Department of Mathematics

College of Science

Salahaddin University-Erbil

Subject: Calculus of Variations

Course Book: Fourth Stage-Second Semester

Lecturer's Name: Dr. Andam Ali Mustafa

Academic Year: 2023-2024

Course Book

1. Course name	Calculus of Variations
2. Lecturer in charge	Andam Ali Mustafa
3. Department / College	Mathematics / Science
4. Contact	**E-mail:** andam.mustafa@su.edu.krd **Tel:** +9647504302367
5. Time (in hours) per week	3 hours
6. Office hours	Thursday 10:30-2:30
7. Course code	
8. Teacher's academic profile	❖ **25/8/2022** PhD in University Roma Tre in Rome, Italy. ❖ **3/3/2015** M.Sc. in the Department of Mathematics, College of Science, Salahaddin University-Erbil, Iraq. ❖ **31/3/2015** Assistant lecturer in the Department of Mathematics, College of Science, Salahaddin University-Erbil, Iraq. ❖ **2/7/2009** B.Sc. in the Department of Mathematics, College of Science, Salahaddin University-Erbil, Iraq. ❖ **2004-2005** Awarded a baccalaureate from the Kurdistan High School, Erbil, Iraq.
9. Keywords	Maximum, Minimum, Functionals, Euler Lagrange Equation, Variational Problems, BVP.

10. Course overview:

The Calculus of Variations course provides a comprehensive exploration of fundamental principles and practical applications in the field. The curriculum commences with an introduction to variational problems, offering historical context and elucidating the motivations behind studying variations in functions.

Building on this foundation, the course delves into core concepts such as the fundamental lemma and the Euler-Lagrange equation, exposing students to different forms of the equation and variations. Real-world applications in physics, including the Brachistochrone problem and the Minimum Surface Area of Rotation, showcase the relevance of these mathematical techniques. The study of constrained extremization follows, addressing isoperimetric problems and Dido's Problem, emphasizing various types of variational problems and their applications in constrained optimization.

The course then navigates through challenges posed by functions with multiple variables, providing techniques and examples that equip students to tackle complex, multivariable problems effectively. Advanced concepts, including variational notation, Hamilton's Principle, and the Legendre Test, are explored in-depth through illustrative examples. The reduction of Boundary Value Problems (BVP) into variational problems is introduced, with practical examples demonstrating the reduction process.

Finally, the course covers direct methods for solving variational problems, offering insights into techniques and applications through relevant examples. The holistic approach of this course aims to blend theoretical foundations with practical problem-solving skills, ensuring students grasp the intricacies of variations in functions and their applications in constrained optimization.

11. Course objective:

The course in Calculus of Variations is designed to provide students with a comprehensive understanding of the fundamental principles and practical applications within this mathematical discipline. The primary objective is to establish a solid foundation in key concepts such as the fundamental lemma and the Euler-Lagrange equation, enabling students to comprehend the intricacies of variational calculus. Real-world applications, particularly in physics, such as the Brachistochrone problem and the Minimum Surface Area of Rotation, are explored to underscore the practical relevance of these mathematical methods.

The course further aims to equip students with the skills necessary to solve problems involving multiple variables, emphasizing effective techniques for handling complex, multivariable functions. Constrained optimization is a focal point, with an emphasis on isoperimetric problems, Dido's Problem, and various types of variational problems, enabling students to apply these concepts in practical scenarios. Advanced topics, including variational notation, Hamilton's Principle, and the Legendre Test, are introduced to provide a nuanced understanding of more complex aspects of variational calculus. The curriculum also covers reduction techniques for transforming Boundary Value Problems (BVP) into variational problems, demonstrating the practical application of these reduction methods. Additionally, students are exposed to direct methods for solving variational problems, fostering alternative approaches to tackle intricate mathematical scenarios.

Overall, the course aims to cultivate strong analytical and problem-solving skills, preparing students for the application of variational calculus principles across a diverse range of mathematical challenges.

12. Student's obligation

- Students reign a commitment to come on time and remain in the classroom for the duration of scheduled classes.
- Nothingness speaks students with each other during lecture.
- All devices must be turned off.
- When teacher ask question, Students will be to raise your hand before answer his question.
- Students own an obligation to write tests and final examinations at the times scheduled by the teacher or the College.

13. Forms of teaching

I give hard copy of My lecture notes to students before coming lecturer time. first, I remember students about previous lecture, and then I start new lecture. At the end of the lecture give a homework for the next lecture. During this proses I use presentation and whiteboard.

14. Assessment scheme

1. Exam: 20% marks **2. Quiz:** 10% marks **3. Homework:** 5% marks

4. Assignments and other activities: %5 marks **5. Final exam:** 60 % marks

15. Student learning outcome:

- Explain some of the concepts of Calculus of variations, a primary area of mathematics, using examples.
- Apply mathematical ideas and concepts within the context of Calculus of variations.
- Solve a range of problems in Calculus of variations.

16. Course Reading List and References:

- Louis Komzsik: Applied Calculus of variations for Engineers, CRC Press, NewYork, 2009.
- C. R. Wylie and L.C Barrett: Advanced Engineering Mathematics, McGrawHill Inc, Singapore -1982.
- B. S. Grewal: Higher Engineering Mathematics, Khanna Publishers, New Delhi, 2005.
- Tyn Myint U: Linear Partial Differential Equations for scientists and Engineers Birkhauser, Boston, 2007.

17. The Topics:	**Lecturer's name**

Chapter 1 (Maxima and Minima, Sufficient condition for having maximum and minimum: (Weierstrass Theorem), Necessary condition for Maximum/Minimum when f is differentiable, Necessary conditions for Maxima/Minima functions of several variables multi-variable functions, Functionals, Examples.**)** **Chapter 2 (**Fundamental Lemma of Calculus of Variations, Euler-Lagrange Equation (Necessary Condition for Extremum), Different Forms of Euler Lagrange Equation, Special Cases, Examples.**)** **Chapter 3 (**Brachistochrone Problem (Shortest Time of Descent Problem), Minimum Surface Area of Rotation.**)** **Chapter 4 (**Constrained Extremization Problem (Isoperimetric Problems, Dido's Problem, Variational Problem, Variational Problem, Problems**)** **Chapter 5 (**Problems with more than one independent variables, Examples.**)** **Chapter 6 (**The Variational Notation, Variation is analogous to derivative in calculus, Hamilton's Principle Second Order Conditions for Extremum. Legendre Test for Extremum, Examples.**)** **Chapter 7 (**Reduction of BVP into Variational Problems, Examples.**)** **Chapter 8 (**Direct Method to Solve Variational Problems, Examples.**)**	**Dr. Andam Ali Mustafa**
18. Practical Topics (If there is any)	

19. Examinations:

Questions in the examination will be arranged the matching mode by way of the examples and exercises that I give delivered in the lecture notes. Sometimes will be have extra mark in examination for worthy students. Many of the questions will take from those books that I mentioned in the References part.

20. Extra notes:

Answers of examination will find in the board's declaration physics department after every examination.

21. Peer review

.