## Coordinate Systems

## Definition

- Geodesy: is the science of measuring and monitoring the size and shape of the Earth including its gravity field and determining the location of points on the Earth's surface. The surface of the earth is shaped by the earth's gravity and most geodetic observations are referenced to the earth's gravity field. The original focus of geodesy has expanded to include applications in ocean and space exploration. Nowadays, geodesy includes the determination of the ocean floor and the surfaces and gravity fields of other celestial bodies like moon. Geodesy become more important and necessary when measurements have to be taken over large distances like several kilometres in each direction, because over large distances the curvature of the earth become more and more and the error resulted from that will increase significantly.


## The Figure of the Earth

- The solid earth

- Approximation as a sphere

- Approximation as an ellipsoid



## Geoid:

- The equipotential surface of the Earth's gravity field which best fits global mean sea level. Geoid is used to describe the gravity shape of the earth and is not the surface of the earth. Geoid has no mathematical shape as it is irregular shape and the best representation of geoid with a mathematical shape is ellipsoid.
- Geoid is used because it is a conventional reference surface of heights, but because it is irregular in shape, it can't be used for determination of coordinates



## Coordinate Reference Systems

- 1-Geographical coordinate system
- In order to define geographical longitude and latitude (КA, $\phi A)$ it is required to define an equator and zero meridian.
- The equator is a plane perpendicular to the spin axis (rotation axis) of the Earth and passing through the centre of mass of the Earth.
- the zero meridian is an arbitrary reference plane in which
 the spin axis is contained.
- Therefore, the geographical latitude of a point is the angle between the equatorial plane and the vertical (i.e. the direction of the gravity vector) at the point.
- The geographical meridian is defined as the plane passing through the spin axis of the Earth and a plane passing through the vertical at the point.
- The zero meridian is agreed to be the geographical meridian through the old telescope at Greenwich observatory.


## The limitations of the geographical coordinate system are:

- The axis of rotation of the Earth is not fixed with respect to the solid mass of the Earth, but it is in a state of continuous motion (polar motion).
- The zero meridian is not passing exactly at a point in Greenwich, but it is defined as the average value of longitudes of some observatories around the globe, the IERS (International Earth Rotation Service) has defined the mean spin axis, reference pole and the zero meridian.
- geographical coordinates are not appropriate measurements of position on the Earth, but they are indicating the inclination of the direction of the local vertical.


## 2- Geodetic (Ellipsoidal) Coordinates

- The geodetic position of a point on the Earth is the ellipsoidal coordinates of the projection of the this point on the reference ellipsoid surface along the normal to the ellipsoid.
- The geodetic latitude ( $\phi \mathrm{G}$ ) can be defined as the inclination of the normal to the equatorial plane, and geodetic meridian as the plane through the normal and the minor axis of the ellipsoid.
- In addition, the geodetic longitude ( $\kappa \mathrm{G}$ ) of a point is defined as the angle between its geodetic meridional plane and an arbitrary zero meridian.
- The main reason of using geodetic coordinate system is that, ellipsoid is the closest geometrical shape to the shape of the Earth



## 3- Cartesian coordinate system

- It is more useful and popular alternative of using angular measurements of longitude and latitude. It is about positioning of any point on, below or above the surface of the Earth with respect to a Cartesian coordinate system.
- Having specified the equator and zero meridian of geodetic or astronomical coordinate systems, then Cartesian coordinate system can be defined (X, Y and Z). The origin of the Cartesian coordinate system could be the centre of the associated reference ellipsoid or the assigned mass centre of the Earth (Geocentric Cartesian coordinates).
- The X axis is in the direction of the zero meridian inside the equatorial plane.
- The Z axis is in the direction perpendicular to the equator.
- The Y axis is perpendicular to the other two.

- The main advantage of using Cartesian coordinate system is that it is defined by the direction of the three axes and the origin without the complication of reference ellipsoid and projection grids.
- Cartesian coordinate system is convenient for aircraft, satellite and space navigation,
- however it is not convenient for most of users, because any increase in the value of (h) doesn't mean an increase in the value of $(\mathrm{Z})$. For example the value of Z remain constant along the equator regardless of the value of (h).


## Celestial coordinate system

- A celestial coordinate system is a system for specifying positions of celestial objects: satellites, planets, stars, galaxies, and so on. Coordinate systems can specify a position in3-dimensional space, or the direction of the object on the celestial sphere, if its distance is not known or not important.
- In astronomy, the celestial sphere is an imaginary sphere of arbitrarily large radius, concentric with Earth. All objects in the observer's sky can be projected upon the inside surface of the celestial sphere. The celestial sphere is a practical tool for spherical astronomy, allowing observers to plot positions of objects in the sky when their distances are unknown or unimportant.


## Celestial sphere



## Height

- In geodetic coordinate system, the third dimension is expressed in ellipsoidal height (h) of any point. It is defined as the linear distance from the point (above or below the ellipsoid) to the ellipsoid along the normal to the ellipsoid.
- The ellipsoidal height is not suitable to be used in practical, because it is not related to a physical reference like geoid which is very important to determine the direction of flow of water.
- Orthometric height (H), height above mean sea level, is more suitable for use in practical, which is defined as the linear distance from a point to a reference equipotential surface of the gravity field of the Earth, measured along the gravity vector.
- The difference between the ellipsoidal (h) and orthometric height (H) is called geoid-ellipsoid separation or geoid height or geoid undulation and denoted by ( N ).
- There are other heights which will be discussed in physical geodesy subject


$$
\mathrm{h}=\mathrm{N}+\mathrm{H}
$$

## Transformation between coordinate

## systems:

- Transformation between coordinate systems is important for users who use more than one coordinate system and since the advent of GPS it become more interested.
- For example, a navigator on a ship is going to the UK and uses GPS for navigation, so he gets the coordinates with respect to WGS84.
- The local charts that he has are in OSGB36 coordinate system. In order to plot his position on the local charts, the navigator needs first to transfer the WGS84 coordinates to corresponding OSGB36 coordinate system.
- To perform transformation between two coordinate system, it is necessary to know the method of transformation between the two systems.
- If two Cartesian coordinate system are parallel, then there are three transformations needed to be performed, $\Delta \mathrm{X}, \Delta \mathrm{Y}$ and $\Delta \mathrm{Z}$.
- If the two coordinate systems were not parallel then there are another three rotational transformations to be performed $\Theta \mathrm{X}$, $\theta Y$ and $\theta Z$.
- If there was scale variation between the two coordinate systems then scale $\boldsymbol{\mu}$ transformation must be added to the transformation process.


## Transformation between Cartesian and geodetic coordinate systems:

- A set of 3-d Cartesian coordinates $\mathbf{X}, \mathbf{Y}$ and $\mathbf{Z}$ can be transformed to corresponding longitude, latitude and height above an ellipsoid of given dimensions ( $\mathbf{a}$ and $\mathbf{e}$ ), whose origin and axes coincide with the Cartesian system and vice versa, using the following formulae:
- Transformation from geodetic to Cartesian system:

$$
\begin{aligned}
& X=(v+h) \cos \phi_{G} \cos \lambda_{G} \\
& Y=(v+h) \cos \phi_{G} \sin \lambda_{G} \\
& Z=\left(v\left(1-e^{2}\right)+h\right) \sin \phi_{G}
\end{aligned}
$$

## Transformation from Cartesian to geodetic system:

$\tan \lambda_{G}=\frac{-}{Y}$
$\tan \phi_{G}=\frac{\mathrm{Z}+\mathrm{e}^{2} v \sin \phi_{\mathrm{G}}}{\sqrt{\left(\mathrm{X}^{2}+\mathrm{Y}^{2}\right)}}$
$\mathrm{h}=\left(\frac{\mathrm{X}}{\cos \phi \cos \lambda}\right)-v$
where
$v=\frac{\mathrm{a}}{\sqrt{\left(1-\mathrm{e}^{2} \sin ^{2} \phi_{\mathrm{G}}\right)}}$
$e=\frac{\sqrt{a^{2}-b^{2}}}{a}$
$e=\sqrt{1-\frac{b^{2}}{a^{2}}}$
$\mathrm{a}=$ semi major axis of ellipsoid
e $=$ eccentricity of ellipsoid
$\mathrm{h}=$ ellipsoidal height of point

## Helmert Transformations

- The most general method of transformation between two geodetic datum is performed using all 7 parameters of geometrical transformation, therefore any variation between the two systems is accommodated. The general transformation formula for this purpose is Helmert transformation and expressed as below
- From the formula, it can be seen that all the 7 parameters exist (3 shifts, 3 rotations and a scale factor) along with the set of coordinates of both systems.

$$
\left[\begin{array}{l}
\mathrm{X}_{2} \\
\mathrm{Y}_{2} \\
\mathrm{Z}_{2}
\end{array}\right]=\left[\begin{array}{c}
\mathrm{X}_{1} \\
\mathrm{Y}_{1} \\
\mathrm{Z}_{1}
\end{array}\right]+\left[\begin{array}{ccc}
\mu & \theta_{\mathrm{Z}} & -\theta_{\mathrm{y}} \\
-\theta_{\mathrm{Z}} & \mu & \theta_{\mathrm{x}} \\
\theta_{\mathrm{y}} & -\theta_{\mathrm{x}} & \mu
\end{array}\right]\left[\begin{array}{c}
\mathrm{X}_{1} \\
\mathrm{Y}_{1} \\
\mathrm{Z}_{1}
\end{array}\right]+\left[\begin{array}{c}
\Delta \mathrm{X} \\
\Delta \mathrm{Y} \\
\Delta \mathrm{Z}
\end{array}\right]
$$

where
$\mathrm{X}_{1}, \mathrm{Y}_{1}, \mathrm{Z}_{1} \quad$ cartesian coordinates in first datum $\mathrm{X}_{2}, \mathrm{Y}_{2}, \mathrm{Z}_{2}$ cartesian coordinates in second datum.

- The rotation angles in the matrix are small in value and expressed in radians. The complete formula include a rotation matrix with sine and cosine expressions of the angles, but because the difference in definition of the axes are small, the above formula is sufficient .
- If geodetic coordinates were provided then it is necessary to convert them to the Cartesian coordinates first, then the above transformation can be applied.
- In order to find the 7 parameters of transformation between to geodetic datum, it is necessary that the coordinates of a set of points in both systems are known. The more the common points are, the better the estimation will be.


## Example of coordinate systems for the same point

- Cartesian X, Y and Z

3851459 m -79556m 5065774 m

- Latitude $\phi$, longitude $\lambda$, and height $h$
$52^{\circ} 56^{\prime} 00^{\prime \prime}$ N10 $1^{\prime} 00^{\prime \prime}$ W 90 m
- National Grid, Easting E and Northing N $354885 \mathrm{~m} \quad 337673 \mathrm{~m}$


## Datum shift between OSGB 36 and ED 50 (European Datum 1950)



## Coordinate systems in Iraq

Datum

Nahrwan 1934
Nahrwan 1967
European 1950
Karbala 1979 Polservice
WGS 1984
IGRS
https://doi.org/10.31272/ jeasd.26.3.8

Spheroid
Clarke 1880 RGS
Clarke 1880 RGS
International 1924
Clarke 1880 RGS
WGS 1984
GRS 1980

| Datum | Karbala 1979 <br> Polservice | IGRS | Differences |
| :---: | :---: | :---: | :---: |
| Spheroid | $\begin{gathered} \text { Clarke } \\ \text { 1880 } \\ \text { RGS } \end{gathered}$ | $\begin{aligned} & \text { GRS } \\ & 1980 \end{aligned}$ |  |
| Semi-major Axis ( m ) | $\begin{gathered} 6378249.1 \\ 45 \end{gathered}$ | 6378137.0 | -112.145 |
| Semi-minor | 6356514.8 | 6356752.3 | 237.4445905 |
| Axis (m) | 69549776 | 14140356 | 8 |
| Inverse Flattening ( m ) | 293.465 | $\begin{gathered} 298.25722 \\ 2101 \end{gathered}$ | 4.792222101 |
| Eccentricity | 0.0068035 | 0.0066943 | -0.00010913 |
| Square ( $\mathbf{e}^{\mathbf{2}}$ ) | 1128 | 8002 | 126 |



