زانکۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

A FRESH INSIGHT INTO THE EVOLUTIONARY STATUS AND THIRD BODY HYPOTHESIS OF THE ECLIPSING BINARIES AD ANDROMEDAE

Research Project

Submitted to the department of Physics in partial fulfillment of the requirements for the degree of BSc. in Physics

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LECTURER NAME

MAY– 2023

بِسْمِ اللّهِ الرَّحْمَنِ الرَّحِيمِ
قَالُواْ سُبْحَانَكَ لاَ عِلْمَ لَنَا إِلاَّ مَا عَلَّمْتَنَا إِنَّكَ أَنتَ الْعَلِيمُ الْحَكِيمُ
صدق الله العظيم

سورة البقرة الاية32

Supervisor Certificate

This research project has been written under my supervision and has been submitted for the award of the degree of BSc. in (Physics).

Signature
Name
Date / /

I confirm that all requirements have been completed.

Signature:
Name:
Head of the Department of . . . . . .
Date / /

This project is dedicated to:

Allah Almighty, my
Creator and my Master,

My great teacher and
messenger, Mohammed
(May Allah bless and grant
him), who taught us the
purpose of life,

My homeland Kurdistan, the warmest womb,

The Salahadin University; my second magnificent home;

My great parents, who never stop giving of themselves in countless ways,

My beloved brothers and sisters;

To all my family, the symbol of love and giving,

My friends who encourage and support me,

All the people in my life who touch my heart.

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LIST OF TABLES

LIST OF FIGURES

CONTENTS

Method on Light curves of selected stars

Result And Dissection On Light Curves Of Selected Star

SUMMARY

New orbital period variation of the eclipsing binary, AD Andromeda, was analyzed based on one CCD photometric times of minimum we have obtained and all available photoelectric and CCD values collected from the literatures. It is discovered that the orbital period of the binary shows a periodic oscillation with a period of 14.38 years and an amplitude of 0.0186 days. The periodic oscillation can be explained by the light-time effect via the presence of a tertiary component in a nearly circular orbit with a small eccentricity of e = 0.30 in the system. Based on the present analysis, it is estimated that the mass of the third body is no less than 1.76(±0.08)$M\odot$, and it should contribute light to the total system. Meanwhile, the photoelectric light curve obtained in yellow light by Ruciński [Ruciński, S.M., 1966. AcA 16, 307] was reanalyzed with the 2003 version of the W–D code. The results show that AD Andromeda is a detached eclipsing binary, and photometric solutions were computed. Based on the analysis, we obtained a small amount of third light in the system (L3V∼0.001), which is too small for the contribution of the tertiary companion star. The low luminosity of the third companion may be explained in two possible ways, either: (1) the third companion might itself be a close double star consisting of two stars of 0.88 solar masses, or (2) it is a dark star such as a neutron star. We think the first possibility is a more likely one than a neutron star companion. New photometric and spectroscopic observations and a detailed investigation of those data are urgently required in the future.

Aims. We aim to derive the absolute parameters of the components of AD And, A interpret their orbital period changes and discuss their evolutionary status.

Methods. New and complete multi-filter light curves of the eclipsing binaries AD was obtained and analysed with modern methods. Using all reliably observed times of minimum light, we examined orbital period irregularities using the least squares method.

Results. For AD And we derive reliable spectral types for their primary stars. Statistical checks of orbital period analysis for all systems are very reassuring in the cases AD And. The light -Time Effect (LITE) results are checked by inclusion of a third light option in the photometric analyses. Light curve solutions provide the means to calculate the absolute parameters of the components of the systems and reliably estimate their present evolutionary status.

Conclusions. AD And is found to be a detached system in which both close stars are of age ∼109 yr and is probably a

CHAPTER ONE

INTRODUCTION

1.1 Binary Stars

The term binary was first used in this context by Sir William Herschel in 1800 (AD) [1]. Binary stars, often called double stars, refer to pairs of stars sufficiently close to each other in space to be gravitationally bound together, orbiting around common center of mass gravitationally bound to each other formed at the same time as has been illustrated in the Fig. (1-1). The two members of a binary star are of unequal brightness [2]. The brighter star is called the primary and the fainter is called the companion [3]. Binary systems are of special interest, because analysis of their orbital characteristics is done by using of Kepler's third law which yields a direct measure of stellar masses [4]. Binary stars which can be resolved with a telescope or inter ferometric methods are known as visual binaries. Most of the known visual binary stars have not completed one whole revolution, but are observed to have travelled along a curved path or a partial arc. At least 80% of all stars in the Milky Way are part of multiple systems (binaries, triplets or more) some are close enough that they are able to transfer matter through tidal forces. These are close or contact binaries according to conservative statistics. There seems to be no obvious preference for particular combinations of brightness, size, or mass differences and a wide range in periods of revolution from less than a day to thousands of years. Likewise, there is a large range in separations from those stars in contact to those separated by thousands of times the Earth to Sun distance. Historically, visual binaries are which those that appear as double stars when seen through a telescope, were discovered to be gravitationally bound by William Herschel around 1800(AD) [1].

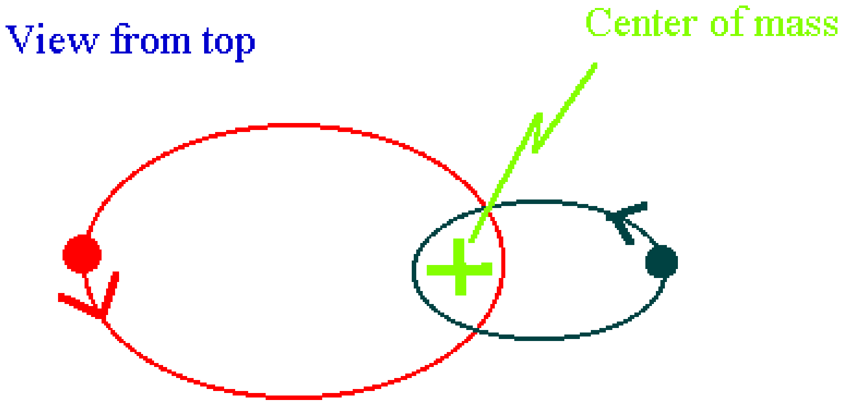


Figure (1-1): Binary stars orbiting their center of gravity [5].

It is noticeable from the Fig. (1-1) that the orbits of two stars of a binary system, relative to the center of mass of the two stars. The more massive star M has a smaller orbit than the less massive star m, but both orbit with the center of mass at one focus of their ellipses. While the Fig. (1-1), shows that the system can be considered equally well as one star (the secondary) orbiting the other (the primary). The dashed line represents the distance between the two stars at a particular moment in time as shown in the above figure, with the same length and direction, to aid in relating the two equivalent ways of visualizing the geometry but it can be considered that one star fixed, with the other star orbiting in a larger ellipse. One can choose either star as the central star, but it is customary to choose the brighter star (usually the more massive, but not always), it is called the primary and the other star is called the secondary. The primary star is at a focus of the larger ellipse [4.The first major catalogue of binary stars was made by Sir William Herschel, in about

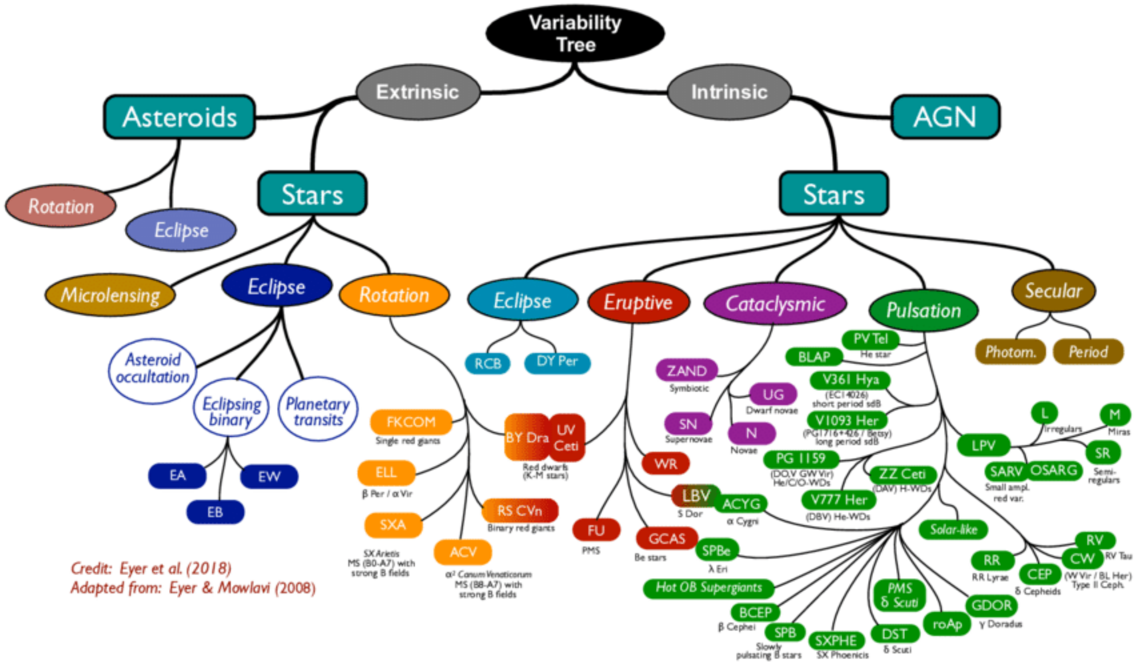
1800(AD) he was the first coined the term "binary star" to describe the union of two stars through their mutual gravitational attraction. William Herschel also differentiated between apparent and real double stars, an apparent double is two stars that appear to be close together but actually are not physically associated because they are separated by a large distance along the line of sight. A real double (physical binary) consists of two stars that are bound together by their mutual gravity and revolve around a common center of mass (2,9).When two stars are far enough apart that they evolve independently, they are called wide pair, each star in a wide pair follows the same career as a single star of the same mass, but when the two stars are close enough to each other that they can transfer matter to one another at the same stage of their development, they are called a close pair.(9) Thromjjn these exchanges of mass in a close pair alter the way that each evolves (2,8). Binary star also can be classified according to how it was learned that there are two stars rather than one. Some methods of recognizing a binary stars system favor the discovery of wide pairs. Others favor the discovery of close pairs (7,9). Approximately two-thirds of all solar field stars are members of binary systems, and recent studies suggest that virtually all stars begin life as member of multiple systems. Consequently, many of all the stars we see at night are actually binaries, comprised of two stars gravitationally bound in orbit one another (10). These Binary systems are important astrophysical laboratories because they allow us to deduce the properties of the constituent stars move a (concretely) than we can with single stars. The physics that governs how stars orbit one another was developed by Newton and Kepler over three hundred years ago, and can be summarized by the equation

$$P^2 = 4\pi^2 / G(M_p+M_s)\, a^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1\text{-}1)$$

Where G is the gravitational constants and M_p and Ms are the masses of the two components (a) is the semi-major axes of the two orbits a= $a_P + a_S$ In cmgs units , G=6.67*10^{-11} dyn cm^2 g^{-2} but these units are not the units of choice, Where mases measured in solar masses distances in AU(11).

1.2 Types of Binary Stars:

For decades, the study of eclipsing variable stars formed an important branch of variable stars or systems as shown in Fig. (1.2) which shows the flow chart for outline scheme of Photometric classification of eclipsing variables and its successive criteria .The most important of the two branches related to eclipsing variables is the branch of binary systems .The problem of binary system classification has come over a wide range of effecting parameters on which or accordingly, the related identification to a certain class is drawn. Both wide and close binary systems have subdivision groups depending on the dominant parameter taken as major factor of classification. It has been found that the majority of eclipsing variable, which are among the most useful representatives of this field are" close " binary system. Increased attention to such system is due to the behavior of their light variation, therefore, it has been customary for many years to divide eclipsing variables into the following groups. This classification includes the following types Kopal (1959):



Credit: Eyer et al. (2018)
Adapted from: Eyer & Mowlavi (2008)

Figure (1-2): Binary stars .

1. 3 Classifications of Binary Stars by Methods of Observation:

Binary stars are classified into four types according to the way in which they were observed: visually, by observation; spectroscopically, by periodic changes in spectral lines; photometrically, by changes in brightness caused by an eclipse; or astrometrically, by measuring a deviation in a star's position. Any binary star can belong to several of these classes; for example, several spectroscopic binaries are also eclipsing binaries.

1.3.1 Optical Double

This is not really a binary star. Two stars just happen to appear along almost the same line of sight. The two stars appear very close in the sky but are at different distances from Earth .They are actually very far apart [11]. The two objects are totally unrelated and are actually moving through space at right angles to each other. They passed at a minimum separation of 9 seconds of arc in 1960 and have been separating ever since [12].

1.3.2 Visual Binaries

True binary star systems (as opposed to purely optical doubles) in which both components are visible and resolvable in a telescopic eyepiece, Astronomers consider a visual binary, assuming initially that the brighter primary component is stationary and the fainter secondary component is orbiting around it [13] . The angular separation of the stars and the angular direction to the secondary can be directly observed. Making use of observations extending over many years or decades, the relative orbit of the secondary can be determined. In most cases, the two stars components differ in brightness [14]. The first binary orbit to be determined was that of W UMa in 1830 as shown in figure (1-3). With the current generation of stellar interferometers, many more binary systems fall into this category, although some researchers call these “interfero metric binaries”. Knowledge of the positions of the stars on the sky, as a function of orbital phase, coupled with RV observations; allow the masses of the stars to be measured directly, along with their luminosity ratio. These stars are therefore good for determining the mass–luminosity relation of stars, but, more importantly, they provide an essentially geometric determination of the distance to the system which is very reliable.

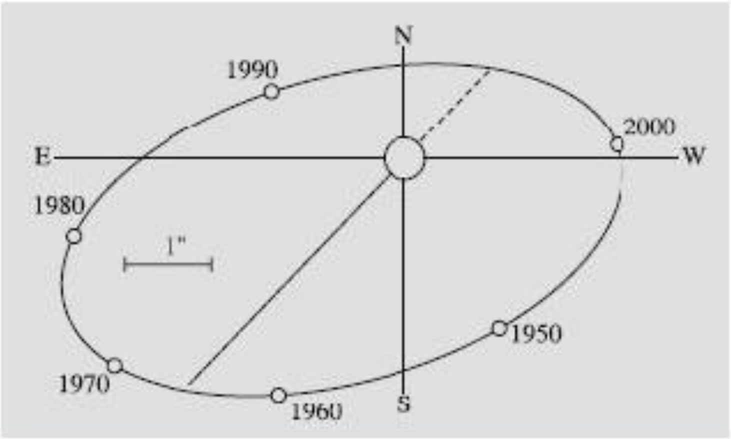


Figure (1-3): The orbit of M Ursae Majoris was the first binary orbit determined observationally in 1830 [15].

1.3.3 Astrometric Binaries

If only one component is visible, because the other is too faint and/or is too close to its brighter companion to be separated through telescopic resolution alone, gravitational effects may help one to prove that the system is a binary. The first astrometric binary was Sirius, which in the 1830's was observed to have an undulating proper motion. It was concluded that it had a small companion, which was visually discovered a few decades later, see Figure (1-4). The companion, Sirius B, was a completely new type of object, a white dwarf the proper motions of nearby stars have been carefully studied in the search for planetary systems. Although for example Barnard's star may have unseen companions, the existence of planetary systems around other O stars was not established by proper motion studies but with spectroscopic observations [16]. Another interesting type of astrometric binary is presented by cases where the components are so close that they are, or have been until recently, irresolvable. Fig: (1-4) shows the proper motion of the Sirius system over 70 years, the slight perturbations or wobble in the bright star, Sirius A, is due to the presence of its much dimmer white dwarf companion, Sirius B [17].

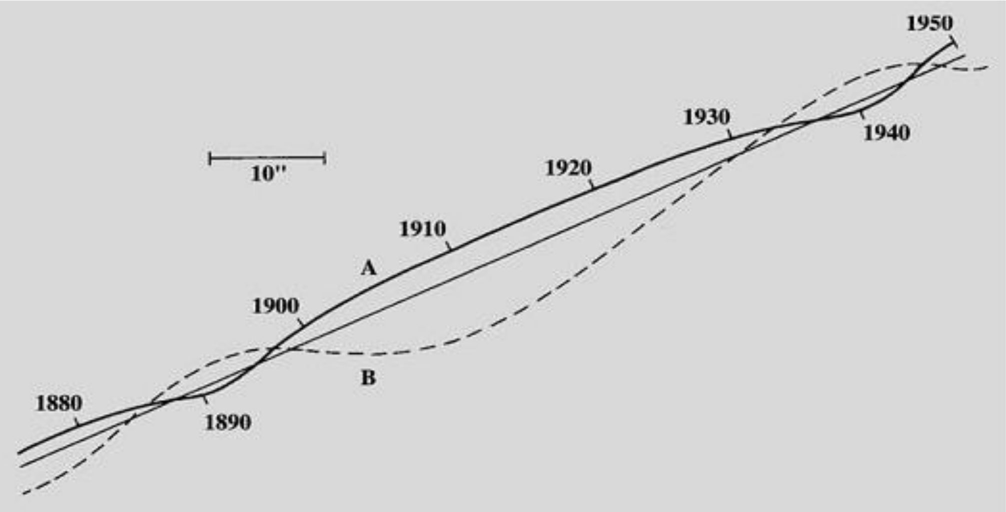


Figure (1-4): The apparent paths of Sirius and its companion in the sky [15].

1.3.4 Spectroscopic Binaries

Spectroscopic binary systems are those for which their binarity is apparent from variation of their radial velocity. The secondary component may also produce spectral lines strong enough to be visible in the spectrum of the system, in which the case of the spectroscopic binary is “double-lined”.

Spectroscopic observations of these systems allows calculation of the orbital period and eccentricity, the mass ratio, and the minimum masses of the components, $M \sin 3i$, where i is the inclination of the orbit relative to the line of sight of the observer. Spectroscopic binaries have too small separation for the components to be seen individually, but the binary nature of the system betrays itself because of the Doppler shifts in the spectrum [18]. This causes the absorption line to be displaced toward the blue end of the spectrum as the primary star approaches the Earth in its orbit, and then to the red end as it recedes [15].The first to be discovered was the brightness star of the double star Mizar, which was itself shown to be a binary in 1889 by E. Pickering [17]. If a binary system is unresolved into its components then the spectrum obtained from it will actually be a combination of the spectra from each of the component stars. As these stars orbit each other, then one star, A, may be moving towards the Earth whilst the other, B, may be moving away. The spectrum from A will therefore be blue-shifted to higher frequencies (shorter wavelengths) whilst B's spectrum will be redshifted. If the stars are moving across our line of sight then no Doppler shifting occurs so the lines stay in their mean positions. As the stars continue orbiting, A will recede so its spectral lines will move towards the red end of the spectrum and B's will move toward the blue.This is shown schematically in figure (1-5).

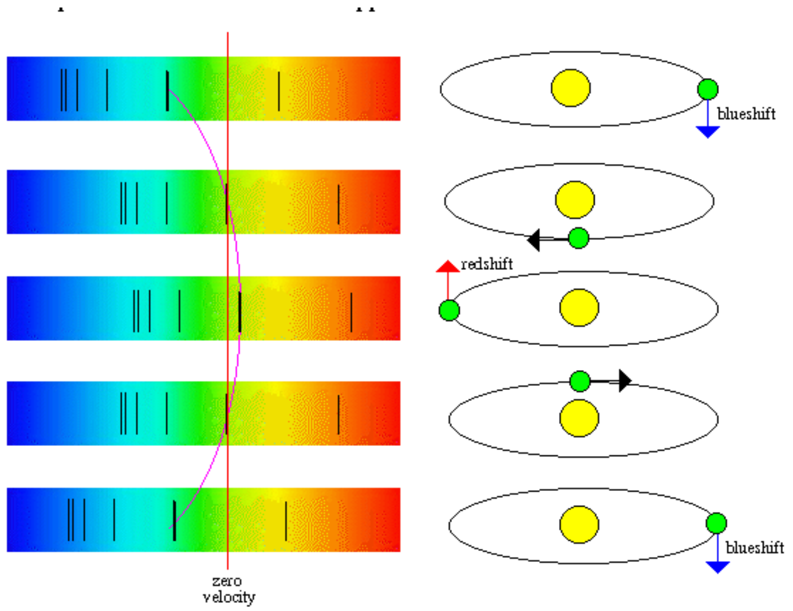


Figure (1-5): Spectroscopic binary star from APOD.

1.3.5 Eclipsing Binaries

Eclipsing Binaries are those binary systems in which the orbital plane of the system is almost side on as seen from the Earth so that as the stars move around the common center of gravity there are eclipses. Stars in an eclipsing binary system are usually so close together that they cannot be separated even in the most powerful telescopes and are usually called eclipsing variables [18]. They are in an eclipsing system because of the way the light from the system varies in a regular and characteristic pattern. An eclipsing binary allows scientists to find out a lot about how binary systems evolve and thus the fate of most stars. A primary eclipse occurs when the brighter star is eclipsed by the fainter star. The secondary eclipse occurs when the fainter star is eclipsed by the brighter star. Assuming the two stars are of the same brightness then both eclipses will be equal. If one of the stars is much fainter than the other the secondary eclipse may not be readily observed by visual observers. If the orbits of the two stars are circular then the secondary eclipse will occur midway between primary eclipses. Suppose that the orbits are elliptical the secondary eclipse may occur earlier or later than midway. A primary eclipse is said to be total if one star is completely obscured by the other star for a period of time. The period of minimum light can be minutes, days or in one case two years if a star is not totally obscured then the eclipse is described as partial. In that case, there is no prolonged period of minimum light. There can also be a prolonged period of minimum light with an annular eclipse Figure (1-6) shows the Schematic picture of a simple eclipsing binary, and its light curve. In the light curve, time increases to the right, as the smaller, brighter star (in black in this diagram) passes in front of the larger, fainter star (left), there is a secondary minimum. t_1, t_2, t_3, and t_4 are the times of the four contacts. As the brighter star passes behind the fainter one (right), there is a primary minimum [19] .

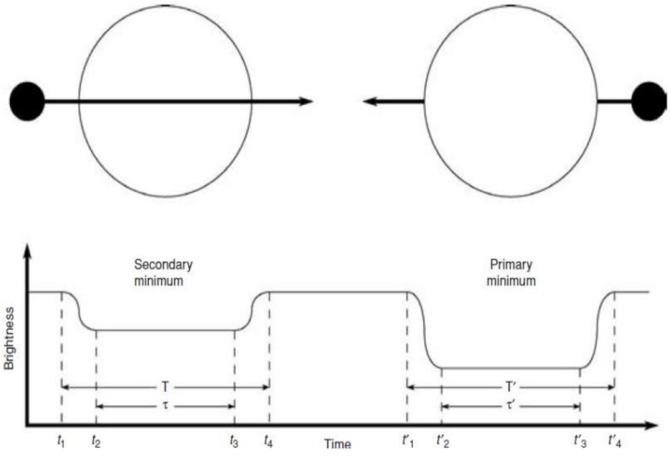


Figure (1-6): Schematic picture of a simple eclipsing binary, and its light curve.[20]

1.4 Types of Eclipsing Binaries

There are three basic types of eclipsing binary which are classified according to the shape of their light curves. The light curve will be the observed magnitudes over the whole period of the system; these types are [21].

1.4.1 Algol Stars

The Algol-type eclipsing variables have been named after β Persei or Algol . During most of the period, the light curve is fairly constant. This corresponds to phases during which the stars are seen separate from each other and the total magnitude remains constant. There are two different minima in the light curve, one of which, the primary minimum, is usually much deeper than the other one. This is due to the brightness difference of the stars. When the larger star, which is usually a cool giant, eclipses the smaller and hotter component, there is a deep minimum in the light curve. When the small, brighter star passes across the disc of the giant, the total magnitude of the system does not change by much [15] . The light curve of Algol shows a significant difference in the depths of their two minima . The shape of the minima depends on whether the eclipses are partial or total. In a partial eclipse the light curve is smooth, since the brightness changes smoothly as the depth of the eclipse varies. In a total eclipse there is an interval during which one component is completely invisible. The total brightness is then constant and the light curve has a flat bottomed minimum. The shape of the minima in

Algol variables thus gives information on the inclination of the orbit. Figure (1-7) shows the light curve of Algol type [19]. The duration of the minima depends on the ratio of the stellar radii to the size of the orbit. If the star is also a spectroscopic binary, the true dimensions of the orbit can be obtained. In that case, the masses and the size of the orbit, and thus also the radii can be determined without having to know the distance of the system [21] .

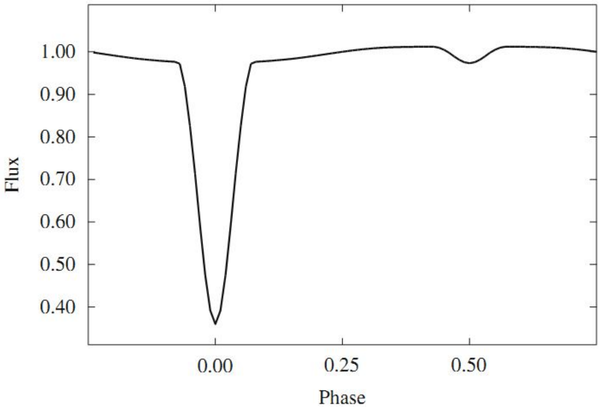


Figure (1-7): Synthetic " Algol "type light curve (V band) [19]

1.4.2 β Lyrae Stars

In the β Lyrae-type binaries, the total magnitude varies continuously. The stars are so close to each other that one of them has been pulled into ellipsoidal shape. Thus the brightness varies also outside the eclipses. The β Lyrae variables can be described as eclipsing ellipsoidal variables. In the β Lyrae system itself, one star has overfilled its Roche lobe and is steadily losing mass to its companion. The mass transfer causes additional features in the light curve [22]. Figure (1-8) shows synthetic β Lyrae type light curve [21 and 23]. The orbital period is in general greater than one day .

In addition, *β* Lyrae type stars has many features which can be summarized as follows:

1-It is continuously variable.

2-It has quiet difference in surface brightness.

3-It has large difference in minima depths [24].

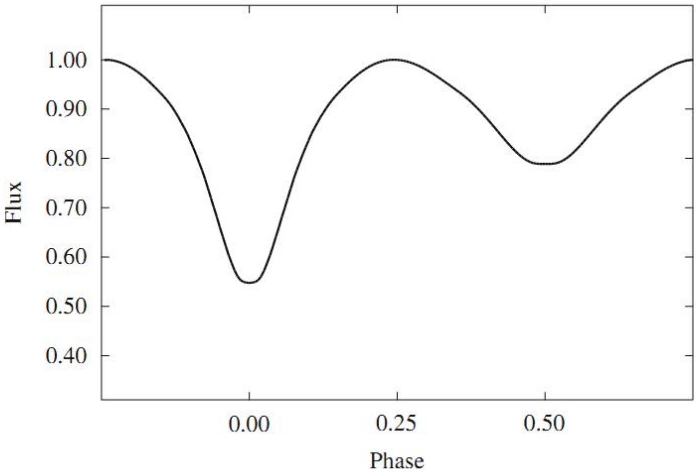


Figure (1-8): Synthetic β Lyrae type light curve (V band) [21 and 23]

Chapter Two

Method on Light curves of selected stars

2.1 Introduction on AD Andromeda

AD Andromeda (BD +47 4207, Vmax =11:2mag) is an interesting system with two nearly identical members, whose spectral type was previously classified as F-type (Koch et al., 1970). Later, it was classified as A0V type by Hill et al. (1975) in their spectral study. The light variability of AD Andromeda was discovered by Guthnick and Preger (1927). It was found that the photographic magnitude ranging from 10:m8 to 11:m7 and with almost the same two minima. The observational data were later analyzed by Gap- oschkin (1932) who gave approximate geometrical elements of the system. Then, it was reobserved visually and photographically by Taylor and Alexander (1940) and Briede (1949). The first photo- electric light curve in yellow light was obtained by Rucin´ ski (1966), five times of light minimum were given in that paper. Rucin´ ski (1966) roughly analyzed the light curve with the method of Russell and Merril (1952), he obtained different results from Gaposchkin's (1932) (based on photographic observations). The light curve ob- tained by Rucin´ ski (1966) was reanalyzed by Giuricin and Mardi- rossian (1981) with Wood's (1972) lightcurve synthesis computer model, they rediscuss the photometric elements of AD Andromeda, the results revealed that both components of the system were practically identical with equal sizes, masses, temperatures, and luminosities, and they gave indeterminate orbital inclination i =81.9 The period variations of AD Andromeda had been investigated by Whitney (1957), Rucin´ ski (1966), and Frieboes-Conde and Herczeg (1973). Based on all visual or photographic light minimum derived before 1973 and only five photoelectric light minimum obtained by Rucin´ ski (1966), Frieboes-Conde and Herczeg (1973) determined the period of the triple system 16.8 yr with an amplitude of A3 = 0.^{d}0275, and a mass function of f (m)= 0:382 M . (estimated the mass of the binary based on F3V + F7V). In the present paper, the light-time effect of AD Andromeda was analyzed based on all available photoelectric and CCD times of light mini- mum. The physical properties of the third body are discussed. Moreover, the light curve obtained by Rucin´ ski (1966) was reanalyzed with the 2003 version of the W–D code. New photometric solutions of the system were derived.

2.2 New CCD photometric observations for AD Andromeda

In order to analyze the period variations of the binary stars and investigate the physical properties of the third body, AD Andromeda was observed on December 23, 2007 with the PI1024 TKB CCD photometric system attached to the 1.0-m reflector at the Yunnan Observatory. The R filter, close to the standard Johnson UBV system, was used. The effective field of view of the photometric system is about 60.5 60.5 at the Cassegrain focus and the size of each pixel is000.38. The integration time is 30 s for each image. The coordinates of the variable star, the comparison star, and the check star are listed in Table 1. The PHOT (measure magnitudes for a list of stars) of the aperture photometry package of IRAF was used to reduce the ob- served images. By using our photometric data, one time of minimum light, HJD 2454458.0635(±0.0002), was determined.

2.3 Observations and data reduction

The photometric observations discussed herein were gathered at the Gerostathopoulion Observatory of the University of Athens in 2008 (for details see Table 1), using the 0.4 m Cass grain telescope equipped with an ST-8XMEI CCD camera and BVRI (Bessell specification) photometric filters. Aperture photometry was applied to the raw data and differential magnitudes were obtained using the software MuniWin v.1.1.23 (Hroch 1998). Exposure times were arranged in order to search for any short- period pulsations. Further details of the comparison and check stars of each programme are given in Table 1.In Table 1, we list reference B and V magnitudes for the mean out-of-eclipse light levels of the three binaries. In the case of AL Cam, its main comparison star GSC 04556-00163 has SIMBAD values of B = 10.30 mag and V = 9.81 mag, so that our observed differential magnitudes lead directly to the values of Table 3. We note that B − V = 0.31 is then consistent with the A8 type classification derived below. For the other two binaries, we obtained the V reference values from Hilditch & Hill (1975) and interpolated from the b − y values given in that reference, checked also against the J magnitude values given by SIMBAD for the comparison stars, using the linear gradient formula (cf. Budding & Demircan 2007, Chap. 3.6).For V388 Her, the resulting B and V magnitudes agree with those given by SIMBAD. For AD And, there is a disparity with the V magnitude (11.2) listed by SIMBAD, but that appears to be an old photographic value and is inconsistent with the expected degree of reddening for that star .

2.4 Spectroscopic analysis

A total of 19 spectroscopic standard stars, proposed by GEMINI Observatory2, ranging from A0 to G8 spectral types, were observed with the same instrumental set-up. Exposure times for the variables were 1800 s. All spectra were calibrated and nor- malized to enable direct comparisons. The spectra were then shifted, using Hβ as reference, to compensate for the relative Doppler shifts of each standard. The spectral region between 4800 Å and 5350 Å, where Hβ and numerous metallic lines are strong was used for the spectral classification. The remaining spectral regions were ignored, because they generally lacked sufficient metallic lines with significant signal-to-noise ratios. The variables' spectra were subtracted from those of each standard, deriving sums of squared residuals in each case. These least squares sums guided us to the closest match between the spectra of variables and standards with a formal error of one subclass. The spectra of the variables were also compared with synthetic spectra, following the same method, by using the SPECTRUM software (Gray & Corbally 1994). The resolution of the synthetic spectra was chosen to be the same as that of the variables' spectra, while parameters such as micro turbulence and macro- turbulence were given typical values for this kind of stars. The effective temperatures and gravity parameters for the synthetic spectra were set to lie around the expected range of values of those of the variables.

Chapter Three

Result And Dissection On Light Curves Of Selected Star

3.1 Light curve analysis

We analysed simultaneously complete LCs of each system using all individual observations, with the PHOEBE v.0.29d software (Prša & Zwitter 2005) that follows, more or less, the method of the 2003 version of the Wilson-Devinney (WD) code (Wilson& Devinney 1971; Wilson 1979, 1990). In the absence of spec- troscopic mass ratios, the “q-search” method was applied in modes 2 (detached system), 4 (semi-detached system with the primary component filling its Roche lobe) and 5 (conventional semi-detached binary) to find feasible (“photometric”) estimates of the mass ratio. This value of q was then set as an adjustable parameter in the subsequent analysis. The temperature of the primaries T1 were assigned values (9800 K for AD And) according to their spec- tral class from the tables of Cox (2000) and were kept fixed during the analysis, while the temperatures of the secondaries T2 were adjusted. The Albedos, A1 and A2, and gravity darkening coefficients, g1 and g2, were set to generally adopted values for the given spectral types of the components (Rucinski 1969; von Zeipel 1924; Lucy 1967). The (linear) limb darkening coefficients, x1 and x2, were taken from van Hamme (1993).

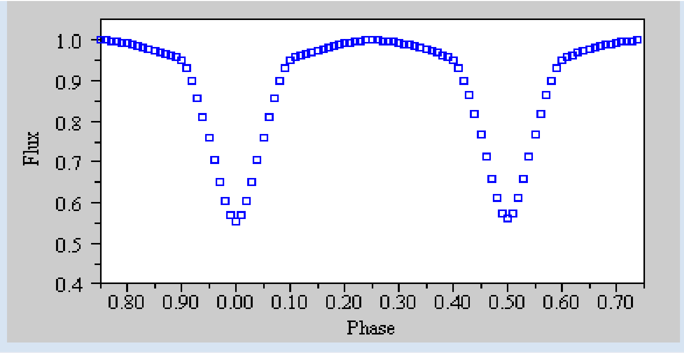
Table (1): The output parameters for AD And:

Where $\Omega_1$andΩ_2=surface potential of the primary and the secondary components ,C_1 and C_2= potential of the primary and the secondary components,f_1,f_2= Fillout of the primary and the secondary components,L_1,L_2=Absolute parameter of components in units of solar Luminosity.

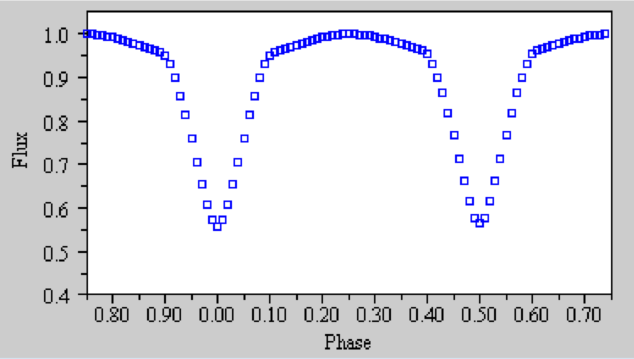
Table (1) : The light curve fit parameters for AD And

Ω1= 4.310000 Ω2 = 4.140000
Ωinner =3.701085 Ωouter = 3.171423
C1 = 3.901118 C2= 3.901118
C inner = 3.901907 C outer= 3.559723
f1= 0.002308 f2= 0.002308
Lagrangian L1 = 0.595817 Lagrangian L2= 1.540608
r1(back)= 0.491415 r2(back) = 0.325065
r1(side) = 0.464740 r2(side) = 0.292304
r1(pole)= 0.435204 r2 (pole) = 0.280426
Surface area 1 = 2.733718 Surface area 2 = 1.145661
Mean radius 1 = 0.463786 Mean radius 2 = 0.299265

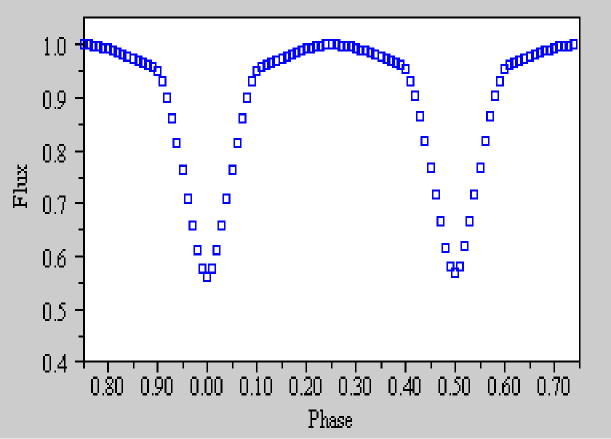
Parameter	**Star1**	**Star2**
Mass Ratio (M2/ M1)	0.970000	
Surface Potential Ω	4.310000	4.140000
Temperature	9800.00	9790.00
Gravity Darkening	0.320	0.320
Limb Darkening	0.418	0.418
Reflection	2.300	2.400
Inclination	82.600	



B



V



R

Figure (3-1): Light curve fitting for B, V and R Filter for AD And .

3.2 Absolute Parameters

The absolute parameters can be determined using Eqs. (3) to (7),[2,3,4,5] for this purpose programs handled using Matlab program. The Table (3) represents the present work as compared with other researches work.

$$\mathbf{A^3 = 74.5\ p^2\ (M_1+M_2} \qquad \text{…… (1)}$$

$$\mathbf{(M_2/M) = q\ (M_1/M)} \qquad \text{…… (2)}$$

$$\mathbf{R_1 = Ar_1,\ R_2 = Ar_2} \qquad \text{..…. (3)}$$

$$\mathbf{L_1 = R_1^2\ T_1^4,\quad L_2 = R_2^2\ T_2^4} \qquad \text{…… (4)}$$

$$\mathbf{M_1 = \frac{1}{1+q} M\ ,\quad M_2 = \frac{q}{1+q} M} \qquad \text{……(5)}$$

$$\mathbf{Q = M_2/M_1} \qquad \text{…… (6)}$$

$$\mathbf{R_{1,2}/R_{sun} = ar_{1,2}} \qquad \text{…… (7)}$$

Table (2): Absolute parameter (in solar units)of the systems for short period binary star

Star	R1	R2	M1	M2	L1	L2	Type	Refrence
AD AND	2.16	1.3496	0.5076	0.4924	43	46	F	Present work
AD AND	2.3	2.4	2.7	2.76	44	47	F	A. Liakos 2012

3.3 Bolometric Magnitudes-$M_{bol1,2}$:

The parameters can be evaluated using the relation given by Kopal and Shapely (1956) or Allen (1976) [11,12].

$$\mathbf{M_{bol(1,2)} = 42.36\text{-}\log T_{1,2}\text{-}5\log(R_{1,2}/R_o)} \qquad \text{…… (8)}$$

The Bolometric Magnitudes can be determined using Equation (8), for this purpose programs handled using Mat lab program. The able (4) represents the present work regarding the adapted axillary parameters for active short period binary system.

$$g_{1,2} = \frac{c_2}{4\lambda T_{1,2}[1-\exp(-c^2/\lambda T_{1,2}} \qquad \text{…… (9)}$$

The reflection coefficients were evaluated the following equations [13]

$$E_{eff1} = t_1[\frac{T_2}{T_1}]^4 \frac{e(c_2/\lambda T_2-1)}{e(c_2/\lambda T_1-1)} \qquad \text{…… (10)}$$

$$E_{eff2} = t_2[\frac{T_1}{T_2}]^4 \frac{e(c_2/\lambda T_1-1)}{e(c_2/\lambda T_2-1)} \qquad \text{…… (11)}$$

Table (3): Adapted auxiliary parameters for active short period binary system.

Star	x_1	x_2	g_1	g_2	E_1	E_2
AD And	0.492	0.493	0.3019	0.3022	4.5244	4.5204
B	0.418	0.419	0.2933	0.2935	4.6577	4.6535
V	0.353	0.354	0.2860	0.2863	4.7762	4.7718
R						

3.4 Age- Mass relationship for CA binaries of stars:

The Age- Mass can be determined using Equation (12), for this purpose programs handled using Mat lab program. The Table (5) represents the present work parameters regarding the age-mass relation. More recently relationship between stellar masses and ages have been used of the form given by Barrado et al (1994).[13]

$$\mathbf{Log(Age) = 9.883-2.965\ log\ [\frac{M}{M_o}]} \qquad \text{……(12)}$$

Table (4): The age-mass relation for other investigation:

Star	Primary log Age	Secondary log Age
AD And	109.0367	110.2542

3.5 The Life Time:

The life time has been for the short period binary systems from the famous mass-luminosity relation and the results are presented in the Table (6).

$$\frac{t1{,}2}{t_{o}}=\frac{1}{(M1{,}2/M_{o})2.5} \qquad \text{…… (13)}$$

The Life time can be determined using Equation (13), for this purpose Mat lab programs handled. The table (6) represents the present results relating life time of short period binary systems.

Table (5): Lifetime of short period binary systems

Star	$t_1/t_{sun}*10^{83}$	$t_2/t_{sun}*10^{83}$
AD And	4.0133	4.0133

3.6 Geometry of Roche Loop:

The unique form of the Roche lobe relies upon at the mass ratio q = M1/M2, and should be evaluated numerically. However, for plenty functions it's miles beneficial to approximate the Roche lobe as a sphere of the identical volume. An approximate system for the radius of this sphere is

r_1/A = max [f_1, f_2],for $0 < q < 20$……………………………………………..(18)

Where f_1= 0.38 + 0.2 log (q) and f_2 = 0.46224 $(q/1+q)^{1/3}$. Function f1 is extra than f2 for q > 0.5228 . The duration A is the orbital separation of the machine and r1 is the radius of the sector whose extent approximates the Roche lobe of mass M1. This components is correct inside to approximately 2%.

(Paczynski, B. (1971) Another approximate components became possible Eggleton’s method and reads as follows:

$r_1/A = 0.49\ q^{2/3}/\ 0.6\ q^{2/3} + \ln (1+q^{1/3})$……………………………………….(19)

This formula gives results up to 1% accuracy over the entire range of the mass ratio q (Eggleton, P. P. (1 May 1983) . The fiq (6) show The shape of the spotted model of the AD And binary system at different phases.

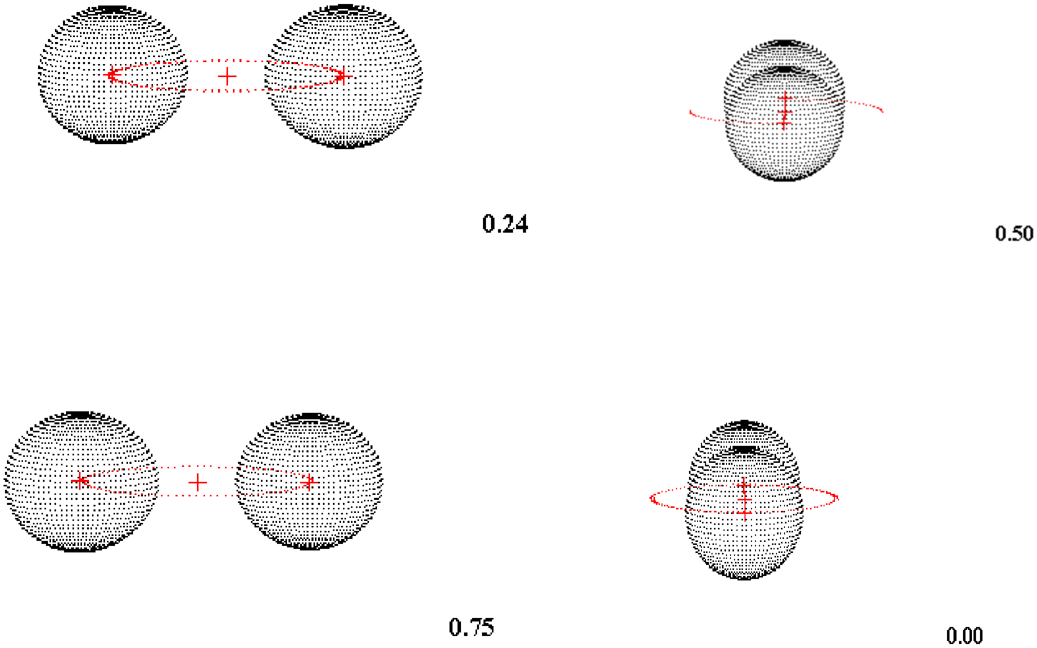


Figure (3-2): Shows the shape of the spotted model of AD And binary star at different phases.

3.7 First Contact Angle:

The first contact angle (θ_1) can be derived from the following equation [14].

$$\theta_1 = \sin^{-1} \sqrt{(r_1 + r_2)^2 \csc^2 i - \cot^2 i} \qquad \text{……} \ (14)$$

$$a = r1 / (r1 + r2) \qquad \text{……} \ (15)$$

$$c = \cos i / (r1 + r2) \qquad \text{……} \ (16)$$

The First contact angle can be determined using Eqs. (14) to (16), for this purpose programs handled using Mat lab program. The Table (8) represents the present first contact angle and type of eclipse.

Table (6): First contact angle and type of eclipse.

Star	θ_1	a	C_o	Type of eclipse
AD And	0.2271	2.5175	0.7454	F5V

3.8 Empirical Relationship between CI ,M_{bol} and T_{eff}:

$M_{sun} = 4.83$, $M_{bol} = M_v + BC\ (T)$.

$B.C._{sun} = -0.07$

$$B-V = -3.684 \log (T) + 14.551 \qquad \text{…} \ (17)$$

For log (T)<3.961

$$B-V = 0.344[\log (T)]^2 - 3.402 \log(T) + 8.037 \qquad \text{…} \ (18)$$

For log (T)>3.961

$$BC = -8.499[\log(T)-4]^4+13.421[\log(T)-4]^3-8.131[\log(T)-4]^2-3.901[\log(T)-4]-0.438 \quad \ldots\ldots(19)$$

Some workers (Mochnacki & Doughty 1972; Binnendijk 1977) use this parameter instead of **Ω** to parameterize the equipotential surfaces. The following relations relate it to **Ω is** [4]:

$$C = 2B1\Omega \pm 2B_2^2 \qquad \ldots\ldots (20)$$

Where

$$B=1/1+q \qquad \ldots\ldots(21)$$

$$B_2=q/1+q \qquad \ldots\ldots (22)$$

The Empirical Relationship between CI ,M_{bol} and T_{eff} can be determined using Eqs. (16) to (22), for this purpose programs handled using Matlab program. The Table (9) represents the present empirical values for primary and secondary stars of the binary system.

Table (7): Empirical values for present work.

Star	Component	BC= M_{bol}-M_v	CI= B-V	M_{bol}	M_v
AD And	Primary	0.1749	25.6169	82.2777	82.1028
	Secondary	-0.1734	25.0349	83.2751	83.1017

RESULTS AND DISCUSSION

In Table 2 we list our results from fitting the O − C diagram for AD And together with early published results. Our best fit solution is shown in Figure 2. Parameters of our solution are almost the same as in previous papers, except for the orbital period P3 of the third body. It is about 2 years shorter (12.1 yr, in contrast to 14.3 yr) than in other solutions. This difference can be explained by the fact that we have used a much longer time interval for an analysis. Liao & Qian (2009) and Liakos et al. (2012) used only minima times from photoelectric and CCD observations and neglected all O − C points obtained before 1990 because of their poorer quality. We also used these older photographic and visual observations even with smaller weight. Moreover, the last CCD minima times cover the full cycle of O − C variations (see Fig. 2). We also detected a secular period change (parameter Q) not mentioned by other authors. The period change corresponds to increase of the period dP/dt =1.06(94) × 10−4 s yr−1 and should be connected with mass transfer from the secondary component to the primary one and/or with the Applegate effect (Applegate 1992), but this is not in agreement with a detached configuration of the system (Liakos et al. 2012). It is necessary to note that the relative statistical error of Q is almost 90%, which degrades its significance. We have also tried a solution with no Q and we surprisingly obtained results with worse statistical significance. We cannot confirm or disprove the secular period changes and only future observations can resolve this problem. Our solution implies a minimal 3rd body mass (for $i_3 = 90^\circ$) of ∼ 2.33 $M_\odot$, using absolute parameters from Liakos et al. (2012). This would indicate that the third light contributes about 15% to the total luminosity of the system, but the third light resulting from light curve analysis (Liakos et al. 2012) is about 3%. This difference can be explained with the assumption that the 3rd star is actually a binary system with two solar-mass components as mentioned by Liakos et al. (2012).

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