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**Department of Mathematics**

**College of Science**

**University of Salahaddin-Erbil**

**Subject: Functional Analysis- First semesters**

**Course Book : Fourth Year Mathematics**

**Lecturer's name : Assist. Prof. Dr. Ahmed Muhammad**

**Academic Year: 2022-2023**

**Course Book**

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| **1. Course name** | Functional Analysis |
| **2. Lecturer in charge** | Ahmed Muhammad  |
| **3. Department/ College** | Mathematics/Science |
| **4. Contact** | e-mail: ahmed.muhammad@su.edu.krdTel: 0750 477 31 16 |
| **5. Time (in hours) per week**  | Theory: 3tutorial: -  |
| **6. Office hours** | Sunday 10-30-12:00 , Tuesday 11-30-12:30 and Thursday 10: 00-12:00 |
| **7. Course code** |  |
| **8. Teacher's academic profile**  | **Education:**2014 : PhD in Mathematics, School of Mathematics, Cardiff University, UK Thesis title: Approximation of Quadratic Numerical Range of Block Operator Matrices.2004: M.Sc in Mathematics, Department of Mathematics, College of Science, Salahaddin University. Thesis title:  Some generalizations of Numerical Range of operators on a complex Hilbert.**Research interests**My research lie in the fields of functional analysis, operator theory and spectral theory. I am particularly interested in numerical range, q-numerical range, c-numerical range, numerical range of matrix polynomials, quadratic numerical range and computation of quadratic numerical range of differential operators and spectral pollution.One of the main achievements of my research to date has been the absence of quadratic numerical range-pollution, which is new result in operator theory because in general, discretization of differential operators may result in spectral pollution. I have shown that the proofof this does not happen for finite difference discretizations of the Hain-L\"{u}st operator is a little more tricky than proving that every point of the quadratic numerical range can be approximated. I require the following concept, which has not been introduced before, and which i expect will be useful in many contexts when dealing with unbounded block operator matrices.**Work experience:**04-09-2002: Assistant lecturer at Department of Mathematics, College of Science, University of Salahaddin- Erbil, Iraq.6-11-2002 to 28-06-2008: Deputy of Department of Mathematics, College of Science,Salahaddin University-Erbil, Iraq.18-10-2005: Lecturer , Department of Mathematics, College of Science, University of Salahaddin- Erbil, Iraq.2004 to 28-06-2008 : Member of the Scientific and Higher study committee of the department of mathematics.2003-2006 : Member of final examination committee of B.Sc of the college.2007 : Member of final examination committee of M.Sc and PhD. of the college.2008 : Member of final examination committee of B.Sc of the college.01-10-2008 up to 2012:  I was PhD student at Cardiff School of Mathematics- Cardiff University, UK.10-06-2012: I have got PhD in Mathematics, School of Mathematics, Cardiff University, UK.2014 : Member of final examination committee of B.Sc of the college.16-03-2022Assistant Professor , Department of Mathematics, College of Science, University of Salahaddin- Erbil, Iraq.**Supervision of Master theses:**1. 01/07/2021- present : Darawan Zrar Muhammad, current Master student; Thesis titled On the computing

S-numerical range of polynomial operator matrices .1. 01/4/2018-12/3/2020 : Berivan Faris

Master Thesis titled Approximation of S- numerical range of operator matrices with a pplications.1. 15/11/2015-27/4/2017 : Walat Jalal Hamad,

Master Thesis titled Approximation of generalized numerical range of operator matrices with a pplications.1. 01/09/2014-13/12/2015: Fiza Abdullah Shareef, Master Thesis titled Approximation of q-numerical range of operator matrices with a pplications.

**Teaching Experience:**I have taught a variety of undergraduate courses in calculus, multivariable calculus, foundation of mathematics, fundamental of mathematics, linear algebra, differential equations, linear programming, abstract algebra, real analysis, functional analysis and complex analysis, as well as graduate courses in operator theory and functional analysis.**Teaching Experience:**I have taught a variety of undergraduate courses in calculus, multivariable calculus, foundation of mathematics, fundamental of mathematics, linear algebra, differential equations, linear programming, abstract algebra, real analysis, functional analysis and complex analysis, as well as graduate courses in operator theory and functional analysis.**Publications**:1. A. Muhammad, Approximation of numerical range of polynomial operators. Journal of Operator and

Matrices 15 (2021), 1073-1087.1. A. Muhammad and F. Shareef, Computing the q-numerical range of di\_erential operators. Hindawi

Journal of Applied Mathematics Vol. 2020, Article, ID 6584805, 12 pages, (2020).1. A. Muhammad and F. Shareef, A numerical investigation of q-numerical range of operator matrices. Far

 east Journal of mathematical science 125 (2020), 1-33. 4- W. Jalal and A. Muhammad, Elliptic numerical range of  matrices. International mathematical Forum 15 (2020), 293-315.1. B. Azeez and A. Muhammad, Some results on S-numerical

 range of operator matrices. Zanko Journal of pure and Applied science 32 (2020), 57-63.1. A. Muhammad and W. Jalal, A numerical investigation of the c-numerical range of di\_erential operator.

Bulletin of Iranian mathematical society 45 (2019), 1755-1775.1. A. Muhammad and M. Marletta, A numerical investigation of the quadratic numerical range of Hain-L\"{u}st operator. International Journal of Computer Mathematics 90 (2013), 2431-2451.
2. A. Muhammad and M. Marletta, Approximation of quadratic numerical range of block operator matrices. Integral Equation and Operator Theory 74 (2012), 151-162.
3. A. Muhammad and M. Marletta, Computation of boundary of quadratic numerical range (Approved to be published).
4. A. Muhammad, Joint Numerical Range of matrix polynomials. Al-Rafiden Journal of Computer Science and Mathematics, Vol.6 No.2 (2009), p.129-136.
5. A .Muhammad, The Numerical range of 6 by 6 Irreducible matrix, Al-Rafiden Journal of Computer Science and Mathematics, Vol.4 No.2 (2007), p.89-98.
6. A. Muhammad, The line segment on the boundary of Numerical range, Zanco, The Scientific journal of Salahaddin University-Erbil, Vol.17 No.1 (2005), p.105-111.
7. A. Muhammad, Elliptical Range of an n-tuple operators on a complex Hilbert space, Zanco, The Scientific journal of Salahaddin University-Erbil, Vol.17 No.1 (2005), p. 113-117.

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| **9. Keywords** | Normed linear space, Banach space, bounded linear operator, closed graph, inner product space, Hilbert space , bounded linear functional, compact operator, spectrum, resolvent set and spectral mapping theorem. |
| **10. Course overview:** Most of the lectures will correspond to particular sections of the books, and studying the books will be very helpful. However, material will often be presented in a different order or from a different perspective, and we'll occasionally discuss topics which are not in the books at all. Thus it is important to attend class and, since you shouldn't expect to understand everything right away, you are strongly encouraged to take notes.  |
| **11. Course objective:**This course introduces students to the basic knowledge of linear functional analysis, an important branch of modern analysis.In addition to these specific topics, an important goal of the course is to acquire more familiarity with abstract mathematical reasoning and proofs in general, as a transition to more advanced mathematical courses. |
| **12. Student's obligation**When preparing your homework, please keep the following in mind:1. You are encouraged to discuss the homework problems with your classmates. The best way to learn is to think hard about a problem on your own until you get really stuck or solve it, then ask someone else how they thought about it. However, when it comes to writing down your solutions, you must do this by yourself, in your own words, without looking at someone else's paper or any other source.
2. Your answers should be written in complete sentences which explain the logic of what you are doing. For example, x^2=4, x=2, x=-2 is not understandable: instead, write since x^2=4, it follows that x=2 or x=-2. If your proof is unreadable it will not receive credit. Also, results of calculations and answers to true/false questions should always be justified.

 Proofs should be complete and detailed. The proofs in the book provide good models, but when in doubt, explain more. You can of course cite theorems that we have already proved in class or from the book. |
| **13. Forms of teaching**Whiteboardwithmajek. |
| **14. Assessment schem**..1. Midterm exam s and other activities %30
2. Tutorial and quiz exams %10
3. Final exam %60
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| **15. Student learning outcome:**On successful completion of the course, students should be able to compare and contrast1. finite and infinite dimensional linear spaces.
2. complete and incomplete linear space.
3. normed and inner product spaces.

In particular, recognize the importance of completeness and discuss how vectors are represented in these spaces; understand the notions of Banach spaces and Hilbert Spaces. State and apply fundamental theorems in these spaces; discuss the dual spacesof some standard Banach spaces; discuss the boundedness of linear operators and the spectra of special linear operators; apply functional analysis in the study of differential equations and optimization problems.As with any upper division math course, it is essential to thoroughly learn and understand the key definitions, remembering all the axioms. If you don't know exactly what a Banach space or a Hilbert space then you have no hope of proving that something is or isn't a Banach or a Hilbert space.In the same way it is necessary to learn the statements of the theorems; it is not necessary to memorize their proofs, however the more you understand and the better your command of the material will be. A useful study aid is to try and summarize the key ideas in the proof in a sentence or two;The material in this course is cumulative (builds upon previous chapters) and gets somewhat harder, so it is essential that you do not fall behind.A key to understanding is to ask your own questions. What is a good example? Why is such and such assumption necessary in a theorem, what happens if I drop it? Does this property imply that property, or is there a counterexample?If you get stuck on something or are confused by a particular concept, you are encouraged to come to my office hours. I will be happy to discuss it with you. However, the more thought you have put into it beforehand, the more productive the discussion is likely to be. |
| **16. Course Reading List and References‌:**1. Erwin Kreyszig: Introductory Functional Analysis with Applications (John-Wiley and Sons, 1978).
2. Angus E. Taylor and David C. Lay: Applications of Functional Analysis and Operator Theory (John-Wiley and Sons, 1986).
3. Angus E. Taylor and David C. Lay: lectures on Functional Analysis and Applications

( World Scientific Publishing Co. Pte. Ltd, 1999).Those are very readable books on the subject, containing lots of good exercises and examples**.**  |
| **17. The Topics:** | **Lecturer's name** |
| **The following is the plan for the course.****Chapter One:**Normed linear spaces - Convergent sequences,- Completeness.Banach spaces- Finite dimensional normed spaces and subspaces.Compactness and finite dimension - Bounded linear operators- Normed spaces of operators, dual space.**Chapter Two:**Compactness and finite dimension - Bounded linear operators- Normed spaces of operators, dual space.**Chapter Three:**Inner product spaces- Hilbert spaces- Orthogonal complements, direct sums- Orthonormal sets and sequences, series related to orthonormal sets and sequences- Total orthonormal sets and sequences.**Chapter Four:** Special polynomials. Riesz's representation theorem- Adjoint operator, self-adjoint, normal and unitary operators- Fundamental theorems for normed and Banach spaces: Hahn-Banach theorem. -Reflexive spaces. Category theorem, uniform boundedness principle. Open mapping theorem- Closed graph theorem.**Chapter Five:** Spectral theory of linear operators | Unfortunately timetables of holidays Will be change that is why I cannot determine a week by week review of this topics. |
| **18. Practical Topics (If there is any)** |  |
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| **19. Examinations:**In general we ask students in exam questions to define some notations and used it to solve problems, for example how to show that the space with it is norm is a Banach space or it is Hilbert space. On the other hand we ask other types of questions like briefly mathematical proofs for True mathematical statements like Theorem, corollary and propositions. |
| **20. Extra notes:**Answers of examination will be find in the board’s declaration Mathematics Department after every examination.  |
| **21. Peer review**   |