Question Bank of Introduction to Number Theory for the Fourth Stage of the First Semester:

1. What is the definition of a prime number?
2. State the Euclidean Algorithm for finding the greatest common divisor (GCD).
3. What is the prime factorization of 36?
4. Determine the GCD of 72 and 120.
5. Find the LCM of 15 and 20.
6. Prove that there are infinitely many prime numbers.
7. What is the remainder when 15 is divided by 7?
8. Solve the congruence equation: $2x \equiv 7 \pmod{11}$.
9. What is the value of $\varphi(15)$, where φ is Euler's totient function?
10. Determine whether 27 is a quadratic residue modulo 7.
11. Find the least positive residue of 3^{14} modulo 17.
12. Use the Chinese Remainder Theorem to solve the system of congruences: $x \equiv 2 \pmod{3}$ $x \equiv 4 \pmod{5}$
13. Show that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a + c \equiv b + d \pmod{m}$.
14. What is the value of 3^{-1} modulo 7?
15. Determine whether 121 is a perfect square.
16. Prove that there are infinitely many perfect numbers.
17. State and prove Wilson's theorem.
18. What is the sum of the first 100 prime numbers?
19. Find all solutions to the equation $x^2 \equiv 1 \pmod{10}$.
20. Determine whether 12 is a primitive root modulo 29.
21. State the Quadratic Reciprocity Theorem.
22. Find the GCD of 144 and 441.
23. Prove that if $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{m}$ for any integer c.
24. What is the value of $\varphi(27)$?
25. Determine the last two digits of 7^{2019}.
26. Solve the congruence equation: $5x \equiv 2 \pmod{11}$.
27. What is the value of 11^{-1} modulo 13?

28. Prove that there are infinitely many prime numbers of the form $4k + 3$.

29. Find the prime factorization of 1001.

30. Use Fermat's Little Theorem to simplify 10^{100} modulo 17.

31. Determine the number of positive divisors of 360.

32. Prove that the sum of two odd integers is even.

33. Find the remainder when 2^{100} is divided by 7.

34. Determine whether 123 is divisible by 9.

35. What is the smallest prime factor of 1001?

36. Prove that the square of any even integer is divisible by 4.

37. Find the largest prime factor of 2021.

38. Determine whether 13 is a primitive root modulo 31.

39. Prove that if a and b are relatively prime, then ac and bc are relatively prime for any integer c.

40. What is the value of $\varphi(49)$?

41. Solve the congruence equation: $3x \equiv 5 \pmod 7$.

42. Find the GCD of 126 and 231.

43. Prove that the sum of two consecutive odd integers is even.

44. Determine the last digit of 2^{2023}.

45. What is the smallest positive integer that leaves a remainder of 1 when divided by 4, 5, and 6?

46. Prove that if $a \equiv b \pmod m$ and $c \equiv d \pmod m$, then $ac \equiv bd \pmod m$.

47. What is the value of $\varphi(100)$?

48. Solve the congruence equation: $4x \equiv 6 \pmod 9$.

49. Find the GCD of 105 and 315.

50. Prove that the product of two consecutive integers is always even.