Chapter 3

RANDOMIZED COMPLETE BLOCK DESIGN (RCBD)

RANDOMIZED COMPLETE BLOCK DESIGN (RCBD)

Description of the Design

- Probably the most used and useful of the experimental designs.

- Takes advantage of grouping similar experimental units into blocks or replicates.

- The blocks of experimental units should be as uniform as possible. يَجِبُ أَنْ تَكُونَ موحّدة بقدر الإمكان

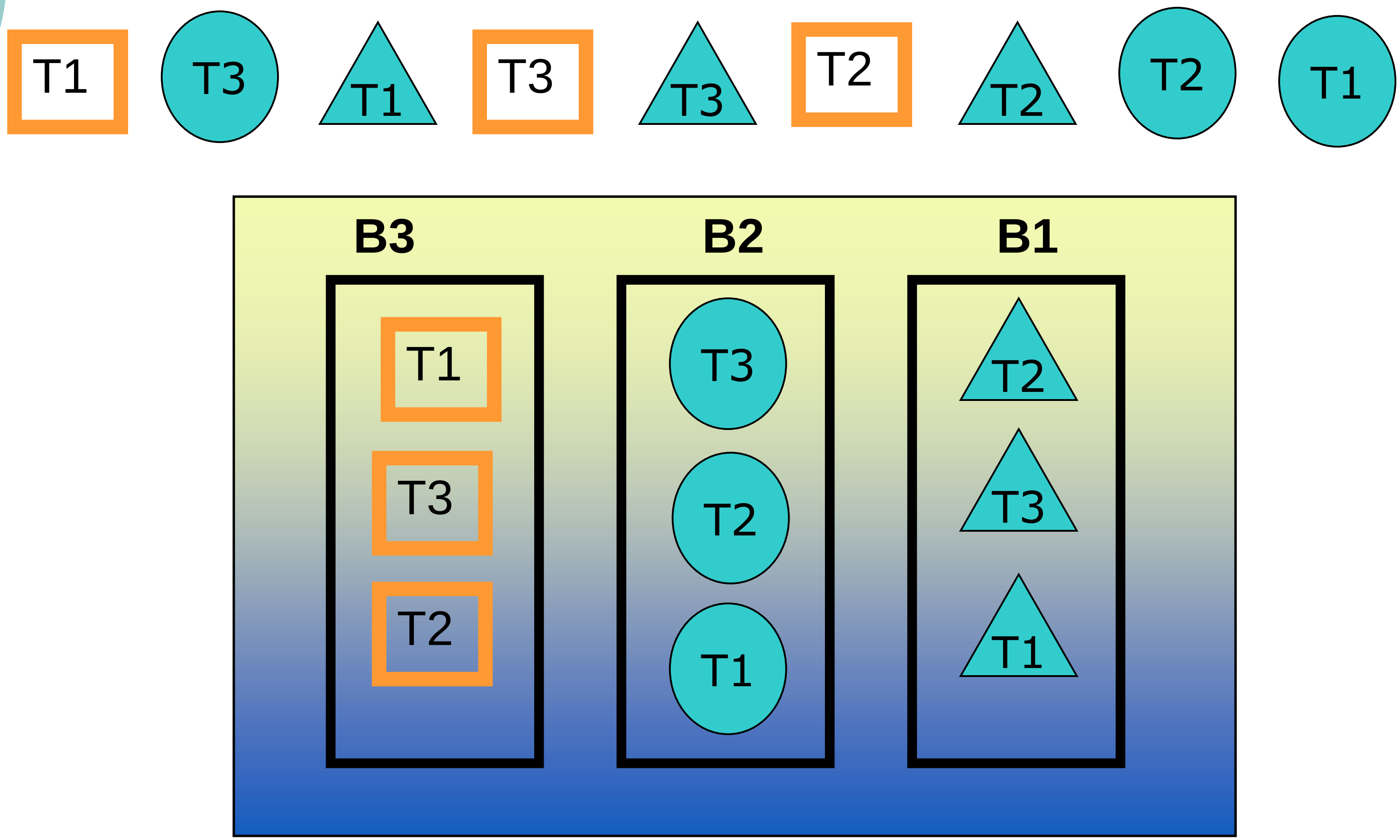
- The purpose of grouping experimental units is to have the units in a block as uniform as possible so that the observed differences between treatments will be largely due to “true” differences between treatments.

Randomization Procedure

- Each replicate is randomized separately.
- Each treatment has the same probability of being assigned to a given experimental unit within a replicate.
- Each treatment must appear at least once per replicate.

Layout of the experiment

$$Number\ of\ block = \frac{Number\ of\ experimen\ unit}{Number\ of\ replication} = \frac{9}{3} = 3\,blocks$$



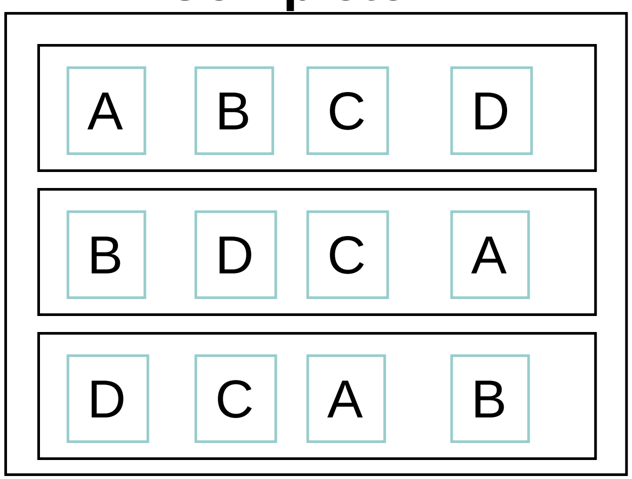
Complete or Incomplete Designs

Can all treatments be accommodated in each block?

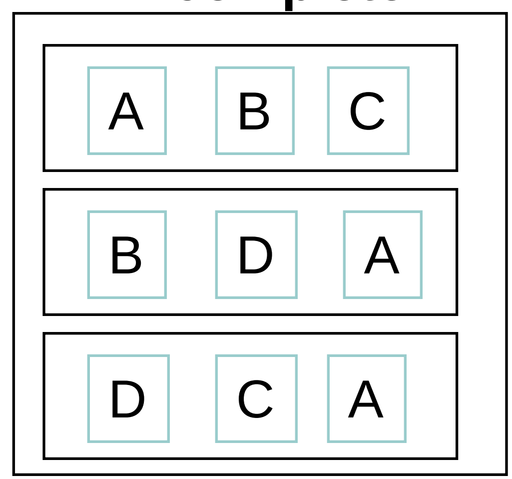
Complete Block Design: Every treatment occurs in each block.

Incomplete Block Design: Not every treatment occurs in each block.

Complete



Incomplete



Advantages of the RCBD

1. Generally more precise than the CRD.

2. No restriction on the number of treatments or replicates.

3. Some treatments may be replicated more times than others.

4. Missing values are easily estimated.

5. This design is more efficient than CRD design

Disadvantages of the RCBD

1. Error df is smaller than that for the CRD (problem with a small number of treatments).

2. If there is a large variation between experimental units within a block, a large error term may result (this may be due to too many treatments).

3. If there are missing data, a RCBD experiment may be less efficient than a CRD

RANDOMIZED COMPLETE BLOCK DESIGN (RCBD)

Define:

y_{ij} = the value of the response variable when block *j* uses treatment *i*

$y_{i\bullet}$ = the mean of the *t* response variable observed when using treatment *i*

= the treatment *i* mean

$y_{\bullet j}$ = the mean of the *b* values of the response variable when using block *j*

= the block *j* mean

y = the mean of all the $b \bullet t$ values of the response variable observed in the experiment

= the overall mean

RANDOMIZED COMPLETE BLOCK DESIGN (RCBD)

Blocks

1 2 3 … b

Treatment Means

Treatments

1
2
.
.
.
t

y_{ij} = response from treatment i and block j

$\bar{y}_{1\bullet}$

$\bar{y}_{2\bullet}$

⋮

$\bar{y}_{t\bullet}$

Block Means

$\bar{y}_{\bullet 1} \quad \bar{y}_{\bullet 2} \quad \cdots \quad \bar{y}_{\bullet b}$

Linear Models for the RCBD

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$$

$i = 1,2,\ldots, t$ $\quad$ $j = 1,2,\ldots, b$

where:

y_{ij} **: The observation in j^{th} block receiving the i^{th} treatment**

μ **: Overall mean**

τ_i **: The effect of the i^{th} treatment**

β_j **: The effect of the j^{th} block**

ε_{ij} **: Random error**

The ANOVA Table, COMPLETE RANDOMIZED BLOCK DESIGN (CRBD)

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F Statistic
Treatments	t-1	SST	$MST = \frac{SST}{t-1}$	$F(trt) = \frac{MST}{MSE}$
Blocks	b-1	SSB	$MSB = \frac{SSB}{b-1}$	$F(blk) = \frac{MSB}{MSE}$
Error	(t-1)×(b-1)	SSE	$MSE = \frac{SSE}{(t-1)(b-1)}$	
Total	(t×b)-1	SSTO		

where $SSTO = SSTret. + SSB + SSE$

$$C.F = \frac{Y..^2}{r \times b}$$

Sum of Squares

- *SST* measures the amount of between-treatment variability

$$SSTreat = b\sum_{i=1}^{t}\left(\bar{y}_{i\bullet} - \bar{y}\right)^2 = \frac{\sum Y_{i.}^2}{b} - \frac{Y_{..}^2}{b \times t}$$

- *SSB* measures the amount of variability due to the blocks

$$SSB = t\sum_{i=1}^{b}\left(\bar{y}_{\bullet j} - \bar{y}\right)^2 = \frac{\sum Y_{.j}^2}{t} - \frac{Y_{..}^2}{b \times t}$$

- *SSTO* measures the total amount of variability

$$SSTO = \sum_{i=1}^{t}\sum_{j=1}^{b}\left(\bar{y}_{ij} - \bar{y}\right)^2 = \sum Y_{ij}^2 - \frac{Y_{..}^2}{b \times t}$$

- *SSE* measures the amount of the variability due to error
- $SS_E = SS_{TO} - SS_{Treat.} - SS_B$

F-Test for Treatment Effects

H_0: No difference between treatment effects

H_a: At least two treatment effects differ

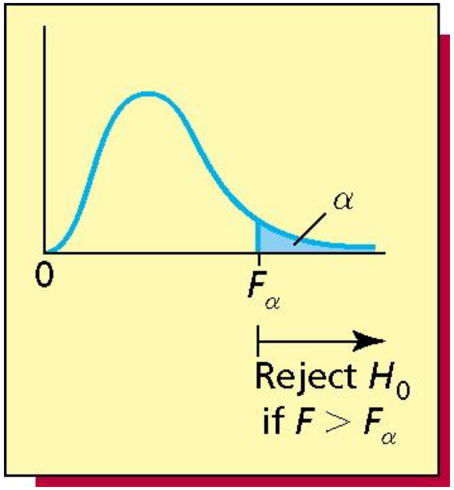
$$\text{Test Statistic: } F = \frac{MST}{MSE} = \frac{SST/(t-1)}{SSE/[(t-1)(b-1)]}$$

Reject H_0 if

$F > F_\alpha$ or

p-value < α

F_α is based on p-1 numerator and $(t$-1$)\times$ (b-1) denominator degrees of freedom



Missing Data

For each missing value in the experiment, you loose one degree of freedom from error and total.

- **Reasons for missing data include:**
  1. **Animal dies**
  2. **Break a test tube.**
  3. **Animals eat grain in the plot.**
  4. **Spill grain sample**
- **The value for a missing plot can be estimated by using the formula:**

$$Y_{ij} = \frac{bB + tT - G}{(t-1)(b-1)}$$

Where:

b : number of blocks

t : number of treatments

B : replicate total of blocks with missing value

T : treatment total of treatment with missing value

G : Experiment total (Y..)

$$Adjust\,SS' = SSt - \frac{[Y_{.j} - (t-1)Y_{ij}]^2}{t(t-1)}$$

Y.j : replicate total of blocks with missing value before estimate

Yij : Missing value

Relative of Efficiency of RCBD

- **Relative Efficiency of RCBD to CRD (how many times as many replicates would be needed for RCBD CRD to have as precise of estimates of treatment means as RCBD does):**

$$RE(\mathrm{RCBD}, CRD) = \frac{MSE_{CRD}}{MSE_{RCBD}}$$

$$\%RE = \frac{(b-1)MSB + b(t-1)MSE}{(bt-1)MSE}$$

OR

$$MSE^{*}_{(CRD)} = \frac{df_{BL}MSBL + (df_t + df_{error})MSE}{df_{Treat.} + df_{BL} + df_{error}}$$

$$\%RE = \frac{(df_{error}+1)(df^{*}_{error(CRD)}+3)MSE^{*}_{(CRD)}}{(df_{error}+3)(df^{*}_{error_{(CRD)}}+1)MSE} \times 100$$

$$df^{*}_{error_{(CRD)}} = df_{BL} + df_{Error}$$

Balanced Incomplete Block Design

These are randomized block designs in which every treatment is not present in every block.

The experimenter is not be able to run all the treatment combinations in each block because:

- **Shortages of Experimental Apparatus or Facilities**
- **The Physical Size of the Block**

Balanced Incomplete Block Design (BIBD)

An incomplete block design in which any two treatments appear together an equal number of times. This happens when all treatment comparisons are equally important and the treatment combinations used in each block should be selected in a balanced manner.

Suppose that there are **α** treatments and that each block can hold exactly **k** where (k < α) treatments. A balanced incomplete block design may be constructed by taking binomial coefficient of α and kblocks and assigning a different combination of treatments to each block.

Assume that there are **α** treatments and **b** blocks. In addition, assume that each block contains **k** treatments, that each treatment occurs **r**times in the design (or is replicated r times), and that there are **N = αr = bk**total observations. Furthermore, the number of times each pair of treatments appears in the same block is

$$\lambda = \frac{r(k-1)}{a-1}$$

r:عدد تكرارات كل معالجة في التجربة
k:عدد القطع التجريبية في القطاع
a:عدد المعالجات

If α= b, the design is said to be symmetric.

The Statistical Model for the BIBD

The statistical model for the BIBD

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$$

where y_{ij} is the ith observation in the jth block, μ is the overall mean, τ_i is the effect of the ith treatment, β_j is the effect of the jth block, and ϵ_{ij} is the NID $(0, \sigma^2)$ random error

The BIBD ANOVA Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments (adjusted)	$\dfrac{k\sum Q_i^2}{\lambda a}$	$a-1$	$\dfrac{SS_{\text{Treatments(adjusted)}}}{a-1}$	$F_0 = \dfrac{MS_{\text{Treatments(adjusted)}}}{MS_E}$
Blocks	$\dfrac{1}{k}\sum y_{.j}^2 - \dfrac{y_{..}^2}{N}$	$b-1$	$\dfrac{SS_{\text{Blocks}}}{b-1}$	
Error	SS_E (by subtraction)	$N-a-b+1$	$\dfrac{SS_E}{N-a-b+1}$	
Total	$\sum\sum y_{ij}^2 - \dfrac{y_{..}^2}{N}$	$N-1$		

$$SS_T = \sum_i \sum_j y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$SS_T = SS_{\text{Treatments(adjusted)}} + SS_{\text{Blocks}} + SS_E$$

$$SS_{\text{Blocks}} = \frac{1}{k} \sum_{j=1}^{b} y_{.j}^2 - \frac{y_{..}^2}{N}$$

$$SS_{\text{Treatments(adjusted)}} = \frac{k \sum_{i=1}^{a} Q_i^2}{\lambda a}$$

$$Q_i = y_{i.} - \frac{1}{k} \sum_{j=1}^{b} n_{ij} y_{.j} \qquad i = 1, 2, \ldots, a$$

Rejection Criteria

$$F_0 > F_{\alpha,(a-1),(N-a-b+1)}$$

Relative Efficiency of BIBI

Steps:

1) Converting BIBID to RCBD and calculating ANOVA table

2) Calculate the Active Error

$$Coefficient\ of\ \mathrm{Re}lative\ \ C.R = \frac{t}{t+1}$$

Where

$$(Active\,Error)A.E = \frac{MSE^{*}}{C.R}$$

2) Calculate the Relative of Efficiency

$$R.E = \frac{MSE}{A.E}$$

Example:

A chemical engineer thinks that the time of reaction for a chemical process is a function of the type of catalyst employed. Four catalysts are currently being investigated. The experimental procedure consists of selecting a batch of raw material, loading the pilot plant, applying each catalyst in a separate run of the pilot plant, and observing the reaction time. Because variations in the batches of raw material may affect the performance of the catalysts, the engineer decides to use batches of raw material as blocks. However, each batch is only large enough to permit three catalysts to be run. Therefore, a randomized incomplete block design must be used. The balanced incomplete block design for this experiment, along with the observations recorded in the following table.

Teratment (Catalyst)	Block (Batch of Raw Material)				
	1	2	3	4	$y_{i.}$
1	73	74	—	71	218
2	—	75	67	72	214
3	73	75	68	—	216
4	75	—	72	75	222
$y_{.j}$	221	224	207	218	$870 = y_{..}$

$a = 4, b = 4, k = 3, \lambda = 2 : N = 12$

$$SS_T = \sum_i \sum_j y_{ij}^2 - \frac{y_{..}^2}{12}$$

$$= 63{,}156 - \frac{(870)^2}{12} = 81.00$$

$$SS_{\text{Blocks}} = \frac{1}{3} \sum_{j=1}^{4} y_{.j}^2 - \frac{y_{..}^2}{12}$$

$$= \frac{1}{3} [(221)^2 + (207)^2 + (224)^2 + (218)^2] - \frac{(870)^2}{12} = 55.00$$

$$Q_1 = (218) - \tfrac{1}{3}(221 + 224 + 218) = -9/3$$

$$Q_2 = (214) - \tfrac{1}{3}(207 + 224 + 218) = -7/3$$

$$Q_3 = (216) - \tfrac{1}{3}(221 + 207 + 224) = -4/3$$

$$Q_4 = (222) - \tfrac{1}{3}(221 + 207 + 218) = 20/3$$

$$SS_{\text{Treatments(adjusted)}} = \frac{k \sum_{i=1}^{4} Q_i^2}{\lambda a}$$

$$= \frac{3[(-9/3)^2 + (-7/3)^2 + (-4/3)^2 + (20/3)^2]}{(2)(4)} = 22.75$$

$$SS_E = SS_T - SS_{\text{Treatments(adjusted)}} - SS_{\text{Blocks}}$$

$$= 81.00 - 22.75 - 55.00 = 3.25$$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Treatments (adjusted for blocks)	22.75	3	7.58	11.66	0.0107
Blocks	55.00	3	—		
Error	3.25	5	0.65		
Total	81.00	11			

$$F_{\alpha,(a-1),(N-a-b+1)} = F_{0.05,3,5} = 5.41$$