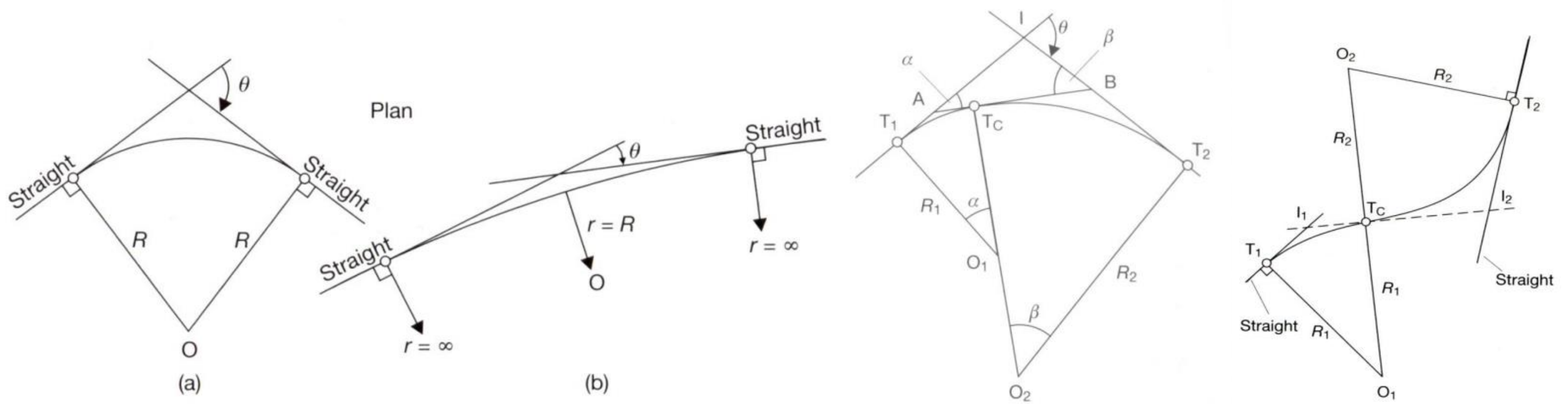
Horizontal and Vertical Curves

Horizontal curves

Horizontal and vertical Curves

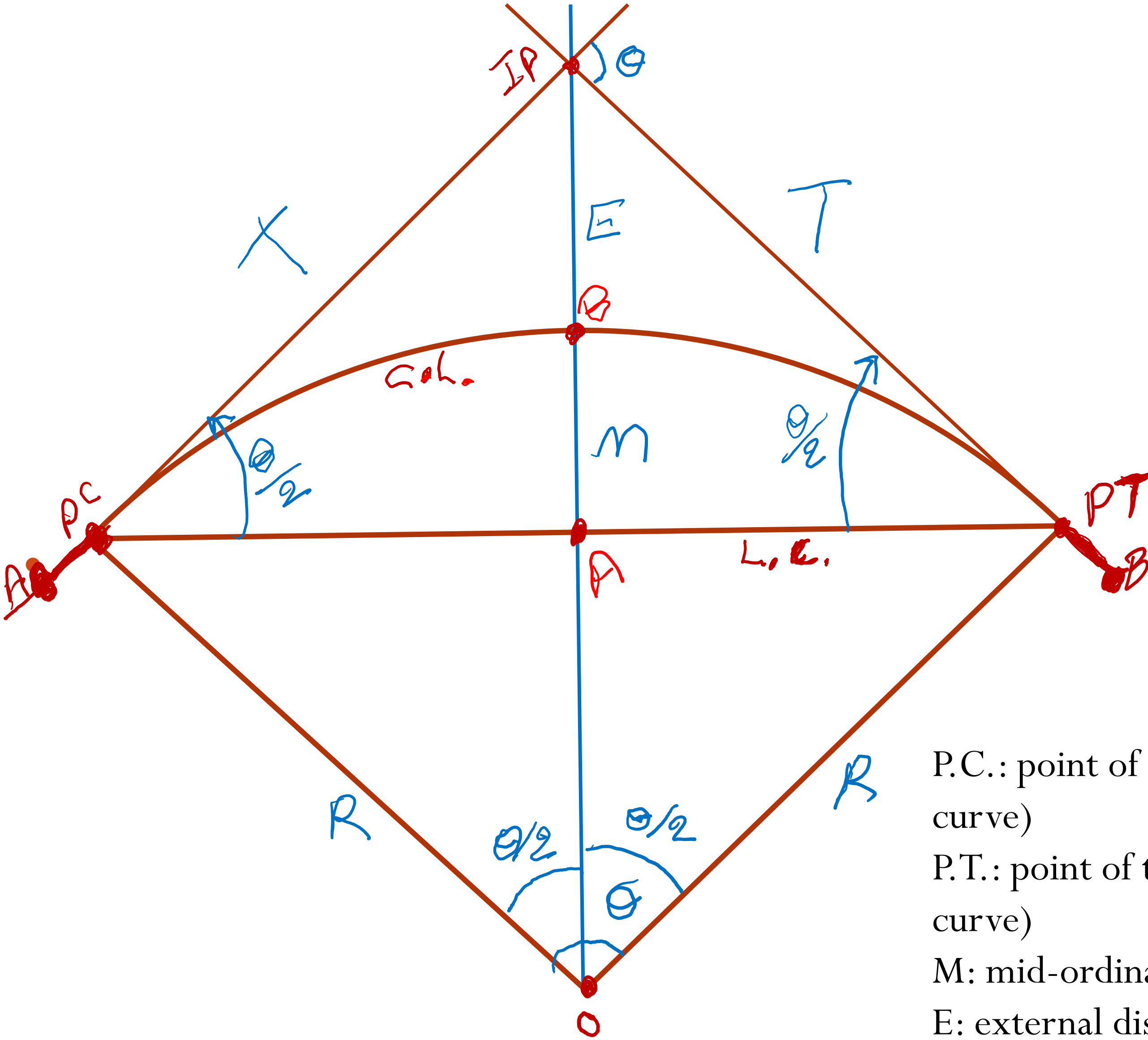
- Straight (tangent) sections of most types of transportation routes, such as highways and railroads are connected by curves in both the horizontal and vertical planes
- Horizontal curve used in horizontal plane and Vertical curve used in vertical plane
- Horizontal curves include circular curves and transition curves



Horizontal curves: (a) circular curve; (b) transition curve. (c) Compound curves (d) reverse curves

- **Types of circular curves**
- Simple circular curves: a circular curve has one radius. This is the most commonly used type of curve
- Compound circular curves: consists of two or more consecutive simple circular curves of different radii without and intervening straight section.
- Reverse circular curves: they consist of two circular curves with their centers lie on different side of the road. Their radii are either equal or different

Terminology of a simple circular curve



IP: point of intersection between the two tangents
A-IP: back tangent
IP-B: forward tangent
θ: deflection angle
O: centre of the curve
R: radius of the curve
T: tangents of the curve
C.L.: curve length
L.C.: long chord

P.C.: point of curvature (start point of the curve)
P.T.: point of tangency (end point of the curve)
M: mid-ordinate (distance between A and B)
E: external distance (between IP and B)

Equations

- $T = R \times \tan\frac{\theta}{2}$
- $L.C. = 2R \times \sin\frac{\theta}{2}$
- $C.L. = R \times \theta^{rad}$
- $M = R\left(1 - \cos\frac{\theta}{2}\right)$
- $E = R(\sec\frac{\theta}{2} - 1)$
- Station PC = Station IP – T
- Station PT = station PC + CL
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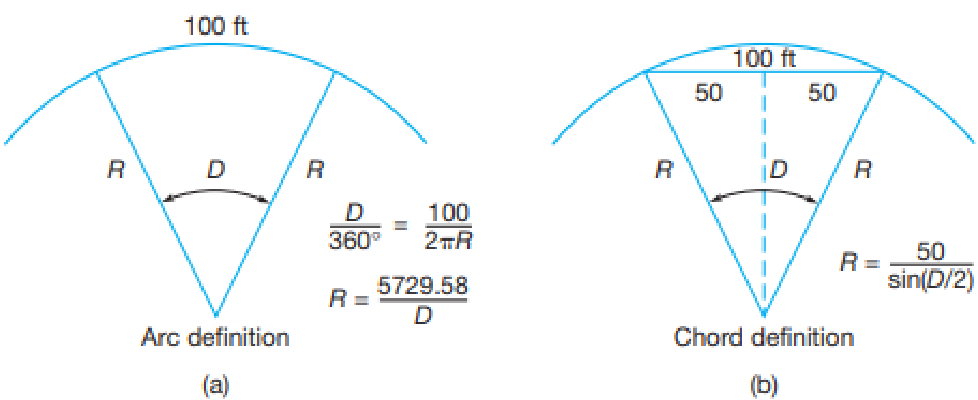
Example

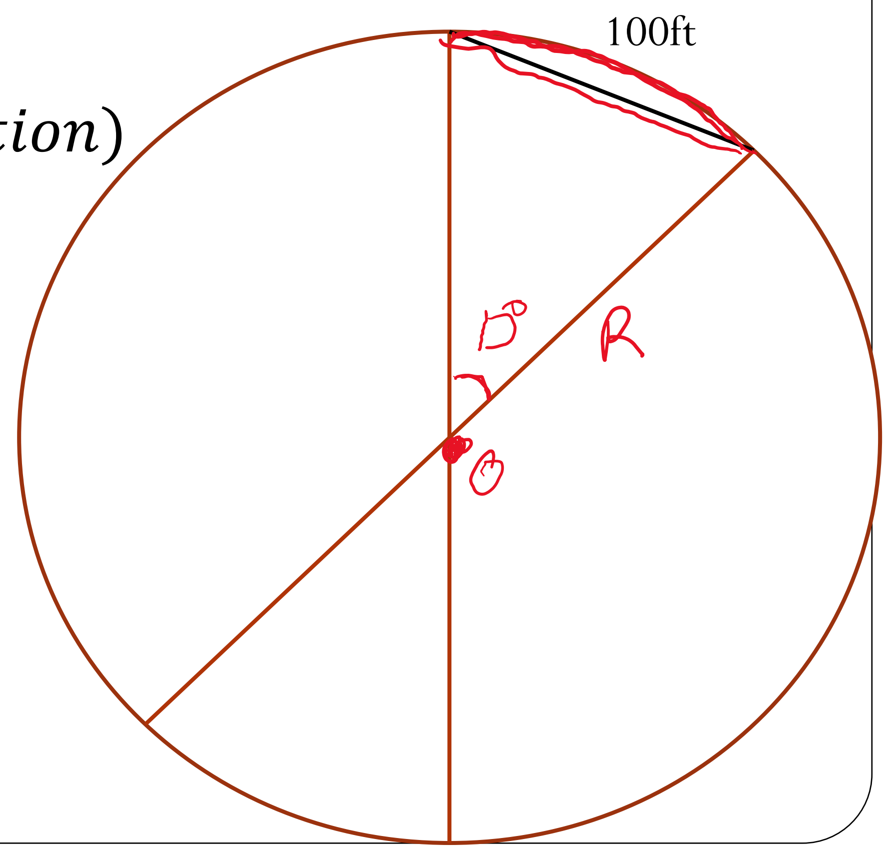
- Calculate the station of PC, PT and long chord, mid-ordinate, external distance given the following data:
- Deflection angle= $16^0 38'$, R=1000m, St IP= 6+26.57

- $T = 1000 \times \tan\frac{16^0 38'}{2} = 146.18m$
- $L.C. = 2 \times 1000 \times \sin\frac{16^0 38'}{2} = 289.29m$
- $C.L. = 1000 \times 16^0 38' \times \frac{\pi}{180} = 290.31$
- $M = 1000 \times \left(1 - \cos\frac{16^0 38'}{2}\right) = 10.52m$
- $E = 1000 \times (\sec\frac{16^0 38'}{2} - 1) = 10.63m$
- Station PC = (6+26.57) – 146.18 = 4+80.39
- Station PT = (4+80.39) + 290.31 = 7+70.70

Degree of curvature D°

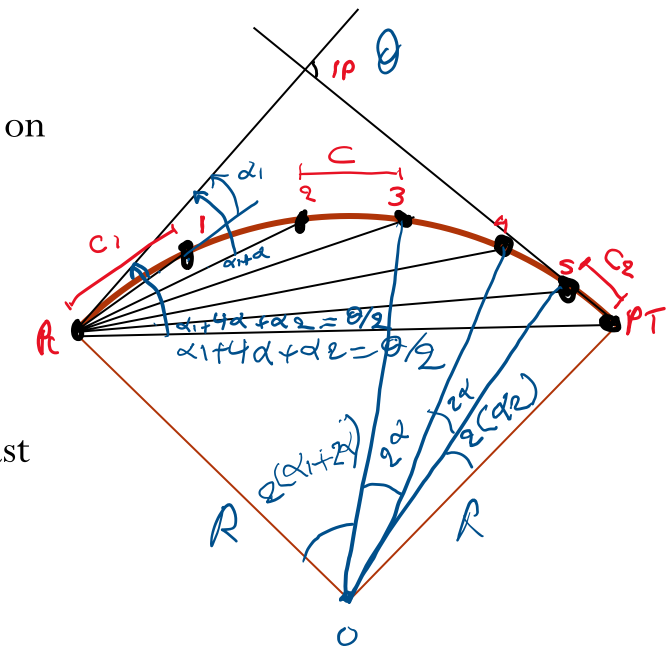
- Curves can be defined by degree of curvature (D°) which is the angle at the center of the curve subtended by:
- An arc of length of 100 ft
- $D° = \dfrac{5729.58}{R}\ in\ feet\ (arch\ definition)$
- A chord of length of 100ft
- $\sin\dfrac{D^0}{2} = \dfrac{50}{R}\ in\ feet\ (chord\ definition)$

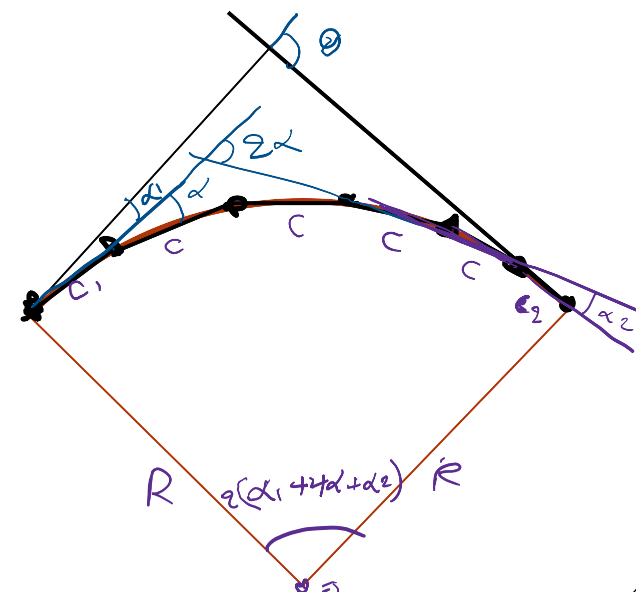




Chord& deflection angle method of layout of horizontal circular curves

- Full stations inside the curve need to be identified once the curve length and station of PC and PT are known.
- First full station comes directly after the station of PC depending on the station intervals given
- last full station is the last on the curve before the station of PC
- There might be other complete stations on the curve where the distance between any 2 of them is C
- C: is the station interval (usually given)
- C1 = first full station – station of PC   C2 = station of PT – last full station
- The angle subtended at the center of the curve for C is α
- The angle subtended at the center of the curve for C1 is $\alpha 1$
- The angle subtended at the center of the curve for C2 is $\alpha 2$
- The angles also represent the amount of deflection for each of the stations from the previous one
- The angles can be used accumulatively to layout the curve from PC
- The angles can be computed using either chord or arc method with almost equal values because of the tiny difference in the results
- Chord method
- $\sin\alpha 1 = \frac{C1}{2R}$ $\sin\alpha 2 = \frac{C2}{2R}$ $\sin\alpha = \frac{C}{2R}$
- Arc method
- $\alpha 1 = \frac{c1}{CL} \times \frac{\theta}{2}$ $\alpha 2 = \frac{c2}{CL} \times \frac{\theta}{2}$ $\alpha = \frac{c}{CL} \times \frac{\theta}{2}$

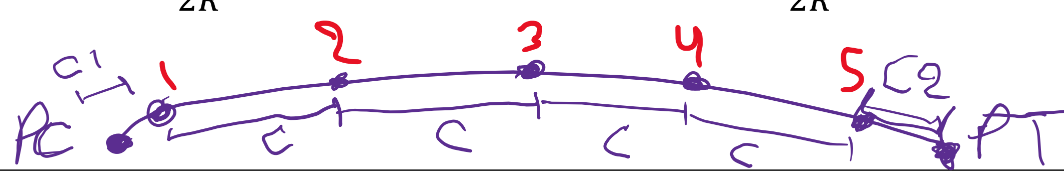




Example

- Prepare the layout table for a horizontal curve with deflection angle of 12°51' and radius of 400m. Given the station interval is 20m and the station of IP is 2+41.78
- $T = R \times \tan\frac{\theta}{2} = 400 \times \tan\frac{12°51'}{2} = \mathbf{45.04m}$
- $L.C. = 2R \times \sin\frac{\theta}{2} = 2 \times 400 \times \sin\frac{12°51'}{2} = \mathbf{89.5m}$
- $C.L. = R \times \theta^{rad} = 400 \times \frac{12°51' \times \pi}{180} = \mathbf{89.71m}$
- Station PC = Station IP – T = (2+41.78) - 45.04 = **1+96.74**
- Station PT = station PC + CL = (1+96.74) + 89.71 = **2+86.45**
- Given the station interval C is **20m**:
- First full station on the curve will be 2+00 (first full station after St PC 1+96.74)
- Last full station on the curve will be 2+80 (last full station before St PT 2+86.45)
- Other full stations on the curve: 2+20, 2+40 and 2+60 so there are 5 full stations in total
- C1 = (2+00) – (1+96.74) = **3.26m** C2 = (2+86.45) – (2+80) = **6.45m**
- **C1+C+C+C+C+C2= 3.26+20+20+20+20+6.45 = 89.71m = CL**

$\sin\alpha 1 = \frac{C1}{2R}$ $\alpha 1 = 0°14'01''$ $\sin\alpha 2 = \frac{C2}{2R}$ $\alpha 2 = 0°27'43''$ $\sin\alpha = \frac{C}{2R}$ $\alpha = 1°25'57''$



Station	Chord length	Deflection angle	Accumulated deflection angle at PC (from IP to the next point)
PC	0	0	0
2+00	3.26	0°14’01”	0°14’01” (α1+0)
2+20	20	1°25’57”	1°39’58” (α1+α)
2+40	20	1°25’57”	3°5’55” (α1+2α)
2+60	20	1°25’57”	4°31’52” (α1+3α)
2+80	20	1°25’57”	5°57’49” (α1+4α)
PT	6.45	0°27’43”	6°25’32” (α1+4α+α2)
Checks you can make	$\sum = LC$	$\sum = \frac{\theta}{2}$	There might be slight difference in $\frac{\theta}{2}$ which is OK

- Calculation of distance from PC to curve points:
- Distance PC to 1st full station (PC – 1) = 2R × sin $\alpha 1$ = **3.26m** (~C1)
- Distance PC to 2nd full station (PC – 2) = 2R × sin (α1+α) = **23.26m** (~C1+C)
- Distance PC to 3rd full station (PC – 3) = 2R × sin (α1+2α) = **43.24m** (~C1+2C)
- Distance PC to 4th full station (PC – 4) = 2R × sin (α1+3α) = **63.2m** (~C1+3C)
- Distance PC to 5th full station (PC – 5) = 2R × sin (α1+4α) = **83.12m** (~C1+4C)
- Distance PC to PT (PC – PT) = 2R × sin (α1+4α+α2) = **89.53m** (~C1+4C+C2)

Procedure of layout of horizontal circular curve

- 1- choose the location of IP
- 2- choose the location of the first tangent and set a point at a distance of about 30m
- 3- set up theodolite at IP and sight to point A. Then clamp the H. motion and measure along this direction the amount of T to locate PC
- 4- transit the telescope and rotate the telescope by the amount of deflection angle then clamp H. motion
- 5- measure along this direction T to locate PT
- 6- move the theodolite to PC after setting up, measure the angle IP-PC-PT to check the amount of half of deflection angle. If there is deviation, repeat the previous steps.
- 7- sight to IP and set the theodolite at FL, set H. circle reading to 0°
- 8- obtain readings of angles of the full stations of the curve and measure the distances to locate the curve points
- The point of intersection of marked meter location with the line of sight is the location of the designated point

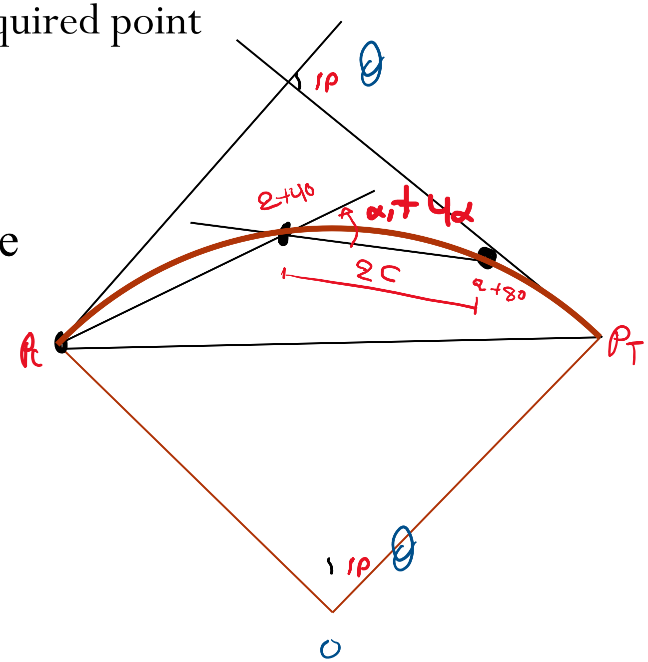
Moving along the curve points to layout other points

- It might be necessary to carry on the layout of some of the curve points from the others.
- The distance and angle between the instrument station and the required point

Is needed to carry out the layout.

- For example, to layout station 2+80 from 2+60 sighting at 2+40

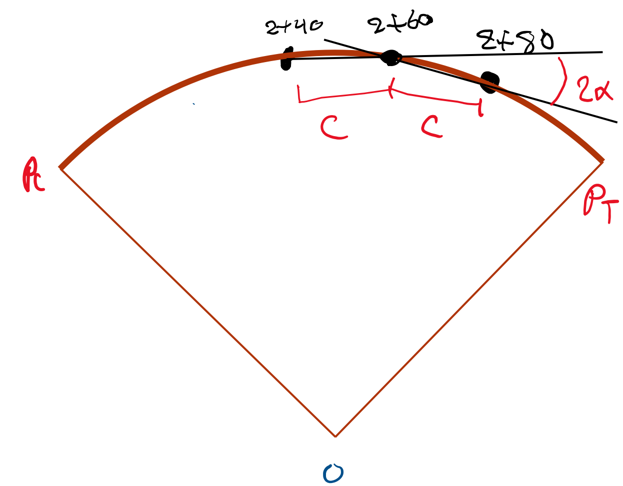
the angle needed will be (2α) and the distance is (2C) between the Two stations and (C) between 2+60 and 2+80



- To layout the station 2+80 from 2+40 sighting PC:
- As a procedure, the instrument is setup at station 2+40
- Sight to PC and clamp H. motion of the theodolite
- Transit the telescope
- Turn the same angle to deflection angle to locate the station 2+80

As it is computed from PC (α1+4α)

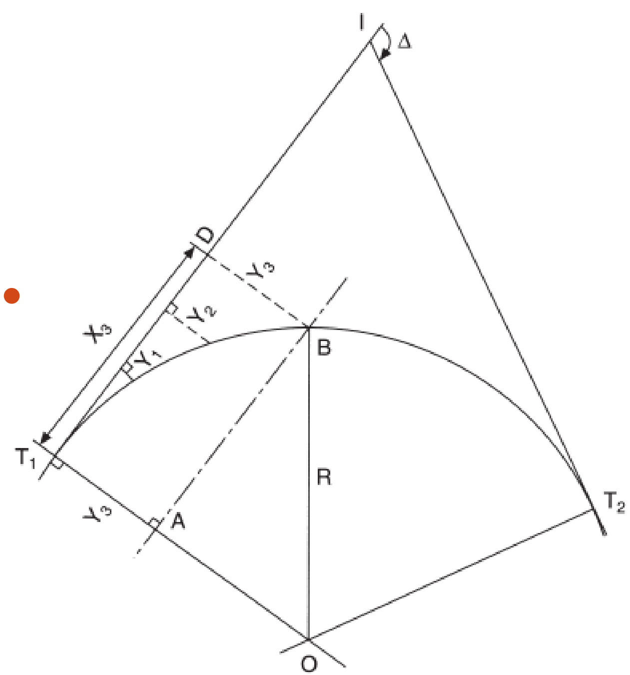
Measure the distance (2C) from 2+40 to 2+80



Setting out using offsets from the tangent

- The position of the curve is located by right-angled offsets Y set out from distances X, measured along each tangent, thereby fixing half the curve from each side.

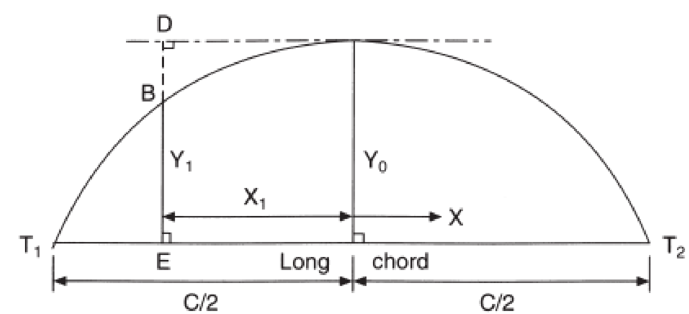
$$Y_i = R - (R^2 - X_i^2)^{\frac{1}{2}}$$

- 

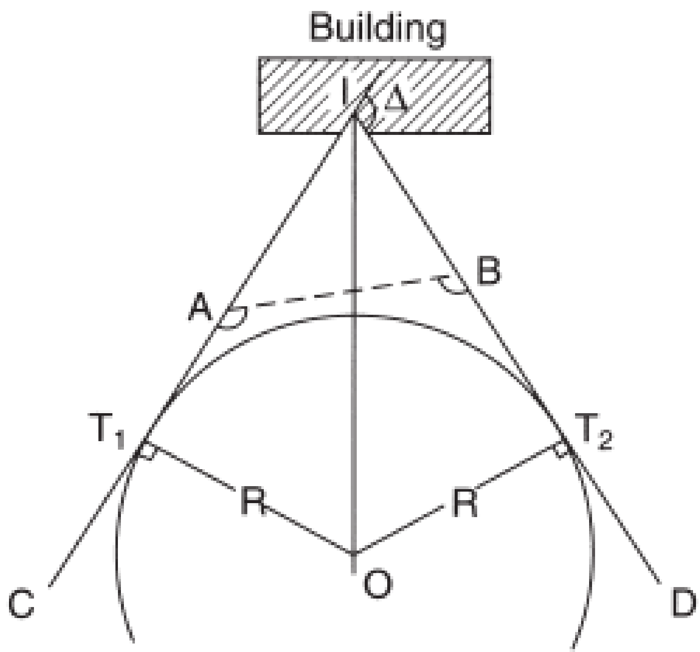
Setting out using offsets from the long chord

- In this case the right-angled offsets Y are set off from the long chord C, at distances X to each side of the center offset Y0. The same calculation can be used to layout the second half from the second tangent

$$Y_i = Y_0 - [R - (R^2 - X^2)^{\frac{1}{2}}]$$

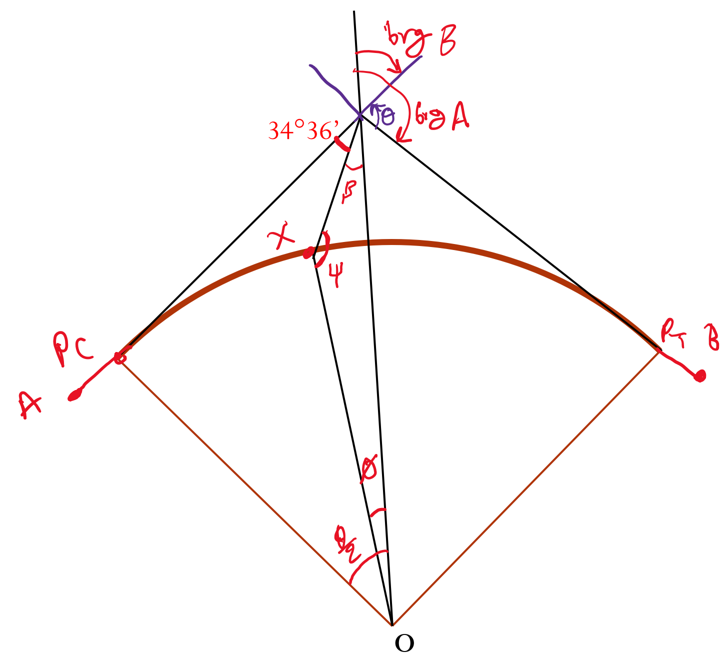
- 

Inaccessible IP

- Sometimes the location of IP is not accessible to setup the theodolite. In that case two points are located on the two tangents (A and B) and the distance between them and the angles at A and B are measured.
- The triangle can be solved for A-IP and B-IP and A-PC and B-PT can be computed
- 

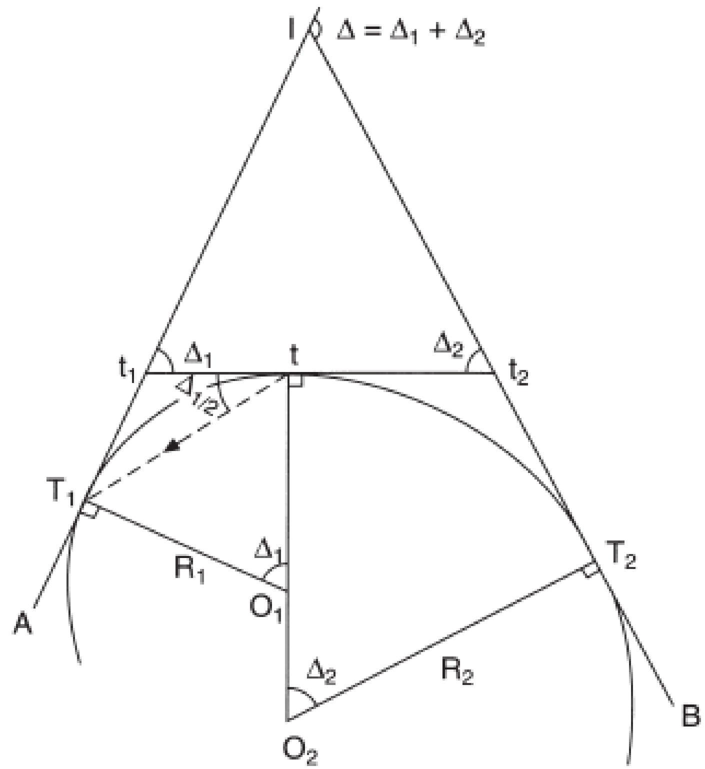
Curve to pass through fixed point

- Example: a circular curve is to connect two straights (A-IP) and (IP-B). The bearings of the lines are 70°42’ and 130°54’ respectively. The curve is to pass through a fixed point “X” such that (IP-X) is 39.72m and the angle between A and X at IP (A-IP-X) is 34°36’. Determine the radius of the curve if the station of IP is 15+78.30.
- $\theta = brgA - brgB = 60°12'$
- $180 - \theta = 119°48'$, $\frac{119°48'}{2} = 59°54'$
- $59°54' - 34°36' = 25°18' = \beta$
- *In triangle IP-X-O, using sin rule*
- $\frac{OX}{\sin\beta} = \frac{IPX}{\sin\varphi} = \frac{IPO}{\sin\Psi}$
- In triangle O-IP-PC $\cos\frac{\theta}{2} = \frac{R}{IPO}$
- So $IPO = \frac{R}{\cos 30°6\prime}$
- Substitute the value of IPO in the sin rule equation:
- $\frac{R}{\sin 25°18\prime} = \frac{39.72}{\sin\varphi} = \frac{\frac{R}{\cos 30°6\prime}}{\sin\Psi}$ taking each two sets at a time:
- $\Psi = 29°36'6$. Given the triangle shape, Ψ=180−29°9′36 = 150°23′54“
- $\varphi = 180 - \Psi - \beta = 4°18'6"$
- So R= 226.39m



Compound curves

- Compound and reverse curves are combinations of two or more circular curves.
- They should be used only for low-speed traffic routes.
- Special formulas have been derived to facilitate computations for such curves.
- A compound curve can be staked with instrument setups at the beginning PC and ending PT, or perhaps with one setup at the point of compound curvature (PCC) where the two curves join.



In the case of the compound curve, the total tangent lengths T_1I and T_2I are found as follows:

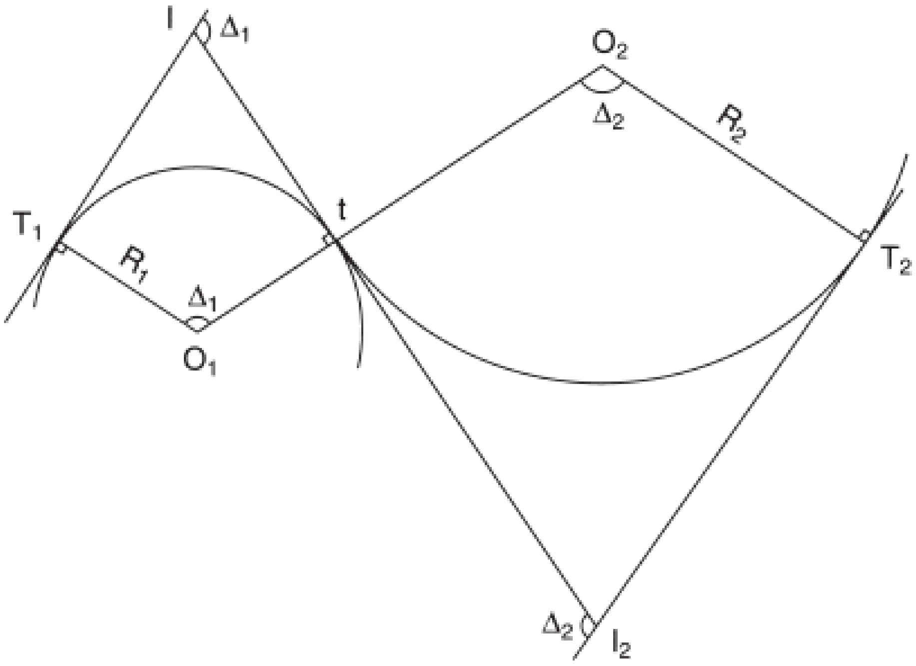
$$R_1 \tan \Delta_1/2 = T_1t_1 = t_1t \quad \text{and} \quad R_2 \tan \Delta_2/2 = T_2t_2 = t_2t, \text{ as } t_1t_2 = t_1t + t_2t$$

then triangle t_1It_2 may be solved for lengths t_1I and t_2I which, if added to the known lengths T_1t_1 and T_2t_2 respectively, give the total tangent lengths.

In setting out this curve, the first curve R_1 is set out in the usual way to point t. The theodolite is moved to t and backsighted to T_1, with the horizontal circle reading $(180° - \Delta_1/2)$. Set the instrument to read zero and it will then be pointing to t_2. Thus the instrument is now oriented and reading zero, prior to setting out curve R_2.

Reverse curves

- Two simple circular curves with equal or different radius. The centerline of each curve lies on one side of the road. The point of connection between the curves is called the point of reverse curve (PRC)



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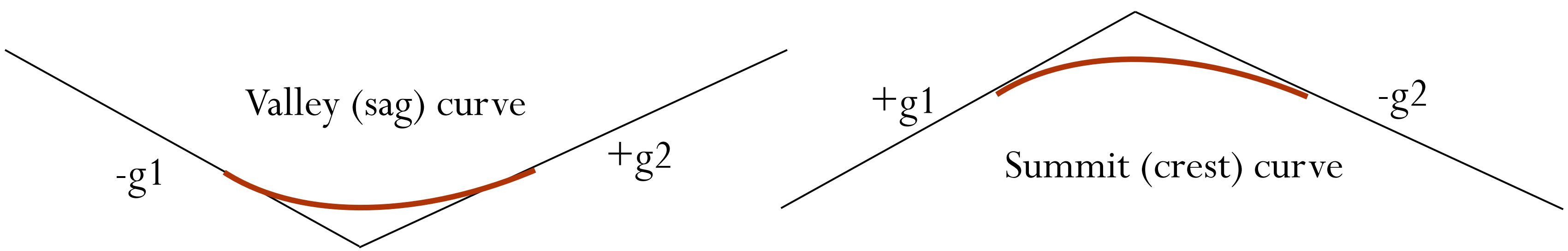
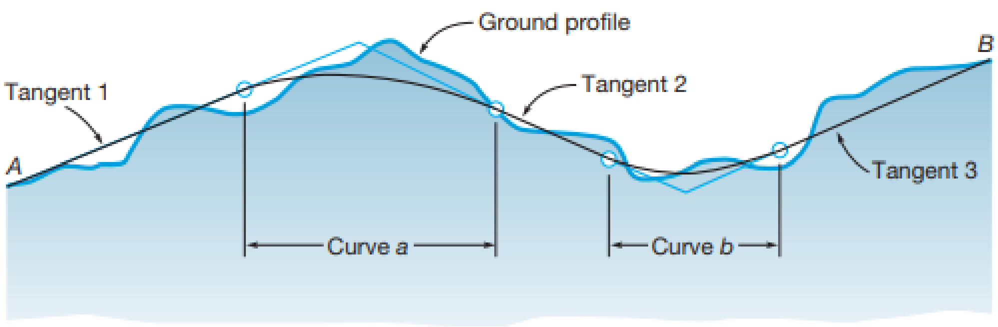
Vertical Curves

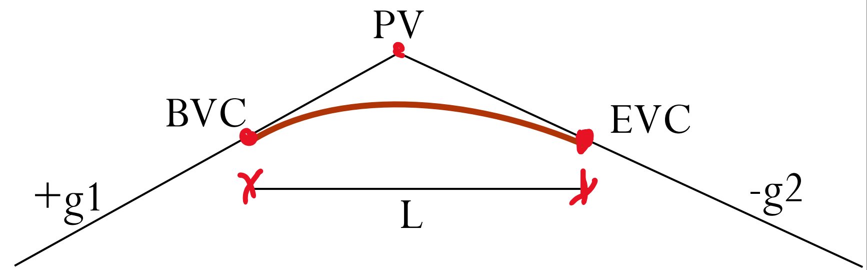
- Curves are needed to provide smooth transitions between straight segments (tangents) of grade lines for highways and railroads. Because these curves exist in vertical planes, they are called vertical curves
- The function of each curve is to provide a gradual change in grade from the initial (back) tangent to the grade of the second (forward) tangent. Because parabolas provide a constant rate of
- change of grade, they are ideal and almost always applied for vertical alignments used by vehicular traffic.
- Two basic types of vertical curves exist, crest and sag:

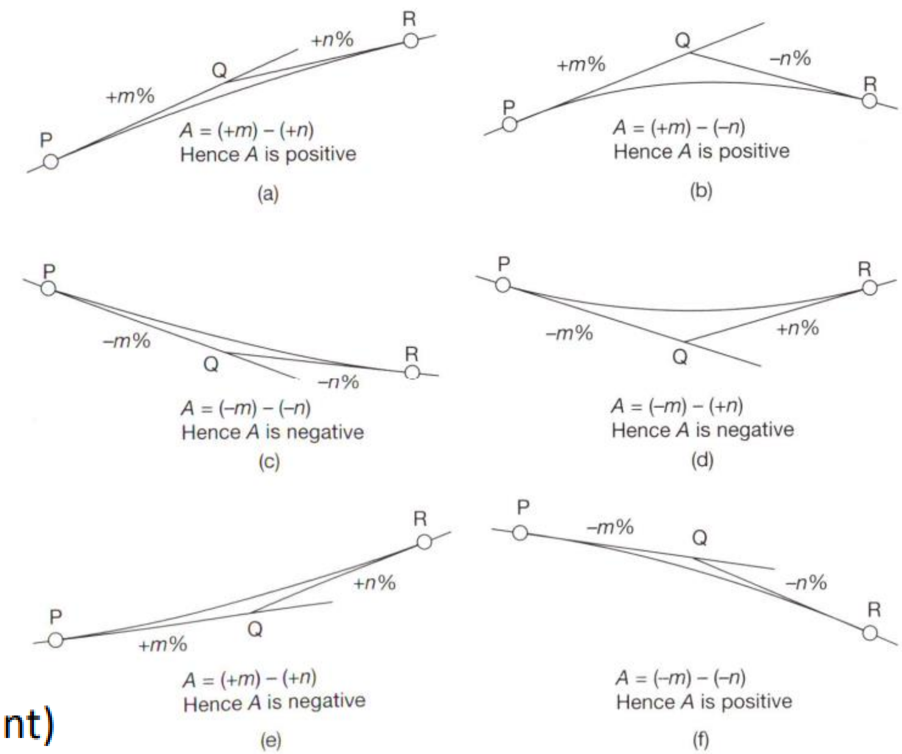




Terminology

- PV: point of intersection of two vertical gradients
- BVC: beginning of vertical curve
- EVC: end of vertical curve
- L: length of vertical curve
- g1: First gradient (entry gradient)
- g2: second gradient (exit gradient)
- Assumptions made for simplicity of calculation:
  - Mid-ordinate=external distance
  - Long chord= curve length
  - BVC-PV=EVC-PV
- In the design, the algebraic difference (A) between the gradients is used considering the sign of the gradient whether positive or negative





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A = m%(entry gradient) – n% (exit gradient)

Formulae

- General formula for vertical curve (parabola)
- $Y = aX^2 + bX + C$
- If X=0 (BVC) then Y = **C** (elevation of BVC)

- The slope of the curve is given by the first derivative of the curve
- $2aX + b$ for BVC (X=0) the value of **b** is **g1**

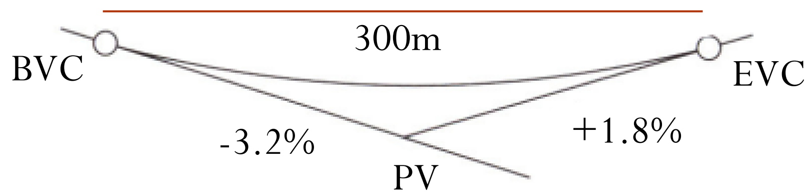
- The second derivative is the rate of change of gradient (2a) so constant rate
- $Y = aX^2 + g1X + elevation\ BVC$
- **a** is computed from g1,g2,L
- **X** is assumed to get values of **Y** as elevations of the curve points.

- $\boldsymbol{X = \frac{-g1}{2a}}$ where X is the distance to the highest or lowest point
- Add the above X value to the equation of vertical curve to get the elevation of that point
- $\boldsymbol{a = \frac{g2-g1}{2L}}$

Example

- Given the following details, determine the **elevations** of the curve points at intervals of **50m** then determine the **station** and **elevation** of the **lowest** point:

L=300m St PV=30+30 g1= -3.2% g2= +1.8% elev. PV=465.92m



- St of BVC= St PV - L/2 = **28+80**
- St of EVC= St PV + L/2 = **31+80**
- Elevation BVC = elev. PV + g1*L/2 = **470.72m**
- Elevation EVC = elev. PV + g2*L/2 = **468.62m**
- $\boldsymbol{a = \frac{g2 - g1}{2L} = \frac{1.8-(-3.2)}{100\times 2\times 300} = +0.0000833}$ at least 7 digits
- First full station = **29+00** last full station = **31+50**
- $Y = aX^2 + g1X + elevation\ BVC$ Y=0.0000833 X^2 – 0.032 X + 470.72
- X of the lowest point = $\boldsymbol{\frac{-g1}{2a} = \frac{-(-0.032)}{2\times 0.0000833} = 192.08m}$
- Y of the lowest point = $0.0000833 \times 192.08^2 - 0.032 \times 192.08 + 470.72 = \boldsymbol{467.64m}$
- Station of the lowest point = St BVC+192.08 = **30+72.08**

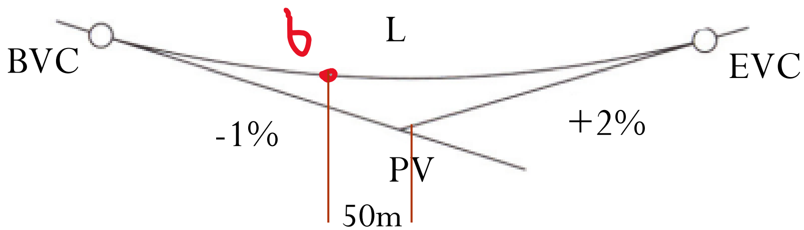
- Amount of fill at each point = elevation of curve point – elevation of gradient level
- Elevation of gradient points for the first half of the curve = aX^2
- Elevation of gradient points for the second half of the curve = elev. EVC – g2*distance from EVC

- Elevation and amount of fill at the lowest point?

Stations	**X**	**aX^2**	**g1X**	**elevations**	**Tangent point elevation**	**Amount of fill, m**
BVC	0	0	0	470.72	470.72	0
29+00	20	0.03	-0.64	470.11	470.08	0.03
29+50	70	0.41	-2.24	468.89	468.48	0.41
30+00	120	1.20	-3.84	468.08	466.88	1.2
30+50	170	2.41	-5.44	467.69	466.28	1.41
31+00	220	4.03	-7.04	467.71	467.18	0.53
31+50	270	6.07	-8.64	468.15	468.08	0.07
BVC	**300**	7.5	-9.6	**468.62**	468.62	0
checks	Last number			Last number		

Example

- A vertical curve is used to connect a falling gradient of **1%** and a rising gradient of **2%** which meet at a valley intersection point with an elevation of **100m**. If the curve is to pass at a station **50m** short of the intersection point at an elevation of **101.7m**, **determine** the length of the curve and location and elevation of the lowest point



- Elevation of BVC = 100+0.01*L/2
- $X_b = \left(\frac{L}{2} - 50\right) \qquad Y_b = 101.7m\ given$
- $a = \frac{0.02-0.01}{2L} = \frac{0.03}{2L}$
- $Y = aX^2 + g1X + elevation\ BVC$
- $101.7 = \frac{3}{200L}\left(\frac{L}{2} - 50\right)^2 - 0.01\left(\frac{L}{2} - 50\right) + \left(100 + 0.01 \times \frac{L}{2}\right)$
- Solving the equation, L=500m
- $a = \frac{0.03}{2L} = 0.00003$
- X of the lowest point $= \frac{-(-0.01)}{2\times 0.00003} = \mathbf{166.67m}$
- $Y = \frac{3}{200L}\left(\frac{L}{2} - 50\right)^2 - 0.01\left(\frac{L}{2} - 50\right) + \left(100 + 0.01 \times \frac{L}{2}\right) = \mathbf{101.7m}$
-