**Question one**

Consider a vector valued function

$$\vec{r}(t) = 2\cos(t)\hat{i} + \sin(t)\hat{j} + t\hat{k}$$

(a) Sketch $\vec{r}(t)$, $\vec{r}(\frac{\pi}{2})$ and $\vec{r'}(\frac{\pi}{2})$.

(b) Find parametric equation for the tangent line to $\vec{r}(t)$ at the point $(0, 1, \frac{\pi}{2})$.

(c) Compute unit tangent vector and unit normal vector of $\vec{r}(t)$.

(d) Compute the curvature $\mathbb{K}(t)$ of $\vec{r}(t)$ at the point $(0, 1, \frac{\pi}{2})$.

**Question two** [3+3+4+7 marks]

Consider a vector field function

$$\vec{F}(x, y) = (xy^2 + 2y)\hat{i} + (x^2y + 2x + 2)\hat{j}$$

(a) Sketch $\vec{F}(x, y)$.

(b) Show that the vector field $\vec{F}(x, y)$ is conservative by checking partial derivative.

(c) Find a potential function $f$ for $F$.

(d) State Fundamental Theorem for the line integrals, then use it to compute the line integral $\int_C \vec{F}.d\vec{r}$ where $C$ is defined by $\vec{r}(t) = e^t\hat{i} + (1 + t)\hat{j}$ where $0 \le t \le 1$.

**Question Three** [12 marks]

For the double integral

$$\int_0^{1/2}\int_{-x}^{x} \frac{1}{\sqrt{x^2+y^2}}\,dydx + \int_{1/2}^{1}\int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\,dydx$$

(a) Sketch the region of integration.

(b) Reverse the double integral in polar coordinate.

(c) Evaluate the integral.

**Question Four** [16 marks]

Consider the line integral

$$I = \oint_C (2xy^2 - x^2y + y^3)\,dx + (2x^2y - x^3 + 4x)\,dy$$

(i) State Greens Theorem.

(ii) Use (i) to evaluate the line integral $I$ if $C$ is the closed triangular path with vertices $(0,0),(2,2),(-2,2)$, oriented counterclockwisely.

**Question Five**

(a) For the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{2+y^3}\, dydx$$

(i) Sketch the region of integration.

(ii) Reverse the order of integration.

(iii) Evaluate the integral.

(b) Find the area of the region bounded by $y = x$, $y = -x$, and $y^2 = x + 2$.

**Question Six**

Set up, do not evaluate the triple integral in rectangular, cylindrical and spherical coordinate to find the volume of the solid in the first octant bounded above by the sphere $x^2+y^2+z^2 = 12$ and bounded below by the cone $z = \sqrt{x^2+y^2}$.

**Question Seven**

Evaluate the following double integrals:(i) $\int_0^1 \int_0^{1-x} cos(1-y)^2 dydx$. (ii) $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \sqrt{x^2+y^2} dydx$

**Question Eight**

Use cylindrical coordinate to compute $\int_0^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{1/2-x^2}}^0 \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{x^2+y^2}} z\ dzdydx$.

**Question Nine**

Use spherical coordinate to compute the volume of the solid $E$ above $xy-$plane outside the cone $z = \sqrt{x^2+y^2}$ and inside the unit sphere $x^2+y^2+z^2 = 1$.

**Question Ten**

Evaluate the following double integrals:(i) $\int_0^1 \int_x^1 sin(y^2)\, dydx$. (ii) $\int\int_D \frac{y}{x^2+y^2}\, dA$ where $D$ is a triangle bounded by $y = x$ $y = 2x$ and $x = 2$.

**Question Eleven**

Use cylindrical coordinate to compute $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{x^2+y^2}}^0\ dzdydx$

**Question Twelev**

Compute

$\int_0^1 \int_0^1 \int_0^{\sqrt{y}} x\, z\, e^{z\, y^2}\ dx\, dy\, dz$

**Question Thireen**

Re-write

$$\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{4}^{4+\sqrt{16-x^2-y^2}} x\, dz\, dy\, dx$$

in terms of cylindrical and spherical coordinates and evaluate the simplest one.

**Question Fourteen**

Verify Green's theorem

$$\oint_C P\, dx + Q\, dy = \int\int_R (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\dot{d}A$$

for the vector function $\vec{F} = x^2\vec{i} - xy\vec{j}$ and $C$ is the triangle having vertices $A(0,3)$, $B(2,0)$ and $C(4,2)$.

**Question Fifteen**

Compute both integrals to verify that the conclusion of the Diveregence Theorem

$$\int\int_S \vec{F}.\dot{d}s = \int\int\int_E div\vec{F}dV$$

holds for the vector function $\vec{F} = y^2z^3\vec{i} + 2yz\vec{j} + 4z^2\vec{k}$ on the solid region $E$ that is enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$.

**Question Sixteen**

Verify Stockes theorem

$$\oint_C \vec{F}.d\vec{r} = \int\int_S Curl\vec{F}.\hat{n}\, ds$$

for the vector function $\vec{F} = 2z\vec{i} + x\vec{j} + y\vec{k}$, where $S$ is the top half surface of the unit sphere and $C$ is the boundary.

**Question Seventeen**

Re-write $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} 2zdzdydx.$ in terms of cylindrical coordinates.

**Question Eighteen**

Find the volume of the region in 3-dimensional space inside the cylindrical $x^2 + y^2 = 1$, above the $xy$-plan and below the plan $x + z = 1$.

**Question Ninteen**

Write down, do not evaluate the triple integral in cylindrical and spherical coordinate

$$\int\int\int_E (x^2 + y^2)\, dv$$

where $E$ is the solid bounded above by surface $z = 9 - x^2 - y^2$ and below by the plane $z = 0$.

**Question Twenty**

Re-write $\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{0}^{2-\sqrt{x^2+y^2}} x\, z\, dz\, dx\, dy.$ in terms of spherical coordinates.

**Question Tweny One**

Compute the line integral $\int_C (xz + 2y)\, ds$, where $C$ is the line segment from $(0, 1, 0)$ to $(1, 0, 2)$.

**Question Tweny two**

Find the area of the region bounded by line $y = x - 3$ and the curve $y^2 = x + 3$.

**Question Tweny three**

Find the area of the region bounded by inside $r^2 = 8cos(2\theta)$ and out side $r = 2$.

**Question Tweny four**

Use spherical coordinate to compute the volume of the solid region bounded the cone $z = \sqrt{3x^2 + 3y^2}$ and the sphere $x^2 + y^2 + z^2 = 9$.

**Question Tweny five**

Find the volume of the region in the first octant bounded by the surface $z = \sin(y)$ and $y = x$.