Question Bank of Numerical Analysis I and Numerical Analysis II for the Third General Stage in the Physics Dep. of the First and Second Semesters:

First: Numerical Analysis I

1. Consider the function $f(x) = x^3 - 2x^2 + x - 1$. Use the bisection method to find a root of this function within the interval [1, 2]. Perform three iterations and show the intermediate steps and approximations.
2. Given the system of linear equations:

$$3x + 2y - z = 1$$

$$2x - 2y + 4z = -2$$

$$-x + 0.5y - z = 0$$

   Use the Gauss elimination method to solve this system. Show each step of the elimination process and the final solution.
3. Use the Newton-Raphson method to find an approximation of the root of the function $f(x) = x^3 - 3x + 1$, starting from the initial guess $x_0 = 1$. Perform two iterations and show the intermediate approximations.
4. Use the bisection method to find a root of the function $f(x) = x^3 - 4x^2 + 5x - 2$ within the interval [0, 2]. Perform four iterations and show the intermediate steps and approximations.
5. Given the system of linear equations:

$$2x + 3y - z = 4$$

$$x - 2y + 3z = 7$$

$$3x + 2y + 4z = 10$$

   Use the Gauss-Seidel method to solve this system. Show each step of the iterative process and the final solution after three iterations.
6. Use the Newton-Raphson method to find an approximation of the root of the function $f(x) = cos(x) - x^2$, starting from the initial guess $x_0 = 1$. Perform three iterations and show the intermediate approximations.

7. Use the secant method to find a root of the equation $f(x) = e^x - x^3 - 5$ within the interval [1, 2]. Perform four iterations and show the intermediate approximations.
8. Given the system of linear equations:

$$4x - y + 3z = 8$$

$$x + 2y - 3z = -1$$

$$2x - y + z = 3$$

Use the Gauss-Jordan elimination method to solve this system. Show each step of the elimination process and the final solution.

9. Use the fixed-point iteration method to find a root of the equation $x = e^{(-x)}$ within the interval [0, 1]. Perform four iterations and show the intermediate approximations.
10. Use the Newton-Raphson method to find an approximation of the root of the function $f(x) = x^3 - 2x - 5$, starting from the initial guess $x_0 = 2$. Perform three iterations and show the intermediate approximations.
11. Use the Newton-Raphson method to find an approximation of the root of the equation $f(x) = x^3 - 4x - 9$, starting from the initial guess $x_0 = 2$. Perform five iterations and show the intermediate approximations.
12. Use the fixed-point iteration method to find a root of the equation $x = cos(x)$ within the interval [0, π/2]. Perform four iterations and show the intermediate approximations.
13. Use the bisection method to find a root of the function $f(x) = x^3 - 6x^2 + 11x - 6$ within the interval [1, 2]. Perform five iterations and show the intermediate steps and approximations.
14. Given the system of linear equations:

$$3x + 2y - z = 1$$

$$2x - 4y + 3z = -5$$

$$-x + y - z = 0$$

Use the Jacobi method to solve this system. Show each step of the iterative process and the final solution after three iterations.

15. Use the secant method to find a root of the equation $f(x) = x^3 - 5x + 2$ within the interval [0, 1]. Perform five iterations and show the intermediate approximations.

16. Given the system of linear equations:

$$2x - 3y + z = 7$$

$$x + 2y + 2z = 10$$

$$-x - y + 3z = 5$$

Use the LU decomposition method to solve this system. Show each step of the decomposition process and the final solution.

17. Use the fixed-point iteration method to find a root of the equation $x = 2 - e^{(-x)}$ within the interval [0, 2]. Perform five iterations and show the intermediate approximations.

18. Use the Newton-Raphson method to find an approximation of the root of the equation $f(x) = 2x^3 - 5x - 3$, starting from the initial guess $x_0 = 1.5$. Perform four iterations and show the intermediate approximations.

19. Use the fixed-point iteration method to find a root of the equation $x = ln(x)$ within the interval [1, 2]. Perform four iterations and show the intermediate approximations.

20. Use the bisection method to find a root of the function $f(x) = x^4 - 3x^2 + 2x - 1$ within the interval [0, 1]. Perform five iterations and show the intermediate steps and approximations.

21. Given the system of linear equations:

$$4x - 2y + 3z = 10$$

$$-x + 3y - 2z = -5$$

$$2x - y + 4z = 3$$

Use the Gauss-Seidel method to solve this system. Show each step of the iterative process and the final solution after four iterations.

22. Use the secant method to find a root of the equation $f(x) = sin(x) - x$ within the interval [1, 2]. Perform five iterations and show the intermediate approximations.

23. Given the system of linear equations:

$$3x - y + 2z = 8$$

$$x + 2y - 4z = -7$$

$$-2x + y - 3z = 1$$

Use the Gauss-Jordan elimination method to solve this system. Show each step of the elimination process and the final solution.

24. Use the fixed-point iteration method to find a root of the equation $x = 2 + 3sin(x)$ within the interval [0, 1]. Perform five iterations

25. Use the fixed-point iteration method to find a root of the equation $x = 2 + 3sin(x)$ within the interval [0, 1]. Perform five iterations and show the intermediate approximations.

26. Use the Newton-Raphson method to find an approximation of the root of the equation $f(x) = 3x^3 - 5x^2 + 2$, starting from the initial guess x0 = 0.5. Perform four iterations and show the intermediate approximations.

27. Use the fixed-point iteration method to find a root of the equation $x = ln(2 + x)$ within the interval [1, 2]. Perform five iterations and show the intermediate approximations.

<u>Second: Numerical Analysis II</u>

28. Approximate the definite integral $\int_0^6 e^{(-x^2)} dx$ using the composite trapezoidal rule with four subintervals. Show the step-by-step process and the final approximation.

29. Solve the initial value problem $dy/dx = x^2 + y, y(0) = 1$, using the fourth-order Runge-Kutta method. Calculate the approximate value of y at x = 0.1, showing the intermediate steps.

30. Approximate the definite integral $\int_0^1 sin(x)\, dx$ using the composite midpoint rule with six subintervals. Show the step-by-step process and the final approximation.

31. Solve the initial value problem $dy/dx = x^2 + y, y(0) = 1$, using the fourth-order Runge-Kutta method. Calculate the approximate value of y at x = 0.5, showing the intermediate steps.

32. Approximate the derivative of the function $f(x) = sin(x)$ at $x = \pi/4$ using the three-point forward difference formula. Show the step-by-step calculations and the final approximation.

33. Approximate the definite integral $\int_0^2 x^2 dx$using the composite Simpson's rule with four subintervals. Show the step-by-step process and the final approximation.

34. Solve the initial value problem $dy/dx = x^2 + y, y(0) = 1$, using the Euler's method with a step size of 0.1. Calculate the approximate value of y at x = 0.4, showing the intermediate steps.

35. Approximate the definite integral $\int_1^3 e^x dx$ using the composite trapezoidal rule with six subintervals. Show the step-by-step process and the final approximation.

36. Approximate the derivative of the function $f(x) = e^{(-2x)}$ at x = 1 using the three-point backward difference formula. Show the step-by-step calculations and the final approximation.

37. Approximate the definite integral $\int_0^1 ln(x)\ dx$ using the composite trapezoidal rule with eight subintervals. Show the step-by-step process and the final approximation.

38. Solve the initial value problem $dy/dx = x^2 + y, y(0) = 1$, using the second-order Runge-Kutta method with a step size of 0.1. Calculate the approximate value of y at x = 0.5, showing the intermediate steps.

39. Approximate the definite integral $\int_0^\pi sin(x)\ dx$ using the composite midpoint rule with eight subintervals. Show the step-by-step process and the final approximation.

40. Approximate the derivative of the function $f(x) = sin(2x)$ at $x = \pi/6$ using the three-point central difference formula. Show the step-by-step calculations and the final approximation.

41. Approximate the definite integral $\int_0^3 x^2 dx$ using the composite Simpson's rule with eight subintervals. Show the step-by-step process and the final approximation.

42. Solve the initial value problem $dy/dx = x^2 + y, y(0) = 1$, using the fourth-order Adams-Bashforth method with a step size of 0.2. Calculate the approximate value of y at x = 1, showing the intermediate steps.

43. Approximate the derivative of the function $f(x) = \sqrt{(x)}$ at x = 4 using the five-point stencil method. Show the step-by-step calculations and the final approximation.

44. Approximate the definite integral $\int_0^2 e^{(-x)} dx$ using the composite trapezoidal rule with ten subintervals. Show the step-by-step process and the final approximation.

45. Solve the initial value problem $dy/dx = x^2 + y, y(0) = 1$, using the fourth-order Runge-Kutta method with a step size of 0.2. Calculate the approximate value of y at x = 0.8, showing the intermediate steps.

46. Approximate the definite integral $\int_0^{\pi/2} cos(x)\, dx$ using the composite midpoint rule with ten subintervals. Show the step-by-step process and the final approximation.

47. Solve the initial value problem $dy/dx = x^2 + y, y(0) = 1$, using the second-order Runge-Kutta method with a step size of 0.2. Calculate the approximate value of y at x = 0.6, showing the intermediate steps.

48. Approximate the derivative of the function $f(x) = e^{(-3x)}$ at x = 0 using the three-point forward difference formula. Show the step-by-step calculations and the final approximation.

49. Approximate the definite integral $\int_0^4 x^3 dx$ using the composite Simpson's rule with twelve subintervals. Show the step-by-step process and the final approximation.

50. Approximate the derivative of the function $f(x) = cos(2x)$ at x = π/3 using the three-point central difference formula. Show the step-by-step calculations and the final approximation.

51. Approximate the definite integral $\int_0^5 e^{(-2x)} dx$ using the composite trapezoidal rule with fourteen subintervals. Show the step-by-step process and the final approximation.