**Comparison of Efficiency and Sensitively between**

**(Three Sigma, Six Sigma and Median Absolute Deviation) by using X-bar Control Chart**

**Abstruct**

The main objective of any productivity operation is to get high quality materials and conform to the specifications so as to meet consumer desires. The quality is a principle in which some may think it is new, but it is as old as human, because the man was still in a permanent search for a good thing, and paying attention to the quality .Quality has great care in the industrialized developed countries being of economies exporter seek to control the foreign markets, scale estimators are very important in many statistical applications. The most common of scale estimators is the sample standard deviation. Many statistical quality control textbooks recommend the use of it for estimating the process standard deviation for a normally distributed random variable. Unfortunately standard deviation is not necessarily the most efficient or meaningful estimator of scale in skewed and leptokurtic distributions and it is notably that standard deviation is non-robust to slight deviations from normality. There are many robust measures of scale available in literature. A robust estimator is an estimator which is insensitive to changes in the underlying distribution and also resistant against the presence of outliers. Several authors have worked on robust control charts and several robust measures are experimented in process control.

The present study introduces the Comparison between another robust measure of dispersion namely Modified (**Six Sigma and median absolute deviation**) based on Shewhart mean control chart.

**Keywords:** quality control chart, mean chart, six sigma, capability, median absolute deviation.

1. **INTRODUCTION**

A control chart is an important aid or statistical device used for the study and control chart of the repetitive Process. Control charts were developed by Dr.W.A. Shewhart and it is based upon the fact that variability does exist in all the repetitive processes. Specifying the control limits is the most important step in designing a control chart. When the limits are narrow, the risk of a point falling beyond the limits increases, and hence increase the false indication that the process is out of control. If the limits are wider, the risk increases the points falling within the limits, falsely indicates that the process is in control (Abu-Shawiesh, 2008). There are many measures of scale available in literature and many control charts are available to control process dispersion or the dispersion of quality characteristics.

**Quality Control Charts** [3] [2]

 A quality control chart (also called process chart) is a graph that shows average for the data (output) or the product fall within the common or normal range of variation if the process is under statistical control. Quality control charts were first invented by Walter A. Shewhart, and developed by him and his associate. He published a complete exposition of control charts in 1931. Which used by Shewhart in the construction of his charts. He concluded that a distribution can be transformed into a normal shape by estimating its mean and standard deviation. Shewhart`s idea was whether the production process is going well and naturally and the points plotted on the chart follow a normal distribution. For these reasons, Shewhart resorted to use the normal distribution in the construction of his charts.

Shewhart control charts consist of three parallel lines which are:

1. Center Line (or target line) of the control chart is the mean, or overall average, of the quality characteristic that is being measured, and symbolized as T.
2. The upper control limit (UCL) is the maximum acceptable variation from the mean for a process that is in a state of control.

 Mathematically expressed as:$ UCL=T+3σ$

1. Lower control limit (LCL) is the minimum acceptable variation from the mean for a process that is in a state of control.

 Mathematically expressed as:$ LCL=T-3σ$



**Figure 1: Normal distribution of continuous variables**

**Classification of control charts** [3] [2]

Control charts may be classified into two main types, which are:

# 1. Variable Control Charts

 These charts are used in process control of products when the items produced are measurable (in one of the units of measurement).

 The most important types of variable control charts can be divided into two types:

**1.1 Shewhart Variable Charts:** the most familiar Shewhart charts are

1. Average – Chart (or $\overbar{x}$- chart)
2. Standard Deviation- Chart (S-chart)
3. Range - Chart (R-chart)
4. Individual – Chart(X- chart).
5. Median chart (Me- chart).

**2.1** **Non-Shewhart Variable Charts:** the most popular

* **Classic Variable Charts:** [3] [2]
1. Cumulative sum [control chart](http://en.wikipedia.org/wiki/Control_chart)(CUSUM-chart)
2. Moving average chart (MA-chart)
3. Moving range chart (MR-chart)
4. Geometric Moving average chart (GMA-chart)

**2. Attributes Control Charts**

Attribute control charts are used when:

(a) Measurements are not possible (e.g., defect such as dented cans).

(b) Measurements are not practical (e.g., lengthy chemical analyses of raw products).

(c) Several characteristics are combined on one chart (e.g., counts of different kinds of defects). In this case, the various characteristics can be lumped together into a single chart, or at most two or three charts, each covering that group of characteristics which reflects their importance such as minor, major, and critical. [3]

**The attribute control charts can be classified into:** [2]

1. Defective or nonconforming chart. p-chart (fraction nonconforming)
2. np-chart (number nonconforming).
3. Defects or nonconformities charts. C-chart (number of nonconformities).
4. U-chart (average number of nonconformities).

The following are charts used in this paper:

* **X-bar chart based on Three sigma(**$3σ)$

Use **x-bar** charts to monitor the changes in the mean of a process,

The center line of the x-bar chart is calculated as follows:



The control limits of the mean chart based on three sigma is calculated as follows:



* **bar-X chart based on Median Absolute Deviation**

The Median Absolute Deviation from the sample median (MAD) is a very robust scale estimator than the sample standard deviation (Abu-Shawiesh, 2008). It measures the deviation of the data from the sample median. The MAD is often used as an initial value for the computation of more efficient robust estimators. Let be a simple random sample of size n observation taken over m subgroups, and then the MAD is defined as [4]:

 MAD = 1.4826MD {|Xi - MD|} …..(2)

Where, MD is the median of. The average of the MAD is computed using:

 …. (3)

 When MAD is used as an estimate of variability, then MAD will be used as a replacement of the standard deviation (S), thus,  (Abu-Shawiesh, 2008). The control limits and central line for the mean control chart based on the MAD are calculated as follows:

UCL= 

CL=  …. (4)

LCL= 

Let A5 =, then the control limits in Eq. 4 will be reduced to Eq.:

UCL= 

CL=  …. (5)

LCL= 

The values of () are computed and are presented in Table (A) in the appendix for various values of n.

* **X-bar chart based on Six Sigma** ()

Six Sigma has been successfully applied in other manufacturing sectors. Developed by Motorola in 1983, from the statistical point of view, the term six sigma is defined as having less than 3.4 defects per million opportunities (DPMO) or a success rate of 99.9997% where sigma is a term used to represent the variation about the process average. If an organization is operating at three sigma levels for quality control [5], this is interpreted as achieving a success rate of 93% or 66,800 defects per million opportunities. Therefore, the six sigma method is a very rigorous quality control concept where many organizations still performs at three sigma level. The generating unit. Will include defining the importance of the quality of critical elements as defined by the consumer, the reduction to the minimum deviation in the operations and improve the capacity and increase the consistency and stability of the manufacturing process,

As well as systems that help to achieve the objectives of 6 Sigma design. If the companies practicing Six Sigma initiatives use the control limits suggested by Shewhart, then no point fall outside the control limits because of the improvement in the quality of the process. Which adopt Six Sigma Initiatives, (RADHAKRISHNAN and BALAMURUGAN, 2016 )

Concepts and Terminologies

**1/ Upper Specification Limit (USL)**

**2/ Lower Specification Limit (LSL)**

**3/ Tolerance Level (TL)**: It is the difference between USL and LSL, TL = USL-LSL

**4/ Process Capability ()**: This is the ratio of tolerance level to six times standard deviation of the process.

 

**5/ Subgroup Size (n):**

In order to make control chart analysis effective, it is essential to pay due regard to the rational selection of the subgroups. It is the choice of the sample size n and the frequency of sampling. It is also the number of observed values in any given sample or subgroup.

**6/ Quality Control Constant**

To construct the Six Sigma based control limits the Quality Control constants such as are introduced in this thesis.



**7/ Calculate () and construction six sigma chart**

 In this section a procedure to construct a six sigma based control chart for mean. Fix the Tolerance level (TL) and process capability () to determine the process standard deviation (). Apply the value of in the control limits  , to get the Six Sigma based control limits for Mean using Range. The value of  is obtained using  and z is a standard normal variate. For a specified TL and  of the process, the value of  (termed as) is calculated from presented in Table (B) for various combinations of TL and. Further the value of is also obtained using the procedure given above and are presented in Table (C), for different values of n. The six sigma based control limits for mean using range are constructed as:

 

 

**Application**

The example provided by Mohammed, S.O. (2013, Page No. 45) is considered here. The following data are the results of width (mm) of (PP Box strapping, Plastic Maudling components, and corrugated Boxes) bases for samples of five measurements.

(For width = 12mm)

Data collection (width)

Table 1: width of (PP Box strapping, Plastic Maudling components, and corrugated Boxes).

|  |  |  |  |
| --- | --- | --- | --- |
| Sample no. | Sample observation | Mean | MAD |
| 1 | 11.55 | 11.91 | 11.43 | 11.3 | 11.42 | 11.522 | 0.177912 |
| 2 | 11.57 | 12.15 | 11.16 | 12.18 | 11.65 | 11.742 | 0.726474 |
| 3 | 11.66 | 11.8 | 12 | 11.76 | 11.55 | 11.754 | 0.14826 |
| 4 | 12.29 | 11.8 | 12.22 | 12 | 11.88 | 12.038 | 0.29652 |
| 5 | 12.2 | 11.87 | 12.2 | 12.55 | 12.25 | 12.214 | 0.07413 |
| 6 | 11.8 | 11.85 | 11.75 | 12.15 | 12.05 | 11.92 | 0.14826 |
| 7 | 11.6 | 11.55 | 11.6 | 11.55 | 11.6 | 11.58 | 0 |
| 8 | 11.55 | 11.6 | 11.55 | 11.6 | 11.55 | 11.57 | 0 |
| 9 | 11.6 | 11.55 | 11.6 | 11.55 | 11.6 | 11.58 | 0 |
| 10 | 11.55 | 11.6 | 11.55 | 11.6 | 11.55 | 11.57 | 0 |
| 11 | 11.6 | 11.55 | 11.6 | 11.55 | 11.6 | 11.58 | 0 |
| 12 | 11.55 | 11.6 | 11.55 | 11.6 | 11.55 | 11.57 | 0 |
| 13 | 11.57 | 11.55 | 11.16 | 11.16 | 11.65 | 11.418 | 0.14826 |
| 14 | 11.8 | 11.66 | 11.76 | 11.55 | 12 | 11.754 | 0.14826 |
| 15 | 12.05 | 12 | 11.8 | 11.66 | 11.55 | 11.812 | 0.29652 |

* **Construction of Control Limits three Sigma for Shewhart (Xbar –Chart)**

CL= 

CL= 

LCL= 

* **Construction of Control Limits (MAD) for Xbar -Chart**

UCL= 

CL= 

LCL= 

(From the Table A, for sample size n =5).

* **Construction of Control Limits Six Sigma for X-bar Chart**

For a given (TL =USL - LSL = 12.55 – 11.16= 1.39) & CP = 1.45, it is found from the Table– B that the value of is 0. 15, the value of 6 is obtained from Table– C for n = 5 as 2.16 and the Six Sigma based control limits for X-bar chart.



 

**Table2.** Control limits of control charts based on scale estimators

|  |  |  |  |
| --- | --- | --- | --- |
|  | Three sigma | MAD | Six sigma |
| LCL | 12.36 | 12.23 | 12.02 |
| CL | 11.70 | 11.70 | 11.70 |
| UCL | 11.06 | 11.19 | 11.38 |
| Out of control | 0 | 0 | 2 |

**Mean chart based on MAD**

0

3

6

9

12

15

Sub groups

11

11.3

11.6

11.9

12.2

12.5

CL = 11.70

UCL = 12.23

LCL = 11.19

**Mean chart based on six sigma**

0

3

6

9

12

15

Sub groups

11.3

11.5

11.7

11.9

12.1

12.3

CL = 11.70

UCL = 12.0.2

LCL = 11.38

 **Figure 3: x-bar chart based on six sigma Figure 2: x-bar chart based on MAD**

**Mean chart based on (3) sigma**

0

3

6

9

12

15

Subgroups

11

11.3

11.6

11.9

12.2

12.5

CL = 11.70

UCL = 12.36

LCL = 11.06

 **Figure 4: x-bar chart based on three sigma**

1. **CONCLUSION**

 When we compare robust methods, six sigma method have better property than the other robust methods, because control chart based on six sigma have the smallest control interval and also have the most number of out of control points.

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APPENDIX

Table B:  Values for a specified Cp and TL Table A: The Control limits factors for X bar chart based MAD

|  |  |
| --- | --- |
| **n** |  |
| 2 | 3.58 |
| 3 | 4.48 |
| 4 | 4.08 |
| 5 | 3.618 |
| 6 | 3.6 |
| 7 | 3.42 |
| 8 | 3.38 |
| 9 | 3.32 |
| 10 | 3.26 |

|  |  |
| --- | --- |
| **CP** | **TL=1.39** |
| 1 | 0.2363 |
| 1.1 | 0.2085 |
| 1.2 | 0.1807 |
| 1.3 | 0.1946 |
| 1.4 | 0.1668 |
| 1.5 | 0.1529 |
| 1.6 | 0.139 |
| 1.7 | 0.139 |
| 1.8 | 0.1251 |
| 1.9 | 0.1251 |
| 2 | 0.1112 |
| 2.1 | 0.1112 |
| 2.2 | 0.1112 |
| 2.3 | 0.0973 |
| 2.4 | 0.0973 |
| 2.5 | 0.0973 |

 Table C: Values for a Specified Subgroup Size (n)

|  |  |
| --- | --- |
| **n** |  |
| 2 | 3.416033 |
| 3 | 2.789179 |
| 4 | 2.4155 |
| 5 | 2.160489 |
| 6 | 1.972247 |
| 7 | 1.825946 |
| 8 | 1.708016 |
| 9 | 1.610333 |
| 10 | 1.527696 |