Question one

In each of the following; first state whether each given statement is true or false. Explain your answer:

(a) Every bounded sequence is convergent.

(b) If $\{a_n\}$ and $\{b_n\}$ are monotonic sequence then $\{a_n b_n\}$ is monotonic sequence.

Question two

Determine whether the given series are convergent or not. Explain your answer:

(a) $a_n = \sum_{n=3}^{\infty} \frac{n}{n^2+4}$

(b) $b_n = \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$

(c) $c_n = \sum_{n=1}^{\infty} \frac{3^n}{n^5}$

Question three

Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n(n+1)}$.

Question four

Find interval of convergence and the sum of the power series as a function of x of $\sum_{n=0}^{\infty} \left(\frac{x^2+1}{3}\right)^n$.

Question five

(a) Use power series to evaluate $\int_{-1}^{0} \frac{ln(x+1)}{x}\, dx$.

(b) Use power series to evaluate $\int_{0}^{1} \frac{e^{x^2}}{x}\, dx$.

Question six

Find and sketch the domain of the function $\psi(x, y) = ln(xy + x - y - 1)$.

Question seven

Consider the function $\psi : D(\psi) \subseteq \mathbb{R}^2 \to \mathbb{R}^2$ given by $\psi(x, y) = \frac{1}{\sqrt{144-4x^2-9y^2}}$:

(a) Find and sketch the domain of $\psi(x, y)$.

(b) Find the interior and the boundary of $\psi(x, y)$.

(c) Is the domain an open region, a closed region, a neither ?

(d) Is the domain bounded or unbounded ?.

Question eight

Show that the following limit

$$\lim_{(x,y)\to(0,0)} \frac{\sin(2x) - 2x + y}{x^3 + y}$$

does not exists:

Question nine

Consider the function $\psi : D(\psi) \subseteq \mathbb{R}^2 \to \mathbb{R}$ given by $\psi(x,y) = x^2+2y$ then prove that $\lim_{(x,y)\to(1,2)} \psi(x,y) = 5$.

Question ten

If $\psi(x,y) = tan^{-1}(\frac{2x}{x^2-y^2})$, then show that

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0.$$

Question eleven

Let $\psi : \mathbb{R}^2 \to \mathbb{R}$ be given by $\psi(x,y) = -x^3 + y^3 - 3x - 12y + 20$. Find all critical points of ψ and classify them.

Question tweleve

Show that the function $u(x,y) = ln(x^2 + y^2)$ satisfies Laplace equation $u_{xx} + u_{yy} = 0$.

Question thirteen

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x,y) = \begin{cases} \frac{(x-1)^2\,(y-2)}{\sqrt{(x-1)^2+(y-2)^2}} & \text{for } (x,y) \neq (1,2) \\ 0 & \text{for } (x,y) = (1,2). \end{cases}$.

Show that the partial derivative $f_x(1,2)$ and $f_y(1,2)$ are exists but $f(x,y)$ is not differentiable at $(1,2)$.

Question fourteen

Consider the function $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ given by $\phi(x,y) = \begin{cases} \frac{x^2+y^2}{|x|+|y|} & \text{for } (x,y) \neq 0 \\ 0 & \text{for } (x,y) = 0. \end{cases}$ Show that $\phi(x,y)$ is continuous at $(0,0)$, but it is not differentiable at $(0,0)$.

Question fifteen

Consider the function $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ given by $\phi(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & \text{for } (x,y) \neq 0 \\ 0 & \text{for } (x,y) = 0. \end{cases}$ Show that the partial derivative $\phi_x(0,0)$ and $\phi_y(0,0)$ are exists but $\phi_x(x,y)$ is not differentiable at $(0,0)$.

Question sixteen

Compute the directional derivative of $\pi(x,y) = 5y - 10x + \cos(x)$, at the point $(0,1)$ in the direction of $-3\hat{i} + 4\hat{j}$ vector.

Question seventeen

Compute the directional derivative of $\pi(x,y,z) = \frac{xe^y}{3z^2+1}$, at the point $(2,-1,0)$ in the direction parallel to the vector $\hat{i} - 2\hat{j} + 3\hat{k}$.

Question eighteen

Consider the function $f : D(f) \subseteq \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x,y) = x^2 - 4x + y^2 - 4y + 16$

(a) Find and classify the critical points of $f(x,y)$.

(b) Find the maximum and minimum values of $f(x,y)$ subject to $x^2 + y^2 = 18$.

Question ninteen

Let $\psi : \mathbb{R}^2 \to \mathbb{R}$ be given by $\psi(x, y) = x^3 + 3xy + y^3$. Find all critical points of ψ.

Question tweny

Consider the function $\tau(x, y) = x^2 + xy - e^y$. Find the gradient of the function $\tau(x, y)$ and then compte the directional derivative of the function $\tau(x, y)$ at the point $P(1, 0)$ in the direction of the vector $\overrightarrow{u} = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

Question tweny one

Compute the directional derivative of $\pi(x, y, z) = x^2 - y^2 + xyz$, at the point $(1, 2, -1)$ in the direction of the vector $v = 3\hat{i} + 4\hat{j} + 12\hat{k}$.

Question tweny two

Let $\psi : \mathbb{R}^2 \to \mathbb{R}$ be given by $\psi(x, y) = x^3 - 3x + y^3 - 3y$. Find all critical points of ψ. For each critical point, use the second derivative test to classify it as either a local minimum, a local maximum, or a saddle point.

Question tweny three

Find the maximum and minimum values of the function $\phi(x, y) = x\, e^y$ subject to $x^2 + y^2 = 2$.

Question tweny four

Find the maximum and minimum values of the function $\phi(x, y) = x^2 - y^2$ subject to $4x^2 + y^2 = 1$.

Question tweny five

Consider the function $\phi : D(\psi) \subseteq \mathbb{R}^3 \to \mathbb{R}^3$ given by $\psi(x, y, z) = x^2 + 2y^2 + 3z^2$ then compute $D_u\phi(1, 1, 1)$ if $u = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.