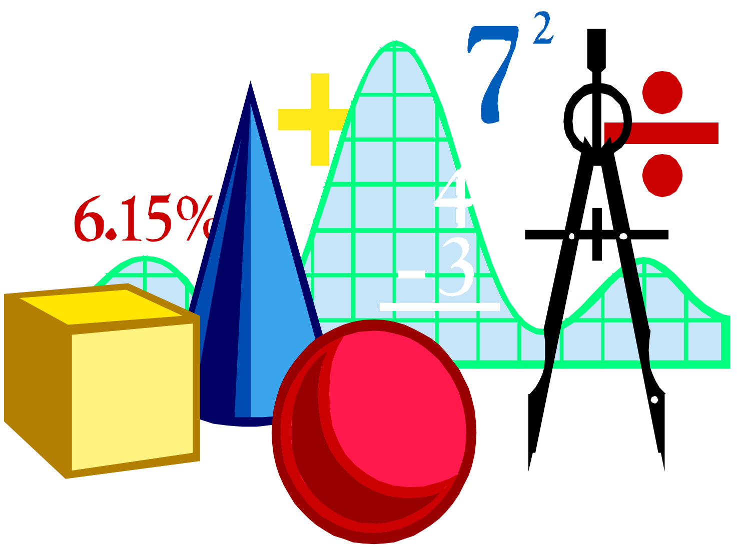
Design and Analysis of Experiments



By

Dr.Akhterkhan S.Hamad

2017 - 2018

Chapter 1

- Introduction to Experimental Design

What is Experimental Design?

Experimental design includes both

- Strategies for organizing data collection
- Data analysis procedures *matched* to those data collection strategies

Classical treatments of design stress analysis procedures based on the analysis of variance (ANOVA)

Other analysis procedure such as those based on hierarchical linear models or analysis of aggregates (e.g., class or school means) are also appropriate

Why Do We Need Experimental Design?

Because of variability

We wouldn't need a science of experimental design

- If all units (students, teachers, & schools) were identical

and

- If all units responded identically to treatments

We need experimental design to control variability so that treatment effects can be identified

The Well-Planned Experiment

- **Simplicity**
  - don’t attempt to do too much
  - write out the objectives, listed in order of priority
- **Degree of precision**
  - appropriate design
  - sufficient replication
- **Absence of systematic error**
- **Range of validity of conclusions**
  - well-defined reference population
  - repeat the experiment in time and space
  - a factorial set of treatments also increases the range
- **Calculation of degree of uncertainty**

Some Definitions

Experiment : Experimentation is the step in the scientific method that helps people decide between two or more competing explanations or hypotheses. These hypotheses suggest reasons to explain a phenomenon, or predict the results of an action

Simple Experiment is an experiment whose study only one variable

Factorial Experiment is an experiment whose design consists of two or more factors, each with discrete possible values or "levels", and whose experimental units take on all possible combinations of these levels across all such factors

- **Treatment** : a condition or set of conditions applied to experimental units in an experiment.
- **Experimental Unit** : the physical entity to which a treatment is randomly assigned and independently applied. or Experimental units are the objects of interest in the experiment. which may be people, parts of people, groups of people, plants, animals.

- **A factor** is a variable that the experimenter has selected for investigation. Or Factor is – an explanatory variable that can take any one of two or more values.
- **Levels**, or settings of each factor. Examples include the oven temperature setting and the amounts of sugar, flour, and eggs
- **Replication** – applying a treatment independently to two or more experimental units. We have included more than one individual in each treatment group

Experimental error: is the variation in the responses among experimental units (e.u.'s) which are assigned the same treatment, and are observed under the same experimental conditions. It is measured by SSE (or MSE).

Systematic errors: an error that is not determined by chance but is introduced by an inaccuracy (as of observation or measurement) inherent in the system .
all of the data is off in the same direction (either to high or too low). Spotting and correcting for systematic error takes a lot of care.

Gross errors : Errors that occur when a measurement process is subject occasionally to large inaccuracies. or

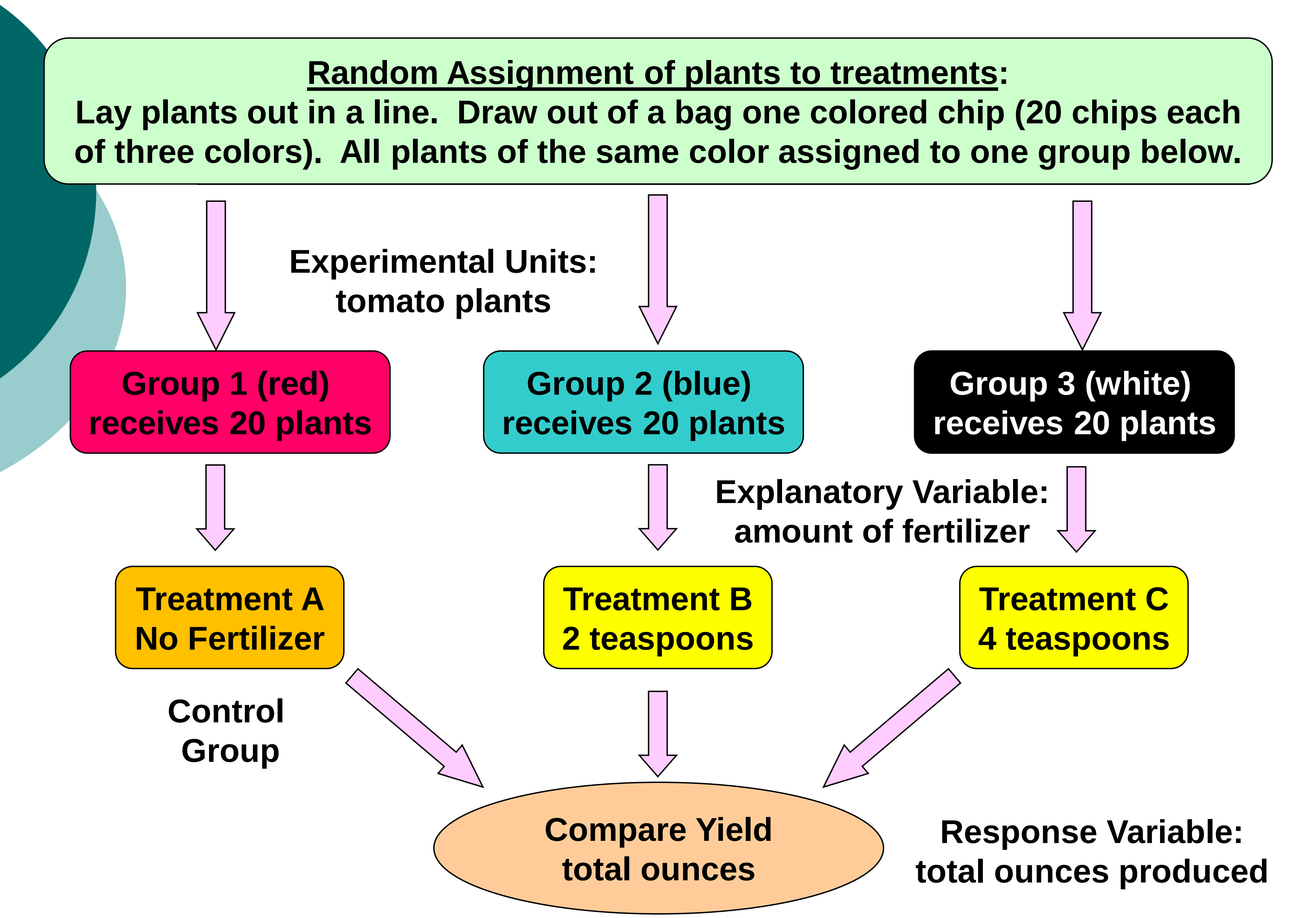
Gross errors are undetected mistakes that cause a measurement to be very much farther from the mean measurement than other measurements.

Example

Draw a picture detailing the following experiment:

A statistics class wants to know the effect of a certain fertilizer on tomato plants. They get 60 plants of the same type. They will have two levels of treatments, 2 and 4 teaspoons of fertilizer. Someone suggests that they should use a control group.

The picture should include enough detail for someone unfamiliar with the problem to understand the problem and be able to duplicate the experiment.


Random Assignment of plants to treatments:
Lay plants out in a line. Draw out of a bag one colored chip (20 chips each of three colors). All plants of the same color assigned to one group below.
Experimental Units: tomato plants
Group 1 (red) receives 20 plants
Group 2 (blue) receives 20 plants
Group 3 (white) receives 20 plants
Explanatory Variable: amount of fertilizer
Treatment A No Fertilizer
Treatment B 2 teaspoons
Treatment C 4 teaspoons
Control Group
Compare Yield total ounces
Response Variable: total ounces produced

Basic Principles of Experimental Design

The basic principles of experimental designs are randomization, replication and local control. These principles make a valid test of significance possible.

(1) Randomization:

The first principle of an experimental design is randomization, which is a random process of assigning treatments to the experimental units. The random process implies that every possible allotment of treatments has the same probability. An experimental unit is the smallest division of the experimental material and a treatment means an experimental condition whose effect is to be measured and compared. The purpose of randomization is to remove bias and other sources of extraneous variation, which are not controllable

Another advantage of randomization (accompanied by replication) is that it forms the basis of any valid statistical test. Hence the treatments must be assigned at random to the experimental units. Randomization is usually done by drawing numbered cards from a well-shuffled pack of cards, or by drawing numbered balls from a well-shaken container or by using tables of random numbers

(2) **Replication:**

The second principle of an experimental design is replication; which is a repetition of the basic experiment. In other words, it is a complete run for all the treatments to be tested in the experiment. In all experiments, some variation is introduced because of the fact that the experimental units such as individuals or plots of land in agricultural experiments cannot be physically identical. This type of variation can be removed by using a number of experimental units. We therefore perform the experiment more than once, i.e., we repeat the basic experiment. An individual repetition is called a replicate. The number, the shape and the size of replicates depend upon the nature of the experimental material. A replication is used

i) To secure more accurate estimate of the experimental error, a term which represents the differences that would be observed if the same treatments were applied several times to the same experimental units;

(ii) To decrease the experimental error and thereby to increase precision, which is a measure of the variability of the experimental error; and

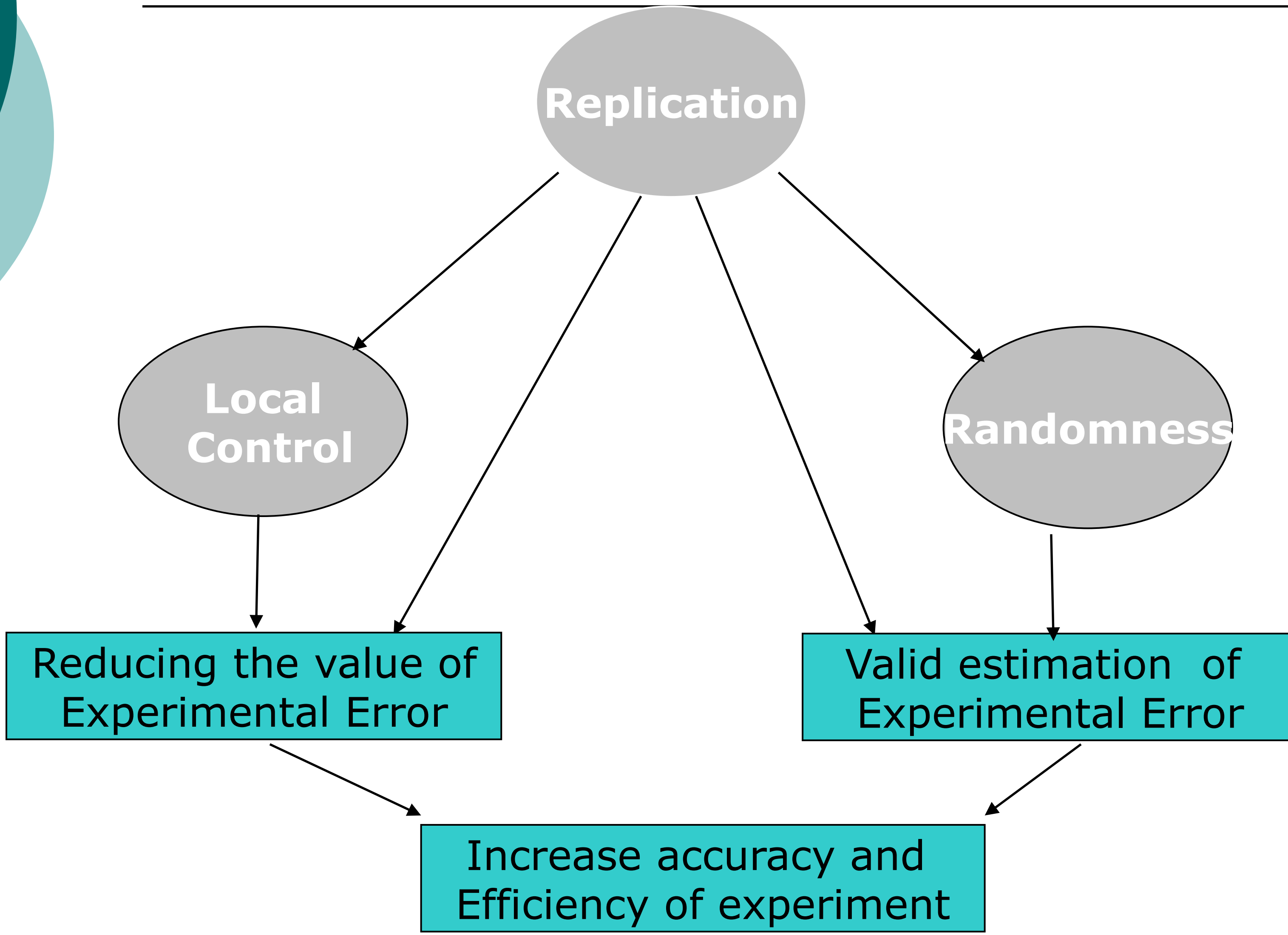
(iii) to obtain more precise estimate of the mean effect of a treatment, since $\sigma^2_{\bar{y}} = \frac{\sigma^2}{n}$, where n denotes the number of replications.

(3) Local Control:

It has been observed that all extraneous sources of variation are not removed by randomization and replication. This necessitates a refinement in the experimental technique. In other words, we need to choose a design in such a manner that all extraneous sources of variation are brought under control. For this purpose, we make use of local control, a term referring to the amount of balancing, blocking and grouping of the experimental units. Balancing means that the treatments should he assigned to the experimental units in such a way that the result is a balanced arrangement of the treatments. Blocking means that like experimental units should be collected together to form a relatively homogeneous group. A block is also a replicate. The main purpose of the principle of local control is to increase the efficiency of an experimental design by decreasing the experimental error.

The point to remember here is that the term local control should not be confused with the word control. The word control in experimental design is used for a treatment. Which does not receive any treatment but we need to find out the effectiveness of other treatments through comparison.

Figure below show the Basic Principles of Experimental Design



Analysis of Variance

- The ANOVA is a technique or statistical methods that sub dividing the total variation in an experiment design to its different resource then testing about them. *To use ANOVA, these assumptions must be satisfied:*

. Assumption 1:- ***<u>Additivity of the Main Effects</u>.***

This assumption means that the value of any observation will be effect by treatments and other effects in the experiment .Which means:-

1) **Treatment effect are constant (fixed).**

Which means inexistence of any interferes between the treatment effect and experimental unit.

2) **The effect of the treatments on the experimental unit** dose not effect on the other experimental unit.

3) **The difference between two treatments** means the difference between the mean of all experimental unit which taken one of the treatment and the mean of all experimental unit which taken another treatment

- **Assumption 2:-** **$\varepsilon_{ij} \sim NID(0, \sigma^2)$.**

  This assumption means that all εij must be normally distributed with zero mean and constant variance.

- **Assumption 3: - Homogeneity of variance.**

  The variances among of all observational units must be the same (are equal for different treatment).

- **Assumption 4:- Independence of means and variance**

This assumption means that μ and σ are uncorrelated.

Terminology

experiment	planned inquiry
treatment	procedure whose effect will be measured
factor	class of related treatments
levels	states of a factor
variable	measurable characteristic of a plot
experimental unit (plot)	unit to which a treatment is applied
replications	experimental units that receive the same treatment
sampling unit	part of experimental unit that is measured
block	group of homogeneous experimental units
experimental error	variation among experimental units that are treated alike

Data Transformation

We convert the data's in the case when one of assumption for ANOVA isn't exist .These transformations methods are;-

- **LOG Transformation**

The log transformation method used when the main effects in the model are multiplicative (not-additive), and when the mean and standard variations are correlated.

$$Y = \log(X). \qquad Y = \log(X + 1)$$

<u>Note</u>:-

- When is zero as values, we add the (1) for each data values.
- We can't able to make this transformation when the negative values are available in data.
  - ***<u>Square Root Transformation</u>***
- These transformation used when the distribution of data is Poisson distribution.

The square root transformation is simply $Y = \sqrt{X}$, although many statisticians recommend the transformation $Y = \sqrt{X + 0.5}$, especially when the variable has one or more 0s. It is often used for counts and for other measures where group means are correlated with within group variances. The square root is often encountered in biology because many biological variables—especially counts—follow a Poisson distribution within groups. Because the mean of a Poisson variable equals the variance of the variable, group means will always be correlated with within-group variances in this case.

The Arcsine Transformation

- Is the most popular transformation used when the data's are in percentage form, for example success rate or death or born rate

Notes: Sometimes called an angular transformation, the arcsine transform equals the inverse sine of the square root of the proportion

$$Y = \text{arcsine}\sqrt{p} = \sin^{-1}\sqrt{p}$$

Where p is the proportion and Y is the result of the transformation. The result may be expressed either in degrees or radians