**Using the gray model technique to study the monthly consumption of electrical power in the city of** [**Sulaymaniyah**](https://www.accuweather.com/en/iq/sulaymaniyah/206978/weather-forecast/206978)

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**Abstruct**

 Electric energy consumption is the form of [energy consumption](https://en.wikipedia.org/wiki/Energy_consumption) that uses [electric energy](https://en.wikipedia.org/wiki/Electric_energy). Electric energy consumption is the actual energy demand made on existing electricity supply it is measured in watt-time, or precisely multiples of watt-hours (written [W·h](https://en.wikipedia.org/wiki/Kilowatt_hour), equal to [watts](https://en.wikipedia.org/wiki/Watt) x hours) which is the universal measure of Energy, Electric and electronic devices consume electric energy to generate desired output (i.e., light, heat, motion, etc.). During operation, some part of the energy—depending on the [electrical efficiency](https://en.wikipedia.org/wiki/Electrical_efficiency)—is consumed in unintended output, such as waste heat. Electric energy consumption forecasting can be regarded as a grey system with unknown and known information, so be analyzed by grey system theory [10]. Grey models require only a limited amount of data to estimate the behavior of unknown systems. This paper focus on the forecasts the electricity power consumption in [Sulaymaniyah](https://www.accuweather.com/en/iq/sulaymaniyah/206978/weather-forecast/206978) city. The result shows that, the model GM (1, 1) has higher forecasting precision and extensive application.

**Keywords:** Electricity power consumption; grey system theory; GM (1, 1) model, direct forecasting

**1 Methodology**

In this section, we focus on the grey system theory and Grey forecasting model. Then, we will give out the introduction of GM (1, 1).

**2.1 Grey forecasting model** [3],[9]

Deng (2002) proposed grey system theory to study the uncertainty of a system .As far as information is concerned, the systems which lack information ,such as structure message, operation mechanism and behavior document, are referred to as grey prediction ,grey relation, grey decision , grey programming and grey control. Grey model is the core of grey system theory , which collect available data to obtain the internal regularity without using any assumptions. The intention is to make forecasting useful for decision and policy makers who need future prediction. In recent years, grey forecasting theory has receiving increasingly attention from scientific researchers. It has been applied to industry, agriculture and scientific studies etc. many fields and achieve satisfied results in prediction. During the process of building grey forecasting model, in essence, it mainly works by accumulating original data to a new data column which has strong regularity and builds a new model; then generate reduction model from this new data column by inverse treatment and finally acquire forecasting model from the above reduction model. Grey forecasting is a quantitative prediction by use of Grey Model, GM (1, n). And GM (1, 1) is the most commonly used grey forecasting model. It is a model composed by first-order differential equation where only contains one variable . And it is an effective forecasting model in real estate sales. After the establishment of a good model, it is necessary to test the accuracy of the model. The mainly accuracy test of grey system theory are relative error method, Correlation method and Posteriori difference method. Here we mainly make use of Posteriori difference method to test the accuracy of model.

**2.2 Fundamental of Grey Theory [3],[4],[5] ,[7]**

**The GM (1, 1) model:** The grey prediction is based on GM (n, m) where n is the order of grey difference equation and m is the number of variables. Among the family of grey prediction model, most of the pervious researchers have focused on GM (1, 1) model in their predictions. GM (1, 1) model ensure a fine agree between simplicity and accuracy of the results.

A non-negative sequence of raw data as:

$ x^{\left(0\right)}=\left\{x^{\left(0\right)}\left(1\right), x^{\left(0\right)}\left(2\right),…,x^{\left(0\right)}\left(n\right)\right\} , n\geq 4$ (2-2-1)

Where, n is the total number (sample size) of modeling data.

Accumulating Generation Operator (AGO) is used to smooth the randomness of primitive sequence. The AGO converting the original sequence into a monotonically increasing sequence. A new sequence X(1) is generated by AGO as:

$ x^{(1)}=\{x^{\left(1\right)}\left(1\right), x^{\left(1\right)}\left(2\right),…,x^{\left(1\right)}\left(n\right)\} , n\geq 4$ (2-2-2)

Where,

$$x^{\left(1\right)}\left(1\right)=x^{\left(0\right)}\left(1\right)$$

$x^{\left(1\right)}\left(k\right)=\sum\_{j=1}^{k}x^{(0)}, \left(k=1,2,…,n\right).$ (2-2-3)

The GM (1, 1) model can be constructed by establishing a first order differential equation for x(1) (k) as:

$\frac{dx^{(1)}k}{dk}+aX^{\left(1\right)}(k)=u$ (2-2-4)

Where parameters **a** and **u** are called the developing coefficient and grey input, respectively.

In practice, parameters **a** and **u** are not calculated directly from Eq. (2-4). Therefore, the solution, also known as time response function, of above equation is given by

$\hat{x}^{\left(1\right)}\left(k+1\right)=\left[x^{\left(0\right)}\left(1\right)-\frac{u}{a}\right]e^{-ak}+\frac{u}{a}$ (2-2-5)

Where, $\hat{x}^{\left(1\right)}\left(k+1\right) $denotes the prediction x at time point k+1 and the coefficients [a,u]T can be obtained by the Ordinary Least Squares (OLS) method:

$\hat{a}=[a,u]^{T}=(B^{T}B)^{-1}B^{T}Y $ (2-2-6)

In that:

 $ Y\_{N}=[X^{\left(0\right)}\left(2\right),X^{\left(0\right)}\left(3\right),…,X^{\left(0\right)}\left(n\right)]^{T}$

$B=\left[\begin{matrix}-\frac{1}{2}(x^{\left(1\right)}\left(1\right)+x^{\left(1\right)}\left(2\right))&1\\-\frac{1}{2}(x^{\left(1\right)}\left(2\right)+x^{\left(1\right)}\left(3\right))&1\\\begin{matrix}.\\.\\-\frac{1}{2}(x^{\left(1\right)}\left(n-1\right)+x^{\left(1\right)}\left(n\right))\end{matrix}&\begin{matrix}.\\.\\1\end{matrix}\end{matrix}\right]$ (2-2-7)

Inverse AGO (IAGO) is used to find predicted values of primitive sequence. By using the IAGO:

$\hat{x}^{\left(0\right)}\left(k+1\right)=[x^{\left(0\right)}\left(1\right)-\frac{u}{a}](1-e^{a})e^{-ak}$ (2-2-9)

Therefore, the fitted and predicted sequence $\hat{ x}^{\left(0\right)}$ is given as:

$\hat{x}^{\left(0\right)}=\left(\hat{x}^{\left(0\right)}\left(1\right),\hat{x}^{\left(0\right)}\left(2\right),…, \hat{x}^{\left(0\right)}\left(n\right),..\right)$ (2-2-10)

and

$$x̂^{(0)}\left(1\right)=x^{\left(0\right)}\left(1\right) , \left(k=2,3,…,n\right).$$

Where

$$\hat{x}^{\left(0\right)}\left(1\right),\hat{x}^{\left(0\right)}\left(2\right),…, \hat{x}^{\left(0\right)}\left(n\right)$$

are called the GM (1, 1) fitted sequence while:

$$\hat{x}^{\left(0\right)}\left(n+1\right),\hat{x}^{\left(0\right)}\left(n+2\right),…,$$

are called the GM (1, 1) forecast values.

Advantages of GM (1,1) mode : [8]

* The possibility of modeling a system under the incomplete information’s,
* Applied to the short time series,
* Apply to build a short-term forecasts,
* Easy calculations.

Weaknesses of GM (1, 1) model :[8]

* Model can be used only for positive realizations of forecasting variable,
* The problem of recognition of a classical random component,
* The problem with the conventional approach to the validation process of the model.

The applicability of GM(1,1) model [8]

* Short time series,
* Short term forecasts,
* Smoothing time series.

**2.3 Model Evaluation and accuracy Indicators**

GM (1, 1) model of the test is divided into three aspects: residual examination correlation degree examination. Posterior difference test .[3]

**Residual Test:[6]**

**Step1:** The size of the residual error inspection, namely the model value and the actual value of the residual point by point inspection is according to the first model to calculate

$x^{\left(0\right)}\left(i+1\right) , $ will be $x^{\left(0\right)}\left(i+1\right)$ regressive generation $x^{\left(0\right)}\left(i\right)$, finally calculated the original sequence $x^{\left(0\right)}\left(i\right)$ and absolute residual sequence and relative residual sequence

 $∆^{\left(0\right) }$= $\left\{∆^{\left(0\right)}\left(i\right) , i=1,2,…,n\right\}$

$∆^{\left(0\right)}\left(i\right)$=$\left|x^{\left(0\right)}\left(i\right)-\hat{x}^{(0)}\left(i\right)\right| $ $i=1,2,……,n$ (2-3-11)

$ ∅\_{(i)}=\left[\frac{∆^{\left(0\right)}\left(i\right)}{x^{\left(0\right)}\left(i\right)}\right]×100\%$ $∅=\{∅\_{i}, i=1,2,……,n\}$

**Step2:** Compute the two –step minimum and maximum of the relative error:

$∆\_{min}=min\left|∆^{\left(0\right)}\left(i\right)\right|$ (2-3-13)

$∆\_{max}=max\left|∆^{\left(0\right)}\left(i\right)\right|$ (2-3-14)

**Step3:** Compute coefficients of grey incidence

$γ\left(\hat{x}^{\left(0\right)}\left(k\right),x^{\left(0\right)}\left(k\right)\right)=\frac{∆\_{min}+p.∆\_{max}}{∆\_{oi}\left(k\right)+p.∆\_{max}}$ (2-3-15)

**Step4:** Compute the degree of grey incidence

$γ\left(\hat{x}^{\left(0\right)},x^{\left(0\right)}\right)$=$\frac{1}{n}\sum\_{i=1}^{n}γ\left(\hat{x}^{\left(0\right)}\left(i\right),x^{\left(0\right)}\right)$ (2-3-16)

**Mean Absolute Percentage Error (MAPE)** [1]

The overall accuracy of this forecasting model can be measured by its means absolute percentage error (MAPE). It is defined as follows:

Minimize: $MAPE=\frac{1}{n}\sum\_{k=2}^{n}\left|\frac{x^{\left(0\right)}\left(k\right)-\hat{x}^{\left(0\right)}\left(k\right)}{x^{\left(0\right)}\left(k\right)}\right|\*100\% $ , $k=1,2,..,n$ (2-3-17)

Where $x^{\left(0\right)}\left(k\right)$ and $\hat{x}^{\left(0\right)}\left(k\right) $are the actual and forecasted values, respectively, and n is the number of forecasts.

**Precision Rate (p)** [5]

Precision Rate, which measures the level of the closeness of the statement of forecast quantity and the actual value, p is defined as follows:

$p = 1 – MAPE$ (2-3-18)

|  |  |
| --- | --- |
| **Precision rank** | **Precision Rate** |
| Highly accurate | 𝑝 ≥ 0.99 |
| Good | 𝑝 ≥ 0.95 |
| Reasonable | 𝑝 ≥ 0.90 |
| Inaccurate | 𝑝 ≤ 0.90 |

 Table (1): Shows the levels of precision rate (p)

**Posterior Difference Test**[9]

Posterior difference inspection is i.e. on residual distribution statistical properties of the inspection.

1. Calculate the average of the original sequence:

$$\overbar{x}^{(0)}=\frac{1}{n}\sum\_{i=1}^{n}x^{(0)}(i)$$

1. Calculating the original sequence $x^{(0)} $mean square error：

$$S\_{1}=\left(\frac{\sum\_{i=1}^{n}\left[x^{\left(0\right)}\left(i\right)-\overbar{x}^{\left(0\right)}\right]^{2}}{n-1}\right)^{\frac{1}{2}}$$

1. Calculating residual mean:

$$\overbar{∆}=\frac{1}{n}\sum\_{i=1}^{n}\overbar{∆}^{(0)}(i)$$

$$S\_{2}=\left(\frac{\sum\_{i=0}^{n}\left[∆^{\left(0\right)}\left(k\right)-\overbar{∆}^{}\right]^{2}}{n-1}\right)^{\frac{1}{2}}$$

1. Calculation variance ratio C:

 $c = \frac{s2 }{s1}$ (2-3-19)

1. Calculation small residual probability:

$$P=P\left\{\left|∆^{\left(0\right)}\left(i\right)-\overbar{∆}^{}\right|<0.6745S\_{1}\right\}$$

Make

$S\_{0}=0.6745S\_{1}$ , $e\_{i}=\left|∆^{\left(0\right)}\left(i\right)-\overbar{∆}^{}\right| , p=p\{e\_{i}<s\_{0}\}$

If for a given $C\_{0}>0$, when $C\_{}>C\_{0}$ , say the model for mean variance ratio qualified model. As for the given $P\_{0}>0$ , when $P>P\_{0}$, say the model for small residual probability qualified model.

Table (2): Show the posterior difference test discriminant reference table

|  |  |  |
| --- | --- | --- |
|  **P** |  **C** |  **Model accuracy** |
| ﹥0.95 | ﹤0.35 | Optimal |
| ﹥0.80 | ﹤0.5 | Qualified |
| ﹥0.70 | ﹥0.65 | Forced to pass the exam |
| ﹤0.70 | ﹥0.65 | The unqualified |

**3 Empirical Analyses**

In order to show the predictive ability of GM (1,1) model, Grey model was used to forecast electricity power consumption as $X^{(0)}$in [Sulaymaniyah](https://www.accuweather.com/en/iq/sulaymaniyah/206978/weather-forecast/206978) city during the year from 2007 to 2016.

The figure below show the data collection form the Statistical unit in Directorate of electricity power consumption in [Sulaymaniyah](https://www.accuweather.com/en/iq/sulaymaniyah/206978/weather-forecast/206978) Governorate



Figure( 1): Show the electricity power consumption in [Sulaymaniyah](https://www.accuweather.com/en/iq/sulaymaniyah/206978/weather-forecast/206978) from Jan 2007 to Dec 2016

Table (3): Shows the mean of the electricity power consumption

|  |  |
| --- | --- |
| **Years** | **Mean** |
| 2007 | 197.4167 |
| 2008 | 184.9167 |
| 2009 | 187.25 |
| 2010 | 376.9167 |
| 2011 | 456.6667 |
| 2012 | 617.1667 |
| 2013 | 661.9167 |
| 2014 | 729.75 |
| 2015 | 753.0833 |
| 2016 | 722.4167 |

**3.1 Model Specifications**

The procedure of Grey model GM (1, 1) can be shown in steps follow:

**Step1:** The table below shows that the original sequence (X (0) (t)) and the structure of accumulated generating sequence (X (1) (t)) by Eq. (2-2-2):

Table (4): Original Sequence structure of accumulated generating sequence

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Time / Month** | **X(0)** | **X(1)** | **Time / Month** | **X(0)** | **X(1)** |
| Jan-07 | 166 | 166 | Jan-12 | 487 | 17325 |
| Feb-07 | 185 | 351 | Feb-12 | 583 | 17908 |
| Mar-07 | 184 | 535 | Mar-12 | 580 | 18488 |
| Apr-07 | 168 | 703 | Apr-12 | 608 | 19096 |
| May-07 | 182 | 885 | May-12 | 531 | 19627 |
| Jun-07 | 290 | 1175 | Jun-12 | 443 | 20070 |
| Jul-07 | 274 | 1449 | Jul-12 | 648 | 20718 |
| Aug-07 | 283 | 1732 | Aug-12 | 752 | 21470 |
| Sep-07 | 173 | 1905 | Sep-12 | 757 | 22227 |
| Oct-07 | 143 | 2048 | Oct-12 | 761 | 22988 |
| Nov-07 | 145 | 2193 | Nov-12 | 742 | 23730 |
| Dec-07 | 176 | 2369 | Dec-12 | 514 | 24244 |
| Jan-08 | 142 | 2511 | Jan-13 | 536 | 24780 |
| Feb-08 | 191 | 2702 | Feb-13 | 636 | 25416 |
| Mar-08 | 149 | 2851 | Mar-13 | 715 | 26131 |
| Apr-08 | 150 | 3001 | Apr-13 | 746 | 26877 |
| May-08 | 136 | 3137 | May-13 | 603 | 27480 |
| Jun-08 | 227 | 3364 | Jun-13 | 490 | 27970 |
| Jul-08 | 230 | 3594 | Jul-13 | 618 | 28588 |
| Aug-08 | 232 | 3826 | Aug-13 | 783 | 29371 |
| Sep-08 | 214 | 4040 | Sep-13 | 821 | 30192 |
| Oct-08 | 197 | 4237 | Oct-13 | 791 | 30983 |
| Nov-08 | 172 | 4409 | Nov-13 | 675 | 31658 |
| Dec-08 | 179 | 4588 | Dec-13 | 529 | 32187 |
| Jan-09 | 114 | 4702 | Jan-14 | 538 | 32725 |
| Feb-09 | 99 | 4801 | Feb-14 | 691 | 33416 |
| Mar-09 | 116 | 4917 | Mar-14 | 791 | 34207 |
| Apr-09 | 120 | 5037 | Apr-14 | 784 | 34991 |
| May-09 | 149 | 5186 | May-14 | 662 | 35653 |
| Jun-09 | 210 | 5396 | Jun-14 | 533 | 36186 |
| Jul-09 | 229 | 5625 | Jul-14 | 701 | 36887 |
| Aug-09 | 228 | 5853 | Aug-14 | 891 | 37778 |
| Sep-09 | 190 | 6043 | Sep-14 | 907 | 38685 |
| Oct-09 | 225 | 6268 | Oct-14 | 849 | 39534 |
| Nov-09 | 293 | 6561 | Nov-14 | 753 | 40287 |
| Dec-09 | 274 | 6835 | Dec-14 | 657 | 40944 |
| Jan-10 | 289 | 7124 | Jan-15 | 639 | 41583 |
| Feb-10 | 301 | 7425 | Feb-15 | 747 | 42330 |
| Mar-10 | 372 | 7797 | Mar-15 | 834 | 43164 |
| Apr-10 | 361 | 8158 | Apr-15 | 822 | 43986 |
| May-10 | 338 | 8496 | May-15 | 718 | 44704 |
| Jun-10 | 312 | 8808 | Jun-15 | 619 | 45323 |
| Jul-10 | 375 | 9183 | Jul-15 | 776 | 46099 |
| Aug-10 | 410 | 9593 | Aug-15 | 780 | 46879 |
| Sep-10 | 440 | 10033 | Sep-15 | 699 | 47578 |
| Oct-10 | 454 | 10487 | Oct-15 | 883 | 48461 |
| Nov-10 | 455 | 10942 | Nov-15 | 825 | 49286 |
| Dec-10 | 416 | 11358 | Dec-15 | 695 | 49981 |
| Jan-11 | 396 | 11754 | Jan-16 | 666 | 50647 |
| Feb-11 | 447 | 12201 | Feb-16 | 740 | 51387 |
| Mar-11 | 455 | 12656 | Mar-16 | 791 | 52178 |
| Apr-11 | 465 | 13121 | Apr-16 | 776 | 52954 |
| May-11 | 417 | 13538 | May-16 | 636 | 53590 |
| Jun-11 | 384 | 13922 | Jun-16 | 787 | 54377 |
| Jul-11 | 441 | 14363 | Jul-16 | 770 | 55147 |
| Aug-11 | 446 | 14809 | Aug-16 | 666 | 55813 |
| Sep-11 | 497 | 15306 | Sep-16 | 700 | 56513 |
| Oct-11 | 495 | 15801 | Oct-16 | 782 | 57295 |
| Nov-11 | 544 | 16345 | Nov-16 | 725 | 58020 |
| Dec-11 | 493 | 16838 | Dec-16 | 630 | 58650 |

Where X(o) a non-negative sequence and *n* is the size of the data. When this sequence is subjected to the Accumulating Generation Operation (AGO), the following sequence X(1) is obtained. It is obvious that X(1) is monotonically increasing figure below explain it:

Fig (2): The accumulated data set

From the fig. (2) it is clear that the functions derived from AGO formulations of original series, the accumulated generation operation (AGO) is one of the most important characteristics of grey theory, and its main purpose is to reduce the randomness of data.

**Step2:** depending on the Eq. (2-2-6), Eq. (2-2-7) and Eq. (2-2-8), by using OLS method the GM (1, 1) model values of the essential terms are presented as:

$$Type equation here.$$

B= $\left[\begin{array}{c}-258.5 1\\-443 1\\-619 1\\-794 1\\-1030 1\\.\\.\\.\\-55480 1\\-56163 1\\-56904 1\\-57657.5 1\\-58335 1\end{array}\right]$ $Y=\left[\begin{array}{c}185\\184\\168\\182\\290\\.\\.\\.\\666\\700\\782\\725\\630\end{array}\right]$ $\hat{a}= \left[\begin{array}{c}a\\u\end{array}\right]= \left[\begin{array}{c}-0.01163\\236\end{array}\right]$

𝑎: Is intercept of the model is started from ($-0.01163$)

𝑢: Is the slop of the model which equal to ($236$)

**Step3:** depending on Eq. (2-2-9), The Grey prediction model GM (1, 1) can be presented as follow:

$$\hat{X}^{\left(1\right)}\left(k+1\right)=\left(166 - \frac{236}{-0.01163} \right)e^{0.01163k}+ \frac{236}{-0.01163}$$

Applying the inverse accumulated generation operation (IAGO) from

Eq. (2-2-10)

$\hat{ X}^{\left(0\right)}\left(k+1\right)$ = $\hat{X}^{\left(1\right)}\left(k+1\right)$ −$\hat{X}^{\left(1\right)}\left(k\right) , k = 1,2, … , m$

**Step 4:** The relative error and absolute percentage error presented as follow:

$$∆^{\left(0\right)}\left(i\right)= \left|x^{\left(0\right)}\left(i\right)- \hat{x}^{\left(0\right)}(i)\right| i=1,2,…, n$$

$$∅ \left(i\right)= \left|\frac{∆^{\left(0\right)}\left(i\right)}{x^{\left(0\right)}\left(i\right)}\right|\*100\% i=1,2,…, n$$

Based on the grey prediction model (2-2-9), calculation simulation to predict the corresponding value and absolute percentage error:

Table (5): Show the actual value, GM (1, 1) value, and absolute percentage error

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Time / Month** | **Actual Value** | **Model Value** | **APE** |  | **Time / Month** | **Actual Value** | **Model Value** | **APE** |
| Jan-07 | 166 | 166 | 0 |  | Jan-12 | 487 | 475.31 | 0.024 |
| Feb-07 | 185 | 239.32 | 0.2936 |  | Feb-12 | 583 | 480.87 | 0.1752 |
| Mar-07 | 184 | 242.12 | 0.3159 |  | Mar-12 | 580 | 486.5 | 0.1612 |
| Apr-07 | 168 | 244.95 | 0.458 |  | Apr-12 | 608 | 492.19 | 0.1905 |
| May-07 | 182 | 247.82 | 0.3616 |  | May-12 | 531 | 497.94 | 0.0623 |
| Jun-07 | 290 | 250.72 | 0.1355 |  | Jun-12 | 443 | 503.77 | 0.1372 |
| Jul-07 | 274 | 253.65 | 0.0743 |  | Jul-12 | 648 | 509.66 | 0.2135 |
| Aug-07 | 283 | 256.62 | 0.0932 |  | Aug-12 | 752 | 515.62 | 0.3143 |
| Sep-07 | 173 | 259.62 | 0.5007 |  | Sep-12 | 757 | 521.66 | 0.3109 |
| Oct-07 | 143 | 262.65 | 0.8367 |  | Oct-12 | 761 | 527.76 | 0.3065 |
| Nov-07 | 145 | 265.73 | 0.8326 |  | Nov-12 | 742 | 533.93 | 0.2804 |
| Dec-07 | 176 | 268.84 | 0.5275 |  | Dec-12 | 514 | 540.18 | 0.0509 |
| Jan-08 | 142 | 271.98 | 0.9154 |  | Jan-13 | 536 | 546.5 | 0.0196 |
| Feb-08 | 191 | 275.16 | 0.4406 |  | Feb-13 | 636 | 552.89 | 0.1307 |
| Mar-08 | 149 | 278.38 | 0.8683 |  | Mar-13 | 715 | 559.36 | 0.2177 |
| Apr-08 | 150 | 281.64 | 0.8776 |  | Apr-13 | 746 | 565.9 | 0.2414 |
| May-08 | 136 | 284.93 | 1.0951 |  | May-13 | 603 | 572.52 | 0.0505 |
| Jun-08 | 227 | 288.26 | 0.2699 |  | Jun-13 | 490 | 579.22 | 0.1821 |
| Jul-08 | 230 | 291.64 | 0.268 |  | Jul-13 | 618 | 585.99 | 0.0518 |
| Aug-08 | 232 | 295.05 | 0.2718 |  | Aug-13 | 783 | 592.85 | 0.2428 |
| Sep-08 | 214 | 298.5 | 0.3949 |  | Sep-13 | 821 | 599.78 | 0.2694 |
| Oct-08 | 197 | 301.99 | 0.533 |  | Oct-13 | 791 | 606.8 | 0.2329 |
| Nov-08 | 172 | 305.52 | 0.7763 |  | Nov-13 | 675 | 613.9 | 0.0905 |
| Dec-08 | 179 | 309.1 | 0.7268 |  | Dec-13 | 529 | 621.08 | 0.1741 |
| Jan-09 | 114 | 312.71 | 1.7431 |  | Jan-14 | 538 | 628.35 | 0.1679 |
| Feb-09 | 99 | 316.37 | 2.1957 |  | Feb-14 | 691 | 635.7 | 0.08 |
| Mar-09 | 116 | 320.07 | 1.7593 |  | Mar-14 | 791 | 643.13 | 0.1869 |
| Apr-09 | 120 | 323.82 | 1.6985 |  | Apr-14 | 784 | 650.66 | 0.1701 |
| May-09 | 149 | 327.61 | 1.1987 |  | May-14 | 662 | 658.27 | 0.0056 |
| Jun-09 | 210 | 331.44 | 0.5783 |  | Jun-14 | 533 | 665.97 | 0.2495 |
| Jul-09 | 229 | 335.31 | 0.4643 |  | Jul-14 | 701 | 673.76 | 0.0389 |
| Aug-09 | 228 | 339.24 | 0.4879 |  | Aug-14 | 891 | 681.64 | 0.235 |
| Sep-09 | 190 | 343.21 | 0.8063 |  | Sep-14 | 907 | 689.61 | 0.2397 |
| Oct-09 | 225 | 347.22 | 0.5432 |  | Oct-14 | 849 | 697.68 | 0.1782 |
| Nov-09 | 293 | 351.28 | 0.1989 |  | Nov-14 | 753 | 705.84 | 0.0626 |
| Dec-09 | 274 | 355.39 | 0.297 |  | Dec-14 | 657 | 714.1 | 0.0869 |
| Jan-10 | 289 | 359.55 | 0.2441 |  | Jan-15 | 639 | 722.45 | 0.1306 |
| Feb-10 | 301 | 363.75 | 0.2085 |  | Feb-15 | 747 | 730.9 | 0.0216 |
| Mar-10 | 372 | 368.01 | 0.0107 |  | Mar-15 | 834 | 739.45 | 0.1134 |
| Apr-10 | 361 | 372.31 | 0.0313 |  | Apr-15 | 822 | 748.1 | 0.0899 |
| May-10 | 338 | 376.67 | 0.1144 |  | May-15 | 718 | 756.85 | 0.0541 |
| Jun-10 | 312 | 381.08 | 0.2214 |  | Jun-15 | 619 | 765.71 | 0.237 |
| Jul-10 | 375 | 385.53 | 0.0281 |  | Jul-15 | 776 | 774.66 | 0.0017 |
| Aug-10 | 410 | 390.04 | 0.0487 |  | Aug-15 | 780 | 783.73 | 0.0048 |
| Sep-10 | 440 | 394.61 | 0.1032 |  | Sep-15 | 699 | 792.89 | 0.1343 |
| Oct-10 | 454 | 399.22 | 0.1207 |  | Oct-15 | 883 | 802.17 | 0.0915 |
| Nov-10 | 455 | 403.89 | 0.1123 |  | Nov-15 | 825 | 811.55 | 0.0163 |
| Dec-10 | 416 | 408.62 | 0.0177 |  | Dec-15 | 695 | 821.05 | 0.1814 |
| Jan-11 | 396 | 413.4 | 0.0439 |  | Jan-16 | 666 | 830.65 | 0.2472 |
| Feb-11 | 447 | 418.23 | 0.0644 |  | Feb-16 | 740 | 840.37 | 0.1356 |
| Mar-11 | 455 | 423.13 | 0.0701 |  | Mar-16 | 791 | 850.2 | 0.0748 |
| Apr-11 | 465 | 428.08 | 0.0794 |  | Apr-16 | 776 | 860.14 | 0.1084 |
| May-11 | 417 | 433.08 | 0.0386 |  | May-16 | 636 | 870.21 | 0.3682 |
| Jun-11 | 384 | 438.15 | 0.141 |  | Jun-16 | 787 | 880.39 | 0.1187 |
| Jul-11 | 441 | 443.27 | 0.0052 |  | Jul-16 | 770 | 890.68 | 0.1567 |
| Aug-11 | 446 | 448.46 | 0.0055 |  | Aug-16 | 666 | 901.1 | 0.353 |
| Sep-11 | 497 | 453.71 | 0.0871 |  | Sep-16 | 700 | 911.64 | 0.3023 |
| Oct-11 | 495 | 459.01 | 0.0727 |  | Oct-16 | 782 | 922.31 | 0.1794 |
| Nov-11 | 544 | 464.38 | 0.1464 |  | Nov-16 | 725 | 933.1 | 0.287 |
| Dec-11 | 493 | 469.82 | 0.047 |  | Dec-16 | 630 | 475.31 | 0.4984 |

Table (5) shows the real value and forecasting precision obtained from GM (1, 1) model with absolute percentage error.

**Step5:** Test of accuracy of the GM (1, 1) model, depending on some statistical measure we find accuracy of models:

**3-2 Model Evaluation and accuracy Indicators**

**First Test:** Residual test, according to Eq. (2-3-11) compute the relative error and absolute percentage error presented shown in table below:

Table (6): Show the relative error and absolute percentage error

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Time / Month | Error | APE | Time / Month | Error | APE |
| Jan-07 | 0.00 | 0 | Jan-12 | 11.69 | 0.024 |
| Feb-07 | -54.32 | 0.2936 | Feb-12 | 102.13 | 0.1752 |
| Mar-07 | -58.12 | 0.3159 | Mar-12 | 93.50 | 0.1612 |
| Apr-07 | -76.95 | 0.458 | Apr-12 | 115.81 | 0.1905 |
| May-07 | -65.82 | 0.3616 | May-12 | 33.06 | 0.0623 |
| Jun-07 | 39.28 | 0.1355 | Jun-12 | -60.77 | 0.1372 |
| Jul-07 | 20.35 | 0.0743 | Jul-12 | 138.34 | 0.2135 |
| Aug-07 | 26.38 | 0.0932 | Aug-12 | 236.38 | 0.3143 |
| Sep-07 | -86.62 | 0.5007 | Sep-12 | 235.34 | 0.3109 |
| Oct-07 | -119.65 | 0.8367 | Oct-12 | 233.24 | 0.3065 |
| Nov-07 | -120.73 | 0.8326 | Nov-12 | 208.07 | 0.2804 |
| Dec-07 | -92.84 | 0.5275 | Dec-12 | -26.18 | 0.0509 |
| Jan-08 | -129.98 | 0.9154 | Jan-13 | -10.50 | 0.0196 |
| Feb-08 | -84.16 | 0.4406 | Feb-13 | 83.11 | 0.1307 |
| Mar-08 | -129.38 | 0.8683 | Mar-13 | 155.64 | 0.2177 |
| Apr-08 | -131.64 | 0.8776 | Apr-13 | 180.10 | 0.2414 |
| May-08 | -148.93 | 1.0951 | May-13 | 30.48 | 0.0505 |
| Jun-08 | -61.26 | 0.2699 | Jun-13 | -89.22 | 0.1821 |
| Jul-08 | -61.64 | 0.268 | Jul-13 | 32.01 | 0.0518 |
| Aug-08 | -63.05 | 0.2718 | Aug-13 | 190.15 | 0.2428 |
| Sep-08 | -84.50 | 0.3949 | Sep-13 | 221.22 | 0.2694 |
| Oct-08 | -104.99 | 0.533 | Oct-13 | 184.20 | 0.2329 |
| Nov-08 | -133.52 | 0.7763 | Nov-13 | 61.10 | 0.0905 |
| Dec-08 | -130.10 | 0.7268 | Dec-13 | -92.08 | 0.1741 |
| Jan-09 | -198.71 | 1.7431 | Jan-14 | -90.35 | 0.1679 |
| Feb-09 | -217.37 | 2.1957 | Feb-14 | 55.30 | 0.08 |
| Mar-09 | -204.07 | 1.7593 | Mar-14 | 147.87 | 0.1869 |
| Apr-09 | -203.82 | 1.6985 | Apr-14 | 133.34 | 0.1701 |
| May-09 | -178.61 | 1.1987 | May-14 | 3.73 | 0.0056 |
| Jun-09 | -121.44 | 0.5783 | Jun-14 | -132.97 | 0.2495 |
| Jul-09 | -106.31 | 0.4643 | Jul-14 | 27.24 | 0.0389 |
| Aug-09 | -111.24 | 0.4879 | Aug-14 | 209.36 | 0.235 |
| Sep-09 | -153.21 | 0.8063 | Sep-14 | 217.39 | 0.2397 |
| Oct-09 | -122.22 | 0.5432 | Oct-14 | 151.32 | 0.1782 |
| Nov-09 | -58.28 | 0.1989 | Nov-14 | 47.16 | 0.0626 |
| Dec-09 | -81.39 | 0.297 | Dec-14 | -57.10 | 0.0869 |
| Jan-10 | -70.55 | 0.2441 | Jan-15 | -83.45 | 0.1306 |
| Feb-10 | -62.75 | 0.2085 | Feb-15 | 16.10 | 0.0216 |
| Mar-10 | 3.99 | 0.0107 | Mar-15 | 94.55 | 0.1134 |
| Apr-10 | -11.31 | 0.0313 | Apr-15 | 73.90 | 0.0899 |
| May-10 | -38.67 | 0.1144 | May-15 | -38.85 | 0.0541 |
| Jun-10 | -69.08 | 0.2214 | Jun-15 | -146.71 | 0.237 |
| Jul-10 | -10.53 | 0.0281 | Jul-15 | 1.34 | 0.0017 |
| Aug-10 | 19.96 | 0.0487 | Aug-15 | -3.73 | 0.0048 |
| Sep-10 | 45.39 | 0.1032 | Sep-15 | -93.89 | 0.1343 |
| Oct-10 | 54.78 | 0.1207 | Oct-15 | 80.83 | 0.0915 |
| Nov-10 | 51.11 | 0.1123 | Nov-15 | 13.45 | 0.0163 |
| Dec-10 | 7.38 | 0.0177 | Dec-15 | -126.05 | 0.1814 |
| Jan-11 | -17.40 | 0.0439 | Jan-16 | -164.65 | 0.2472 |
| Feb-11 | 28.77 | 0.0644 | Feb-16 | -100.37 | 0.1356 |
| Mar-11 | 31.87 | 0.0701 | Mar-16 | -59.20 | 0.0748 |
| Apr-11 | 36.92 | 0.0794 | Apr-16 | -84.14 | 0.1084 |
| May-11 | -16.08 | 0.0386 | May-16 | -234.21 | 0.3682 |
| Jun-11 | -54.15 | 0.141 | Jun-16 | -93.39 | 0.1187 |
| Jul-11 | -2.27 | 0.0052 | Jul-16 | -120.68 | 0.1567 |
| Aug-11 | -2.46 | 0.0055 | Aug-16 | -235.10 | 0.353 |
| Sep-11 | 43.29 | 0.0871 | Sep-16 | -211.64 | 0.3023 |
| Oct-11 | 35.99 | 0.0727 | Oct-16 | -140.31 | 0.1794 |
| Nov-11 | 79.62 | 0.1464 | Nov-16 | -208.10 | 0.287 |
| Dec-11 | 23.18 | 0.047 | Dec-16 | -314.01 | 0.4984 |

According to Eq. (2-3-11) compute the two–step minimum and maximum of the relative error:

Δ𝑚𝑖𝑛= 𝑚𝑖𝑛|Δ (0) (𝑖)| = 0.0000

Δ𝑚𝑎𝑥= 𝑚𝑎𝑥|Δ (0) (𝑖)| = 148.932

From Eq. (2-3-16) compute coefficients of grey incidence shows in bellow:

$$γ \left(\hat{x}^{\left(0\right)}\left(k\right), x^{\left(0\right)}\left(k\right)\right)= \frac{∆\_{min}+p.∆\_{max}}{∆\_{oi}\left(k\right)+ p.∆\_{max}}$$

Of which the distinguishing coefficient p is 0.5

Table (7): Show the coefficients of grey incidence

|  |  |  |  |
| --- | --- | --- | --- |
| $$γ \left(\hat{x}^{\left(0\right)}\left(k\right), x^{\left(0\right)}\left(k\right)\right)$$ | $$γ \left(\hat{x}^{\left(0\right)}\left(k\right), x^{\left(0\right)}\left(k\right)\right)$$ | $$γ \left(\hat{x}^{\left(0\right)}\left(k\right), x^{\left(0\right)}\left(k\right)\right)$$ | $$γ \left(\hat{x}^{\left(0\right)}\left(k\right), x^{\left(0\right)}\left(k\right)\right)$$ |
| 1 | 0.411913 | 0.864327 | 0.732148 |
| 0.578217 | 0.400994 | 0.421677 | 0.262363 |
| 0.561647 | 0.327076 | 0.44333 | 0.255148 |
| 0.491793 | 0.378602 | 0.391352 | 0.329806 |
| 0.530827 | 0.560957 | 0.692571 | 0.612257 |
| 0.654644 | 0.477783 | 0.550637 | 0.566007 |
| 0.785361 | 0.513506 | 0.349929 | 0.471551 |
| 0.738381 | 0.542673 | 0.239563 | 0.822248 |
| 0.462281 | 0.949141 | 0.24036 | 0.440591 |
| 0.383607 | 0.868097 | 0.242003 | 0.501914 |
| 0.381499 | 0.658198 | 0.263565 | 0.657133 |
| 0.4451 | 0.518773 | 0.739891 | 0.336686 |
| 0.364232 | 0.87607 | 0.876448 | 0.982377 |
| 0.469438 | 0.788651 | 0.472572 | 0.952347 |
| 0.365304 | 0.621278 | 0.323613 | 0.442302 |
| 0.361304 | 0.576169 | 0.292522 | 0.479507 |
| 0.333333 | 0.593008 | 0.709572 | 0.847041 |
| 0.54863 | 0.909803 | 0.454936 | 0.371378 |
| 0.54713 | 0.810616 | 0.699395 | 0.311421 |
| 0.541514 | 0.721341 | 0.28141 | 0.425924 |
| 0.46844 | 0.700262 | 0.251845 | 0.557112 |
| 0.41495 | 0.668513 | 0.287885 | 0.469491 |
| 0.358026 | 0.822382 | 0.54929 | 0.241246 |
| 0.364022 | 0.578982 | 0.44712 | 0.443642 |
| 0.272589 | 0.970359 | 0.451826 | 0.381583 |
| 0.255162 | 0.968021 | 0.573828 | 0.240547 |
| 0.267344 | 0.632354 | 0.334928 | 0.26027 |
| 0.26759 | 0.67419 | 0.358335 | 0.346717 |
| 0.294249 | 0.483284 | 0.952254 | 0.263537 |
| 0.380115 | 0.762575 | 0.358988 | 0.191686 |

According to Eq. (2-3-16) compute the degree of grey incidence as

$γ \left(\hat{x}^{\left(0\right)}, x^{\left(0\right)}\right)= \frac{1}{n}\sum\_{t=1}^{n}γ \left(\hat{x}^{\left(0\right)}\left(i\right), x^{\left(0\right)}\left(i\right)\right)=0.619643 $

From above result it is clear that the model is qualified because

$γ \left(\hat{x}^{\left(0\right)}, x^{\left(0\right)}\right)= 0.619643>0.6 when p=0.5 $

**Second Test:** Mean Absolute Percentage Error (MAPE), depending on Eq. (2-2-17):

$$MAPE= \frac{1}{n} \sum\_{k=2}^{n}\left|\frac{x^{\left(0\right)}\left(k\right)- \hat{x}^{\left(0\right)}\left(k\right) }{x^{\left(0\right)}\left(k\right)}\right|\*100\%=0.297$$

𝑀𝐴𝑃𝐸 = $0.00297 $ < 0.05, MAPE is less than (0.05) depending on the grades of MAPE is a good model with acceptable accuracy.

Then by Eq. (2-3-18):

𝑃𝑟𝑒𝑐𝑖𝑠𝑖𝑜𝑛 (𝑝) = 100 − 𝑀𝐴𝑃𝐸 = 100 – $0.297$ = 99.703

From the table (1) it is clear that the Grey model GM (1, 1) depending the above tests has a good model with acceptable accuracy. The forecasted value of (jan-2017 Dec-2021) from GM (1, 1) model shown in table below:

Table (8): Represent the forecasting values

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Years****Months** | **2017** | **2018** | **2019** | **2020** | **2021** |
| Jan. | 944.01 | 1085.4 | 1248 | 1434.9 | 1649.8 |
| Feb. | 955.06 | 1098.1 | 1262.6 | 1451.6 | 1669 |
| Mar. | 966.23 | 1110.9 | 1277.3 | 1468.6 | 1688.6 |
| Apr. | 977.53 | 1123.9 | 1292.3 | 1485.8 | 1708.3 |
| May | 988.97 | 1137.1 | 1307.4 | 1503.2 | 1728.3 |
| Jun. | 1000.5 | 1150.4 | 1322.7 | 1520.8 | 1748.5 |
| Jul. | 1012.2 | 1163.8 | 1338.1 | 1538.6 | 1769 |
| Aug. | 1024.1 | 1177.5 | 1353.8 | 1556.6 | 1789.7 |
| Sep. | 1036.1 | 1191.2 | 1369.6 | 1574.8 | 1810.6 |
| Oct. | 1048.2 | 1205.2 | 1385.7 | 1593.2 | 1831.8 |
| Nov. | 1060.4 | 1219.3 | 1401.9 | 1611.8 | 1853.2 |
| Dec. | 1072.8 | 1233.5 | 1418.3 | 1630.7 | 1874.9 |

Table (9): Show the comparison mean and standard deviation between last 5 years with 5 years forecasting values

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Years** | **Mean****Current value** | **Years** | **Mean****Forecasting value** | **Diff.** | **Sign** |
| 2012 | 617.1667 | 2017 | 1007.175 | 390.008 | + |
| 2013 | 661.9167 | 2018 | 1158.025 | 496.108 | + |
| 2014 | 729.75 | 2019 | 1331.475 | 601.725 | + |
| 2015 | 753.0833 | 2020 | 1530.883 | 777.8 | + |
| 2016 | 722.4167 | 2021 | 1760.142 | 1037.7 | + |

**4 Conclusions and Recommendations**

**4-1 Conclusions**

1. GM(1,1) is an appropriate approach for electricity power consumption prediction because it can use a limited number of samples to construct a prediction model without statistical assumptions.
2. In recent years energy consumption is increasing due to increase in world population, living standards, industrialization in developing countries. [Sulaymaniyah](https://www.accuweather.com/en/iq/sulaymaniyah/206978/weather-forecast/206978) is one of the fastest developing countries in the Iraq in terms of increase in energy power consumption. Thus, a precise forecast in energy planning process is very important for policy makers
3. In this study, a new method based on GM (1,1) has been proposed to forecast the time series. It has been used to analyze the prediction of the monthly electricity power consumption of [Sulaymaniyah](https://www.accuweather.com/en/iq/sulaymaniyah/206978/weather-forecast/206978) by using the modelling and the test data. The analysis results showed the success of the method. Subsequently it has been used to forecast the monthly electricity power consumption until 2021, and the forecasting values have been used to determine the maximum requirements of electricity production sources. The forecasting values indicated that the maximum requirements of the sources appeared in (6,7,8 and 12) for every year. On the other hand, the forecasting results show that the maximum electricity requirements would achieve the 1874.9 kWh in 2021.
4. If we compare the average use of the amount of electricity power consumption in the last five years with the average use of the amount of electricity power consumption predicted, we see that the average amount of electricity power consumption use was increase normally.
	1. **Recommendations**
5. In the future, following works may be focus on many sectorial areas, namely industrial, residence, transportation and agricultural by using GM, Artificial Neural Networks (ANN) and some meta-heuristic methods such as ant colony optimization, genetic algorithm, annealing simulation and so on. In addition.
6. These results from the model studies are very important for energy policy makers and planners about [Sulaymaniyah](https://www.accuweather.com/en/iq/sulaymaniyah/206978/weather-forecast/206978) electricity power investments.
7. The obtained results are important for government decision makers, investors and related institutions. The average annual electricity power consumption shows the total usage although it does not represent the maximum usage at any time in the year. Hence, prediction of monthly electricity power consumption may help governments in:
8. Determination of maximum capacity of investments
9. Planning the future plant investments
10. Making arrangement of maintenance operations
11. Managing the sustainable electricity supply
12. Managing import and export policies.
13. Future studies may focus on daily forecasting.

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