$$y(t) = c_1x_1 + c_2x_2 + \ldots + c_nx_n + d_1u_1 + \ldots + d_ru_r$$

$$\begin{array}{ccccccccccccccc}
y_1 & = & c_{11}x_1 & + & c_{12}x_2 & + & \ldots & + & c_{1n}x_n & + & d_{11}u_1 & + & \ldots & + & d_{1r}u_r \\
y_2 & = & c_{21}x_1 & + & c_{22}x_2 & + & \ldots & + & c_{2n}x_n & + & d_{21}u_1 & + & \ldots & + & d_{2r}u_r \\
\vdots & & \vdots & & & & & & & & & & & & \\
y_m & = & c_{m1}x_1 & + & c_{m2}x_2 & + & \ldots & + & c_{mn}x_n & + & d_{m1}u_1 & + & \ldots & + & d_{mr}u_r
\end{array}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \ldots & c_{1n} \\ c_{21} & c_{22} & \ldots & c_{2n} \\ \vdots & & & \vdots \\ c_{m1} & c_{m2} & \ldots & c_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & \ldots & d_{1r} \\ d_{21} & & d_{2r} \\ \vdots & & \vdots \\ d_{m1} & \ldots & d_{mr} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

where y is a column vector of the output variables $y_i(t)$, C is an m×n matrix of the constant coefficients $c_{ij}$ that weight the state variables, and D is an m × r matrix of the constant coefficients $d_{ij}$ that weight the system inputs. For many physical systems the matrix D is the null matrix, and the output equation reduces to a simple weighted combination of the state variables:

$$\mathbf{y} = \mathbf{Cx}.$$

Definition:-

State Space model for a generic 2nd Order ODE:-

$$ku = a\frac{dy^2}{dt} + b\frac{dy}{dt} + cy$$

$$x_1 = y \quad , \quad \dot{x}_1 = x_2 = \frac{dy}{dt} \quad , \quad \dot{x}_2 = \frac{dy^2}{dt^2}$$

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{X}} = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{-b}{a} & \frac{-c}{a} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} \frac{k}{a} \\ 0 \end{bmatrix}}_{B} \underbrace{u}_{u}$$

$$Ku = a_n \underbrace{\frac{d^n y}{dt^n}}_{X_{n+1}} + a_{n-1}\frac{d^{n-1} x}{dt^{n-1}} + \dots + a_1 \underbrace{\frac{dy}{dt}}_{X_2} + a_0 \underbrace{y}_{X_1}$$

$$ku = a_n\dot{X}_n + \dots\dots a_3\dot{X}_3 + a_2\dot{X}_2 + a_1\dot{X}_1 + a_0 X_1$$

$$\begin{bmatrix}\dot X_1\\ \vdots \\ \vdots \\ \dot X_n\end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & & & \\ \vdots & & & \ddots & & \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & & & & +\frac{a_n}{a_n}\end{bmatrix} \begin{bmatrix}X_1\\ \\ \\ X_n\end{bmatrix} + \begin{bmatrix} -\frac{b}{a_n}\\ \\ \\ -\frac{z}{a_n}\end{bmatrix} u$$

Ex: $4^{th}$ order state space model.

$$Ku = a_4\frac{d^4y}{dt^4} + a_3\frac{d^3y}{dt^3} + a_2\frac{d^2y}{dt^2} + a_1\frac{dy}{dt} + ay$$

$X_1 = y$

$X_2 = \frac{dX_1}{dt} = \dot X_1$

$X_3 = \frac{dX_2}{dt} = \dot X_2$

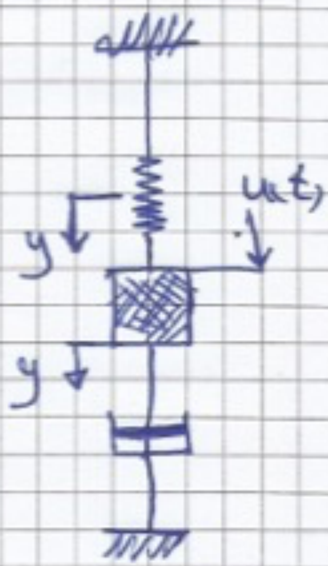
$X_4 = \frac{dX^3}{dt} = \dot X_3$

$$Ku = a_4\dot X_4 + a_3X_4 + a_2X_3 + a_1X_2 + aX_1$$

$$\begin{bmatrix}\dot X_1\\ \dot X_2\\ \dot X_3\\ \dot X_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\frac{a}{a_4} & -\frac{a_1}{a_4} & -\frac{a_2}{a_4} & -\frac{a_3}{a_4}\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ \frac{k}{a_4}\end{bmatrix} u$$

$$y = \underbrace{\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}}_{c} \underbrace{\begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix}}_{X}$$

Ex$_1$:- Obtain the S.S.E for the shown System.




$F_s = ky \uparrow$

$F_d = By \uparrow$

$\sum F = m.a$

$u(t) - ky - B\dot{y} = m\ddot{y}$

∴ $m\ddot{y} + B\dot{y} + ky = u(t)$

* step one:-

$$\frac{d^2y}{dt^2} + \frac{B}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{1}{m}u(t) \quad -①$$

* (the shape should the equation bein)

* step two:-

$x_1 = y$ , $x_2 = \dot{y}$

$x_2 = \dot{x}_1$ , $\dot{x}_2 = \ddot{y}$

For the input. $\dot{X} = AX + BU$

$$\dot{X}_2 + \frac{B}{m} X_2 + \frac{K}{m} X_1 = \frac{1}{m} U(t)$$

$$\dot{X}_2 = \frac{-B}{m} X_2 - \frac{K}{m} X_1 + \frac{1}{m} U(t) \quad \text{--- (1)}$$

$$\dot{X}_1 = X_2 \quad \text{--- (2)}$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{B}{m} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} U(t)$$

For the out put $\quad y = CX + DU$

from $\Rightarrow X_1 = y$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + [0]\, U(t)$$