that describe the system. This set of n equations defines the derivatives of the state variables to be a weighted sum of the state variables and the system inputs.

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & \ldots & b_{1r} \\ b_{21} & & b_{2r} \\ \vdots & & \vdots \\ b_{n1} & \ldots & b_{nr} \end{bmatrix}\begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

where the state vector x is a column vector of length n, the input vector u is a column vector of length r, A is an n × n square matrix of the constant coefficients aij, and B is an n × r matrix of the coefficients bij that weight the inputs.
A system output is defined to be any system variable of interest. A description of a physical system in terms of a set of state variables does not necessarily include all of the variables of direct engineering interest.
An important property of the linear state equation description is that all system variables may be represented by a linear combination of the state variables xi and the system inputs ui.
An arbitrary output variable in a system of order n with r inputs may be written: