Using the Δ-definition

1) Find an equation for the tangent line to the curve at the given point.

a- $y = 4 - x^2$, (-1, 3)

b- $y = \frac{1}{x^3}$, $(-2, -\frac{1}{8})$

c- $y = x^3$, (-2, -8)

d- $f(x) = \sqrt{x + 1}$, (8, 3)

2) Find the slope of the curve at the point indicated.

a- $y = x^3 - 2x + 7$, $x = -2$

b- $y = \frac{1}{x-1}$, x=3

3) Find an equation of the straight-line having slope 1/4 that is tangent to the curve $y = \sqrt{x}$.

4) An object is dropped from the top of a 100-m-high tower. Its height above ground after t seconds is $100 - 4.9t^2$ m. How fast is it falling 2 s after it is dropped?

5) What is the rate of change of the area of a circle ($A = \pi r^2$) with respect to the radius when the radius is $r = 3$?

6) find the indicated derivatives.

a- $\frac{dy}{dx}$ $if\ y = 2\,x^3$

b- $\frac{dz}{dw}$ if $z = \frac{1}{\sqrt{w^2-1}}$

c- $\frac{dp}{dq}$ if $p = q^{\frac{3}{2}}$

Use Differentiation Rules

1) Differentiate the following powers of x.

a- x^3

b- $x^{\sqrt{2}}$

c- $\sqrt{x^{2+\pi}}$

2) Does the curve $y = x^4 - 2\,x^2 + 2$ have any horizontal tangent lines?

3) The curve $y = a x^2 + bx + c$ passes through the point (1, 2) and is tangent to the line $y = x$ at the origin. Find a, b, and c.
4) Find all points (x, y) on the graph of $f(x) = 3x^2 - 4x$ with tangent lines parallel to the line $y = 8x + 5$.
5) Find all points (x, y) on the graph of $y = x/(x - 2)$ with tangent lines perpendicular to the line $y = 2x + 3$.
6) The number of liters of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30 - t)^2$. How fast is the water running out at the end of 10 min? What is the average rate at which the water flows out during the first 10 min?
7) The volume $V = (4/3)\pi r^3$ of a spherical balloon changes with the radius.

a. At what rate (m^3/m) does the volume change with respect to the radius when $r = 2$ m?

b. By approximately how much does the volume increase when the radius changes from 2 to 2.2 m?

8) find dy/dx.
a- $y = -10x + 3 \cos x$
b- $f(x) = \sin x \tan x$
c- $y = \sqrt{x} \sec x + 3$
d- $y = \frac{cosx}{sin^2 x}$
e- $y = (\sec x + \tan x)(\sec x - \tan x)$
f- $y = \frac{cosx}{x} + \frac{x}{cosx}$
g- $f(x) = x^3 \sin x \cos x$
h- $y = \sec x \csc x$
i- $y = (1 + \csc x) \cos x$
j- $y = \frac{3x + \tan x}{x \sec x}$

9) Find y'' if
a- $y = \csc x$.
b- $y = \sec x$.

10) In Exercises a-f, given $y = f(u)$ and $u = g(x)$, find dy/dx.
a- $y = \sin u$, $u = 3x + 1$
b- $y = \tan u$, $u = \pi x^2$
c- $y = -sec\ u$, $u = \frac{1}{x} + 7x$

d- $y = \sqrt{3x^2 - 4x + 6}$

e- $y = 5\,cos^{-4}\,x$

f- $y = \sec(\tan x)$

11) Find dy/dx if $y^2 = x^2 + \sin xy$

12) A cube's surface area increases at the rate of 72 cm^2/s. At what rate is the cube's volume changing when the edge length is $x = 3$ cm?

13) When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm / min. At what rate is the plate's area increasing when the radius is 50 cm?

14) A spherical balloon is inflated with helium at the rate of 100 $\pi\ m^3$/min. How fast is the balloon's radius increasing at the instant the radius is 5 m? How fast is the surface area increasing?

15) A balloon is rising vertically above a level, straight road at a constant rate of 0.3 m/s. Just when the balloon is 20 m above the ground, a bicycle moving at a constant rate of 5 m/s passes under it. How fast is the distance $s(t)$ between the bicycle and balloon increasing 3 s later?

16) Find the linearization of

a- $f(x) = \cos x$ at $x = \pi/2$

b- $f(x) = \sqrt{1 + x}$ at $x = 3$

17) In Exercises a–e, find the absolute maximum and minimum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates.

a- $f(x) = 4 - x^3$, $-2 \le x \le 1$

b- $f(x) = -\frac{1}{x}$, $-2 \le x \le -1$

c- $g(x) = \sqrt{4 - x^2}$, $-2 \le x \le 1$

d- $f(\theta) = \sin\theta$, $-\pi/2 \le \theta \le 5\pi/6$

e- $f(t) = 2 - |t|$, $-1 \le t \le 3$

18) In Exercises a–c, identify the coordinates of any local and absolute extreme points and inflection points. Graph the function.

a- $y = x^2 - 4x + 3$

b- $y = (x - 2)^3 + 1$

c- $y = x(6 - 2x)^2$

19) What is the smallest perimeter possible for a rectangle whose area is 16 cm^2, and what are its dimensions?

20) A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

21) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.

22) Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius 10 cm. What is the maximum volume?

23) Determine the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 3.

24) What values of a and b make $f(x) = x^3 + ax^2 + bx$ have?

a. a local maximum at $x = -1$ and a local minimum at $x = 3$?

b. a local minimum at $x = 4$ and a point of inflection at $x = 1$?

25) Find the point on the graph of $y = 20x^3 + 60x - 3x^5 - 5x^4$ with the largest slope.