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**Department of Mathematics**

**College of Science**

**University of Salahaddin-Erbil**

**Subject: Multivariable Calculus - First semesters**

**Course Book : Second Year Mathematics**

**Lecturer's name : Assist. Prof. Dr. Ahmed Muhammad**

**Academic Year: 2022-2023**

**Course Book**

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| **1. Course name** | Multivariable Calculus | |
| **2. Lecturer in charge** | Ahmed Muhammad | |
| **3. Department/ College** | Mathematics/Science | |
| **4. Contact** | e-mail: ahmedrange82@gmail.com  Tel: 0750 477 31 16 | |
| **5. Time (in hours) per week** | Theory: 3  tutorial: 1 | |
| **6. Office hours** | Monday 9-00-10:00, Tuesday 11:30-12:30 and Thursday 10-00-11:30 | |
| **7. Course code** |  | |
| **8. Teacher's academic profile** | **Education:**  2014 : PhD in Mathematics, School of Mathematics, Cardiff University, UK  Thesis title:  Approximation of Quadratic Numerical Range of Block Operator Matrices.  2004: M.Sc in Mathematics, Department of Mathematics, College of Science, Salahaddin University.  Thesis title:  Some generalizations of Numerical Range of operators on a complex Hilbert.  **Research interests**  My research lie in the fields of functional analysis, operator theory and spectral theory. I am particularly interested in numerical range, q-numerical range, c-numerical range, numerical range of matrix polynomials, quadratic numerical range and computation of quadratic numerical range of differential operators and spectral pollution.  One of the main achievements of my research to date has been the absence of quadratic numerical range-pollution, which is new result in operator theory because in general, discretization of differential operators may result in spectral pollution. I have shown that the proof  of this does not happen for finite difference discretizations of the Hain-L\"{u}st operator is a little more tricky than proving that every point of the quadratic numerical range can be approximated. I require the following concept, which has not been introduced before, and which i expect will be useful in many contexts when dealing with unbounded block operator matrices.  **Work experience:**  04-09-2002:  Assistant lecturer at Department of Mathematics, College of Science, University of Salahaddin- Erbil, Iraq.  6-11-2002 to 28-06-2008:  Deputy of Department of Mathematics, College of Science,Salahaddin University-Erbil, Iraq.  18-10-2005:  Lecturer , Department of Mathematics, College of Science, University of Salahaddin- Erbil, Iraq.  2004 to 28-06-2008 :  Member of the Scientific and Higher study committee of the department of mathematics.  2003-2006 :  Member of final examination committee of B.Sc of the college.  2007 :  Member of final examination committee of M.Sc and PhD. of the college.  2008 :  Member of final examination committee of B.Sc of the college.  01-10-2008 up to 2012:  I was PhD student at Cardiff School of Mathematics- Cardiff University, UK.  10-06-2012:  I have got PhD in Mathematics, School of Mathematics, Cardiff University, UK.  2014 :  Member of final examination committee of B.Sc of the college.  16-03-2022  Assistant Professor , Department of Mathematics, College of Science, University of Salahaddin- Erbil, Iraq.  **Supervision of Master theses:**   1. 01/07/2021- present : Darawan Zrar Muhammad, current Master student; Thesis titled On the computing   S-numerical range of polynomial operator matrices .   1. 01/4/2018-12/3/2020 : Berivan Faris   Master Thesis titled Approximation of S- numerical range of operator matrices with a pplications.   1. 15/11/2015-27/4/2017 : Walat Jalal Hamad,   Master Thesis titled Approximation of generalized numerical range of operator matrices with a pplications.   1. 01/09/2014-13/12/2015: Fiza Abdullah Shareef, Master Thesis titled Approximation of q-numerical range of operator matrices with a pplications.   **Teaching Experience:**  I have taught a variety of undergraduate courses in calculus, multivariable calculus, foundation of mathematics, fundamental of mathematics, linear algebra, differential equations, linear programming, abstract algebra, real analysis, functional analysis and complex analysis, as well as graduate courses in operator theory and functional analysis.  **Publications**:   1. A. Muhammad, Approximation of numerical range of polynomial operators. Journal of Operator and   Matrices 15 (2021), 1073-1087.   1. A. Muhammad and F. Shareef, Computing the q-numerical range of di\_erential operators. Hindawi   Journal of Applied Mathematics Vol. 2020, Article, ID 6584805, 12 pages, (2020).   1. A. Muhammad and F. Shareef, A numerical investigation of q-numerical range of operator matrices. Far   east Journal of mathematical science 125 (2020), 1-33.  4- W. Jalal and A. Muhammad, Elliptic numerical range of  matrices. International mathematical Forum 15  (2020), 293-315.   1. B. Azeez and A. Muhammad, Some results on S-numerical   range of operator matrices. Zanko Journal of  pure and Applied science 32 (2020), 57-63.   1. A. Muhammad and W. Jalal, A numerical investigation of the c-numerical range of di\_erential operator.   Bulletin of Iranian mathematical society 45 (2019), 1755-1775.   1. A. Muhammad and M. Marletta, A numerical investigation of the quadratic numerical range of Hain-L\"{u}st operator. International Journal of Computer Mathematics 90 (2013), 2431-2451. 2. A. Muhammad and M. Marletta, Approximation of quadratic numerical range of block operator matrices. Integral Equation and Operator Theory 74 (2012), 151-162. 3. A. Muhammad and M. Marletta, Computation of boundary of quadratic numerical range (Approved to be published). 4. A. Muhammad, Joint Numerical Range of matrix polynomials. Al-Rafiden Journal of Computer Science and Mathematics, Vol.6 No.2 (2009), p.129-136. 5. A .Muhammad, The Numerical range of 6 by 6 Irreducible matrix, Al-Rafiden Journal of Computer Science and Mathematics, Vol.4 No.2 (2007), p.89-98. 6. A. Muhammad, The line segment on the boundary of Numerical range, Zanco, The Scientific journal of Salahaddin University-Erbil, Vol.17 No.1 (2005), p.105-111. 7. A. Muhammad, Elliptical Range of an n-tuple operators on a complex Hilbert space, Zanco, The Scientific journal of Salahaddin University-Erbil, Vol.17 No.1 (2005), p. 113-117. | |
| **9. Keywords** | **Three dimensional space, function of two variables, limit, continuity, partial derivative, directional derivative, Lagrange multiplier, double integral.** | |
| **10. Course overview:**  Most of the lectures will correspond to particular sections of the books, and studying the books will be very helpful. However, material will often be presented in a different order or from a different perspective, and we'll occasionally discuss topics which are not in the books at all. Thus it is important to attend class and, since you shouldn't expect to understand everything right away, you are strongly encouraged to take notes. | | |
| **11. Course objective:**  This course introduces students to the basic knowledge of the followings:  Sequences, Notation and Formulas for Sequences, convergence and divergence Sequences,  Bounded Sequences, monotonic Sequences, sub Sequences, Series – The Idea and Notation,  Convergence Series, geometric Series, divergence Series, p- Series, integral Series, comparison  test, limit of comparison Test, alternating series, absolute and conditional convergence, raito test,  root test, power series, interval and radius of convergence of power series, Taylor series, Maclaurin series.  Three coordinates system, function of multivariable, limit of function of two variable, continuity, partial derivative, directional derivative, maximum and minimum values, Lagrange multiplier, double integrals. | | |
| **12. Student's obligation**  When preparing your homework, please keep the following in mind:   1. You are encouraged to discuss the homework problems with your classmates. The best way to learn is to think hard about a problem on your own until you get really stuck or solve it, then ask someone else how they thought about it. However, when it comes to writing down your solutions, you must do this by yourself, in your own words, without looking at someone else's paper or any other source. 2. Your answers should be written in complete sentences which explain the logic of what you are doing. For example, x^2=4, x=2, x=-2 is not understandable: instead, write since x^2=4, it follows that x=2 or x=-2. If your proof is unreadable it will not receive credit. Also, results of calculations and answers to true/false questions should always be justified.   Proofs should be complete and detailed. The proofs in the book provide good models, but when in doubt, explain more. You can of course cite theorems that we have already proved in class or from the book. | | |
| **13. Forms of teaching**  Whiteboardwithmajek**.** | | |
| **14. Assessment schem**..   1. Midterm exam s and other activities %30 2. Tutorial and quiz exams %10 3. Final exam %60 | | |
| **15. Student learning outcome:**  One successful completion of the course, students should be able to learn and understand the key definitions, remembering all the axioms and the same way it is necessary to learn the statements of the theorems; it is not necessary to memorize their proofs, however the more you understand and the better your command of the material will be. A useful study aid is to try and summarize the key ideas in the proof in a sentence or two;  The material in this course is cumulative (builds upon previous chapters) and gets somewhat harder, so it is essential that you do not fall behind.  A key to understanding is to ask your own questions. What is a good example? Why is such and such assumption necessary in a theorem, what happens if I drop it? Does this property imply that property, or is there a counterexample?  If you get stuck on something or are confused by a particular concept, you are encouraged to come to my office hours. I will be happy to discuss it with you. However, the more thought you have put into it beforehand, the more productive the discussion is likely to be. | | |
| **16. Course Reading List and References‌:**   1. Stanley I. Grossman: Calculus , New York, London 1986. 2. Robert Wrede and Murray R. Spiegel. Schum's outline of theory and problem's of advanced calculus, second edition. New York, London 1963   Those are very readable books on the subject, containing lots of good exercises and examples**.** | | |
| **17. The Topics:** | | **Lecturer's name** |
| **The following is the plan for the course.**  **Chapter One:**  Sequences, Notation and Formulas for Sequences, convergence and divergence Sequences,  Bounded Sequences, monotonic Sequences, sub Sequences, Series – The Idea and Notation,  Convergence Series, geometric Series, divergence Series, p- Series, integral Series, comparison  test, limit of comparison Test, alternating series, absolute and conditional convergence, raito test,  root test, power series, interval and radius of convergence of power series, Taylor series, Maclaurin series.  **Chapter Two:**  Three dimensional system (3-D) :  We 'll start the chapter off with a fairly short discussion introducing the 3-D coordinate system and the conventions that we'll be using. We will also take a brief look at how the different systems can be change the graph of an equation.  **Chapter Three**  In calculus we concentrated on functions of one variable. In Advanced calculus we will extend our knowledge of calculus into functions of two or more variables.  Here is a list of topics in this chapter:   1. **Limit** :   taking limit of function of several variables.   1. **Continuity:**   In this section we will test some function for continuity.  A function will be continuous at a point if the graph doesn't have any holes or breaks at that points.   1. **Partial derivative** :   in this section we will introduced the idea of partial derivatives as well as the standard notations and how to compute them.   1. **Higher order partial derivatives** :   we will take a look at higher order partial derivative in this section.   1. **Differentials** :   In this section we extend the idea of differentials to functions of several variables.   1. **Chain Rule** :   Here we will look at the chain rule for functions of several variables.   1. **Directional derivatives**:   we will introduce the concept of directional derivatives in this section. We will also see how to compute them and see a couple of nice facts pertaining to directional derivatives.   1. **Maximum and minimum values**. 2. **Lagrange Multipliers** .     **Chapter Four:**  **Multiple integrals:**  Here is a list of topics covered in this chapter:  Double integrals: we will define double integral in this section.   1. **Double integrals over rectangular regions:**   In this section we will start looking how we actually compute double integrals.   1. **Double integrals over general regions:**   Here we will look at the most general double integrals.   1. **Double integrals in polar coordinates :**   In this section we will take a look at evaluating double integrals using polar coordinates.  **Chapter One:**  **Triple integrals:**  Here we will define the triple integral as well as how we evaluate them.   1. **Triple integrals in cylindrical coordinates**:   we will evaluate triple integrals using cylindrical coordinates in this sections.   1. **Triple integrals in spherical coordinates** :   in this section we will evaluate triple integrals using spherical coordinates.  **Chapter Two:**  **Line integrals:**  Here is a list of topics covered in this chapter:   1. **Vector fields:**   in this section we introduce the concepts of a vector fields.   1. **Line integrals – part I** :   Here we will start looking at line integrals . In particular we will look at line integrals with respect to arc length.   1. **Line integrals – part II :**   we will continue looking at line integrals in this section . Here we will be looking at line integrals with respect to x,y and / or z.   1. **Line integrals of vector fields:**   Here we will look at a third type of line integrals , line integrals of vector fields.   1. **Fundamental Theorem for line Integrals**:   In this section we will look at a version of the fundamental theorem of Calculus for line integrals of vector fields.   1. **Conservative vector fields:**   Here we will take a somewhat Detailed look at conservative vector fields and how to find potential functions.   1. **Green's Theorem :**   We will give Green's theorem as well as an interesting application of Green's theorem.   1. **Curl and Divergence** :   In this section we will introduce the concepts of curl and the divergence of a vector field. We will also give two vector forms of Green's theorem. | | Unfortunately timetables of holidays  Will be change that is why I cannot determine a week by week review of this topics. |
| **18. Practical Topics (If there is any)** | |  |
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| **19. Examinations:**  In general we ask students in exam questions to define some notations and used it to solve problems, and we ask other types of questions like briefly mathematical proofs for  True mathematical statements. | | |
| **20. Extra notes:**  Answers of examination will be find in the board’s declaration Mathematics Department after every examination. | | |
| **21. Peer review** | | |