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ABSOLUTE PROPERTIES OF THE SPOTTED ECLIPSING BINARY STAR CV BOOTIS

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This research project has been written under my supervision and has been submitted for the award of the degree of B.Sc. in (Physics)

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Sebar

Content

Abstract

We present new V -band differential brightness measurements as well as new light curve measurements of the detached, circular, double-lined eclipsing binary system CV Boo. These data along with other observations from the literature are combined to derive improved absolute dimensions of the stars for the purpose of testing various aspects of theoretical modeling. Despite complications from intrinsic variability we detect in the system, and despite the rapid rotation of the components, we are able to determine the absolute masses and radii to better than 1.3% and 2%, respectively. We obtain MA = 1.032 ± 0.013M⊙ and RB = 1.262 ± 0.023 R⊙ for the hotter, larger, and more massive primary (star A), and MB = 0.968 ± 0.012M⊙ and RB = 1.173 ± 0.023 R⊙ for the secondary. The estimated effective temperatures are 5760±150K and 5670±150K. The intrinsic variability with a period ∼1% shorter than the orbital period is interpreted as being due to modulation by spots on one or both components. This implies that the spotted star(s) must be rotating faster than the synchronous rate, which disagrees with predictions from current tidal evolution models according to which both stars should be synchronized. We also find that the radius of the secondary is larger than expected from stellar evolution calculations by ∼10%, a discrepancy also seen in other (mostly lower-mass and active) eclipsing binaries. We estimate the age of the system to be approximately 9 Gyr. Both components are near the end of their main-sequence phase, and the primary may have started the shell hydrogen-burning stage.

Key words. stars: fundamental parameters { stars: binaries: close { stars: binaries: eclipsing { stars: evolution {stars: late type { techniques: photometric

Chapter One

Introduction

1.1 Binary Stars

The term binary was first used in this context by Sir William Herschel in 1800 (AD) [1]. Binary stars, often called double stars, refer to pairs of stars sufficiently close to each other in space to be gravitationally bound together, orbiting around common center of mass gravitationally bound to each other formed at the same time as has been illustrated in the Fig. (1-1). The two members of a binary star are of unequal brightness [2]. The brighter star is called the primary and the fainter is called the companion [3]. Binary systems are of special interest, because analysis of their orbital characteristics is done by using of Kepler's third law which yields a direct measure of stellar masses [4]. Binary stars which can be resolved with a telescope or inter geometric methods are known as visual binaries. Most of the known visual binary stars have not completed one whole revolution, but are observed to have travelled along a curved path or a partial arc. At least 80% of all stars in the Milky Way are part of multiple systems (binaries, triplets or more) some are close enough that they are able to transfer matter through tidal forces. These are close or contact binaries according to conservative statistics. There seems to be no obvious preference for particular combinations of brightness, size, or mass differences and a wide range in periods of revolution from less than a day to thousands of years. Likewise, there is a large range in separations from those stars in contact to those separated by thousands of times the Earth to Sun distance. Historically, visual binaries are which those that appear as double stars when seen through a telescope, were discovered to be gravitationally bound by William Herschel around 1800(AD) [1].

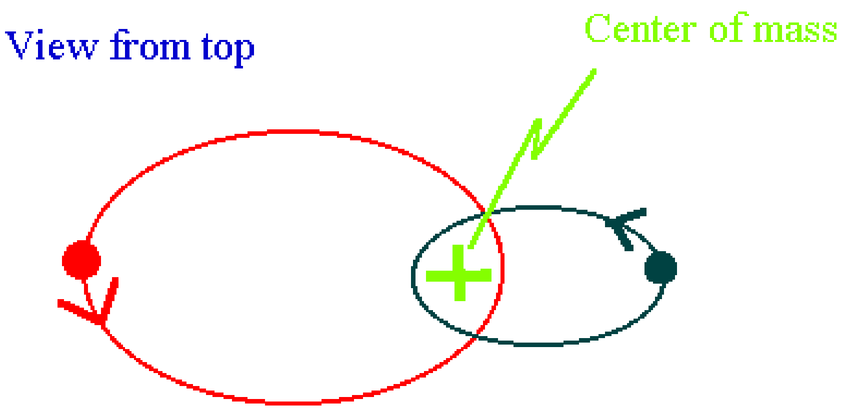

Figure (1-1): Binary stars orbiting their center of gravity [5].

It is noticeable from the Fig. (1-1) that the orbits of two stars of a binary system, relative to the center of mass of the two stars. The more massive star M has a smaller orbit than the less massive star m, but both orbit with the center of mass at one focus of their ellipses. While the Fig. (1-1), shows that the system can be considered equally well as one star (the secondary) orbiting the other (the primary). The dashed line represents the distance between the two stars at a particular moment in time as shown in the above figure, with the same length and direction, to aid in relating the two equivalent ways of visualizing the geometry but it can be considered that one star fixed, with the other star orbiting in a larger ellipse. One can choose either star as the central star, but it is customary to choose the brighter star (usually the more massive, but not always), it is called the primary and the other star is called the secondary. The primary star is at a focus of the larger ellipse [4.The first major catalogue of binary stars was made by Sir William Herschel, in about 1800(AD) he was the first coined the term "binary star" to describe the union of two stars through their mutual gravitational attraction. William Herschel also differentiated between apparent and real double stars, an apparent double is two stars that appear to be close together but actually are not physically associated because they are separated by a large distance along the line of sight. A real double (physical binary) consists of two stars that are bound together by their mutual gravity and revolve around a common center of mass (2,9).When two stars are far enough apart that they evolve independently, they are called wide pair, each star in a wide pair follows the same career as a single star of the same mass, but when the two stars are close enough to each other that they can transfer matter to one another at the same stage of their development, they are called a close pair.[9] Thromjjn these exchanges of mass in a close pair alter the way that each evolves [2,8]. Binary star also can be classified according to how it was learned that there are two stars rather than one. Some methods of recognizing a binary stars system favor the discovery of wide pairs. Others favor the discovery of close pairs [7,9]. Approximately two-thirds of all solar field stars are members of binary systems, and recent studies suggest that virtually all stars begin life as member of multiple systems. Consequently, many of all the stars we see at night are actually binaries, comprised of two stars gravitationally bound in orbit one another [l0]. These Binary systems are important astrophysical laboratories because they allow us to deduce the properties of the constituent stars move a (concretely) than we can with single stars. The physics that governs how stars orbit one another was developed by Newton and Kepler over three hundred years ago, and can be summarized by the equation

$P^2 = 4\pi^2 / G(M_p+M_s)\, a^3$…………………………………………..(1-1)

Where G is the gravitational constants and M_p and Ms are the masses of the two components (a) is the semi-major axes of the two orbits a= $a_P + a_S$ In cmgs units , G=6.67*10^{-11} dyn cm^2 g^{-2} but these units are not the units of choice, Where mases measured in solar masses distances in AU(11).

1.2 Types of Binary Stars:

For decades, the study of eclipsing variable stars formed an important branch of variable stars or systems as shown in Fig. (1.2) which shows the flow chart for outline scheme of Photometric classification of eclipsing variables and its successive criteria .The most important of the two branches related to eclipsing variables is the branch of binary systems .The problem of binary system classification has come over a wide range of effecting parameters on which or accordingly, the related identification to a certain class is drawn. Both wide and close binary systems have subdivision groups depending on the dominant parameter taken as major factor of classification. It has been found that the majority of eclipsing variable, which are among the most useful representatives of this field are" close " binary system. Increased attention to such system is due to the behavior of their light variation, therefore, it has been customary for many years to divide eclipsing variables into the following groups. This classification includes the following types Kopal (1959):

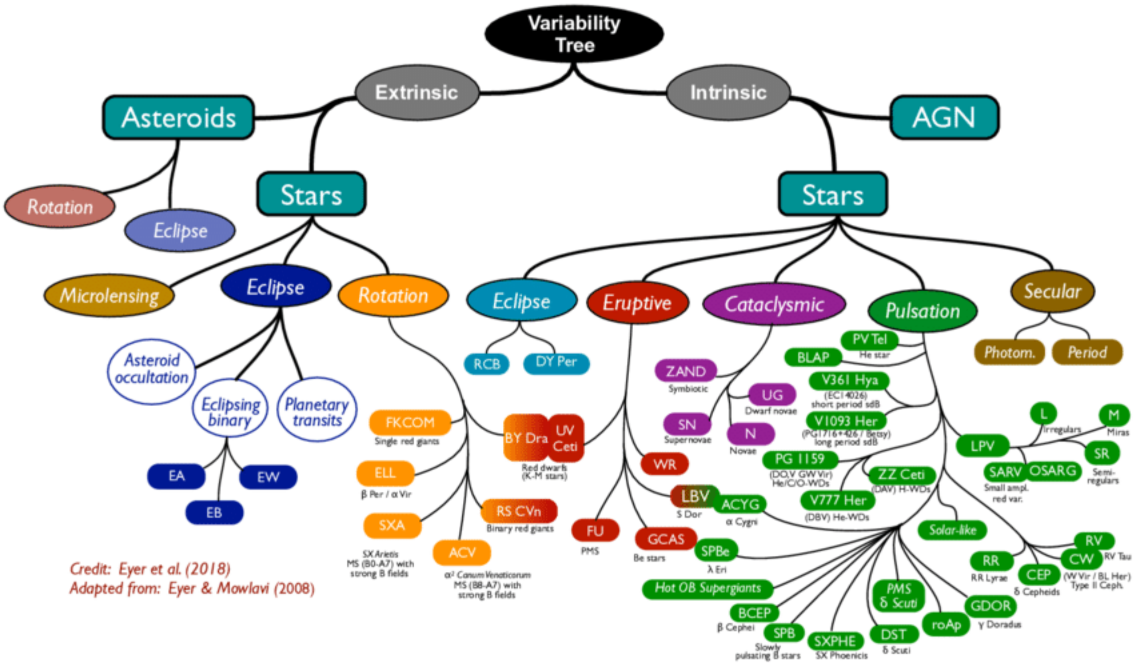

Figure (1-2): Binary stars tree .

1. 3 Classifications of Binary Stars by Methods of Observation:

Binary stars are classified into four types according to the way in which they were observed: visually, by observation; spectroscopically, by periodic changes in spectral lines; photometrically, by changes in brightness caused by an eclipse; or astrometrically, by measuring a deviation in a star's position. Any binary star can belong to several of these classes; for example, several spectroscopic binaries are also eclipsing binaries.

1.3.1 Optical Double

This is not really a binary star. Two stars just happen to appear along almost the same line of sight. The two stars appear very close in the sky but are at different distances from Earth .They are actually very far apart [11]. The two objects are totally unrelated and are actually moving through space at right angles to each other. They passed at a minimum separation of 9 seconds of arc in 1960 and have been separating ever since [12].

1.3.2 Visual Binaries

True binary star systems (as opposed to purely optical doubles) in which both components are visible and resolvable in a telescopic eyepiece, Astronomers consider a visual binary, assuming initially that the brighter primary component is stationary and the fainter secondary component is orbiting around it [13] . The angular separation of the stars and the angular direction to the secondary can be directly observed. Making use of observations extending over many years or decades, the relative orbit of the secondary can be determined. In most cases, the two stars components differ in brightness [14]. The first binary orbit to be determined was that of W UMa in 1830 as shown in figure (1-3). With the current generation of stellar interferometers, many more binary systems fall into this category, although some researchers call these “interfero metric binaries”. Knowledge of the positions of the stars on the sky, as a function of orbital phase, coupled with RV observations; allow the masses of the stars to be measured directly, along with their luminosity ratio. These stars are therefore good for determining the mass–luminosity relation of stars, but, more importantly, they provide an essentially geometric determination of the distance to the system which is very reliable.

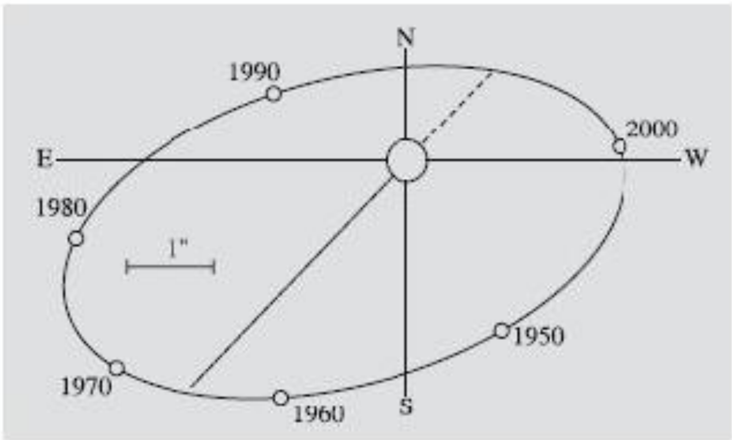

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Figure (1-3): The orbit of M Ursae Majoris was the first binary orbit determined observationally in 1830 [15].

1.3.3 Astrometric Binaries

If only one component is visible, because the other is too faint and/or is too close to its brighter companion to be separated through telescopic resolution alone, gravitational effects may help one to prove that the system is a binary. The first astrometric binary was Sirius, which in the 1830's was observed to have an undulating proper motion. It was concluded that it had a small companion, which was visually discovered a few decades later, see Figure (1-4). The companion, Sirius B, was a completely new type of object, a white dwarf the proper motions of nearby stars have been carefully studied in the search for planetary systems. Although for example Barnard's star may have unseen companions, the existence of planetary systems around other O stars was not established by proper motion studies but with spectroscopic observations [16]. Another interesting type of astrometric binary is presented by cases where the components are so close that they are, or have been until recently, irresolvable. Fig: (1-4) shows the proper motion of the Sirius system over 70 years, the slight perturbations or wobble in the bright star, Sirius A, is due to the presence of its much dimmer white dwarf companion, Sirius B [17].

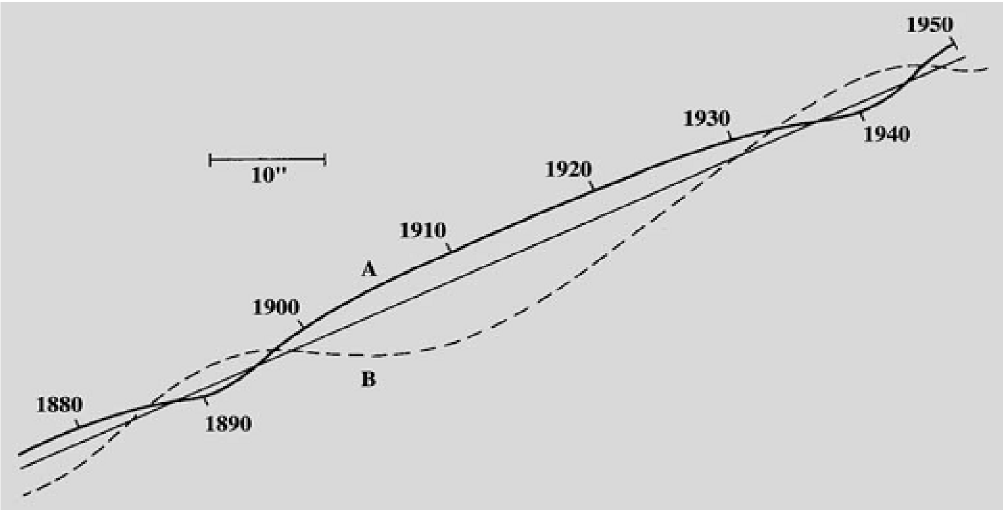


Figure (1-4): The apparent paths of Sirius and its companion in the sky [15].

1.3.4 Spectroscopic Binaries

Spectroscopic binary systems are those for which their binarity is apparent from variation of their radial velocity. The secondary component may also produce spectral lines strong enough to be visible in the spectrum of the system, in which the case of the spectroscopic binary is “double-lined”. Spectroscopic observations of these systems allows calculation of the orbital period and eccentricity, the mass ratio, and the minimum masses of the components, $M \sin3i$, where i is the inclination of the orbit relative to the line of sight of the observer. Spectroscopic binaries have too small separation for the components to be seen individually, but the binary nature of the system betrays itself because of the Doppler shifts in the spectrum [18]. This causes the absorption line to be displaced toward the blue end of the spectrum as the primary star approaches the Earth in its orbit, and then to the red end as it recedes [15].The first to be discovered was the brightness star of the double star Mizar, which was itself shown to be a binary in 1889 by E. Pickering [17]. If a binary system is unresolved into its components then the spectrum obtained from it will actually be a combination of the spectra from each of the component stars. As these stars orbit each other, then one star, A, may be moving towards the Earth whilst the other, B, may be moving away. The spectrum from A will therefore be blue-shifted to higher frequencies (shorter wavelengths) whilst B's spectrum will be redshifted. If the stars are moving across our line of sight then no Doppler shifting occurs so the lines stay in their mean positions. As the stars continue orbiting, A will recede so its spectral lines will move towards the red end of the spectrum and B's will move toward the blue.This is shown schematically in figure (1-5).

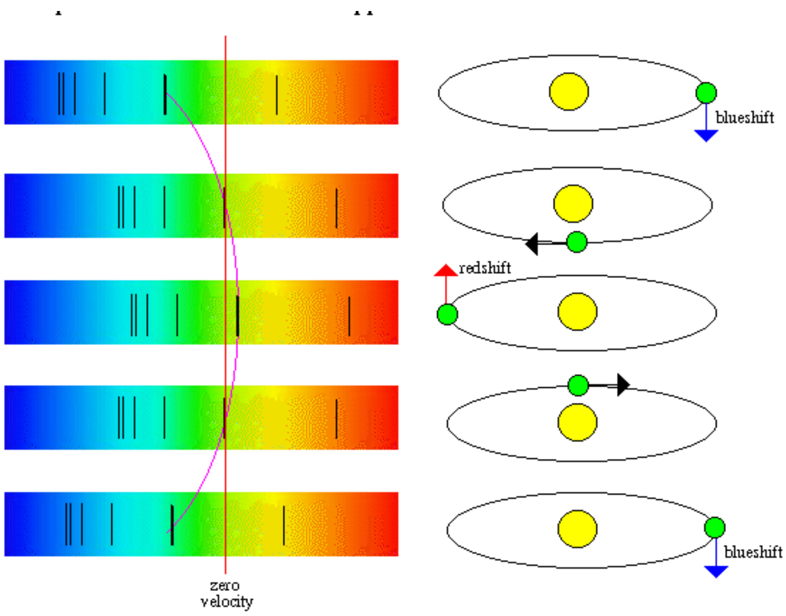


Figure (1-5): Spectroscopic binary star from APOD.

1.3.5 Eclipsing Binaries

Eclipsing Binaries are those binary systems in which the orbital plane of the system is almost side on as seen from the Earth so that as the stars move around the common center of gravity there are eclipses. Stars in an eclipsing binary system are usually so close together that they cannot be separated even in the most powerful telescopes and are usually called eclipsing variables [18]. They are in an eclipsing system because of the way the light from the system varies in a regular and characteristic pattern. An eclipsing binary allows scientists to find out a lot about how binary systems evolve and thus the fate of most stars. A primary eclipse occurs when the brighter star is eclipsed by the fainter star. The secondary eclipse occurs when the fainter star is eclipsed by the brighter star. Assuming the two stars are of the same brightness then both eclipses will be equal. If one of the stars is much fainter than the other the secondary eclipse may not be readily observed by visual observers. If the orbits of the two stars are circular then the secondary eclipse will occur midway between primary eclipses. Suppose that the orbits are elliptical the secondary eclipse may occur earlier or later than midway. A primary eclipse is said to be total if one star is completely obscured by the other star for a period of time. The period of minimum light can be minutes, days or in one case two years if a star is not totally obscured then the eclipse is described as partial. In that case, there is no prolonged period of minimum light. There can also be a prolonged period of minimum light with an annular eclipse Figure (1-6) shows the Schematic picture of a simple eclipsing binary, and its light curve. In the light curve, time increases to the right, as the smaller, brighter star (in black in this diagram) passes in front of the larger, fainter star (left), there is a secondary minimum. t_1, t_2, t_3, and t_4 are the times of the four contacts. As the brighter star passes behind the fainter one (right), there is a primary minimum [19] .

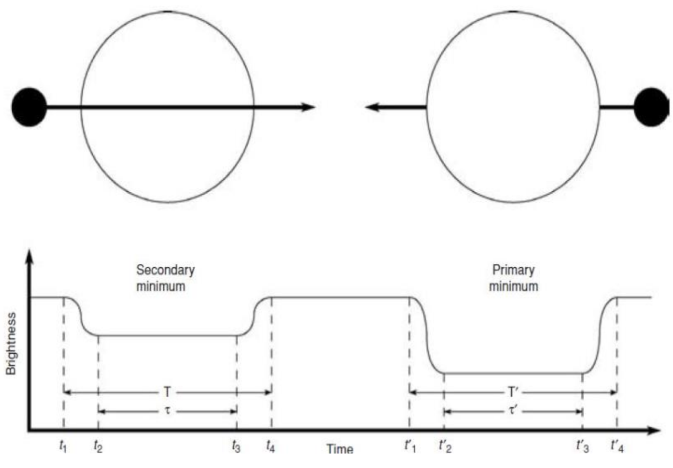


Figure (1-6): Schematic picture of a simple eclipsing binary, and its light curve.[20]

1.4 Types of Eclipsing Binaries

There are three basic types of eclipsing binary which are classified according to the shape of their light curves. The light curve will be the observed magnitudes over the whole period of the system; these types are [21].

1.4.1 Algol Stars

The Algol-type eclipsing variables have been named after β Persei or Algol . During most of the period, the light curve is fairly constant. This corresponds to phases during which the stars are seen separate from each

other and the total magnitude remains constant. There are two different minima in the light curve, one of which, the primary minimum, is usually much deeper than the other one. This is due to the brightness difference of the stars. When the larger star, which is usually a cool giant, eclipses the smaller and hotter component, there is a deep minimum in the light curve. When the small, brighter star passes across the disc of the giant, the total magnitude of the system does not change by much [15] . The light curve of Algol shows a significant difference in the depths of their two minima . The shape of the minima depends on whether the eclipses are partial or total. In a partial eclipse the light curve is smooth, since the brightness changes smoothly as the depth of the eclipse varies. In a total eclipse there is an interval during which one component is completely invisible. The total brightness is then constant and the light curve has a flat bottomed minimum. The shape of the minima in Algol variables thus gives information on the inclination of the orbit. Figure (1-7) shows the light curve of Algol type [19]. The duration of the minima depends on the ratio of the stellar radii to the size of the orbit. If the star is also a spectroscopic binary, the true dimensions of the orbit can be obtained. In that case, the masses and the size of the orbit, and thus also the radii can be determined without having to know the distance of the system [21] .

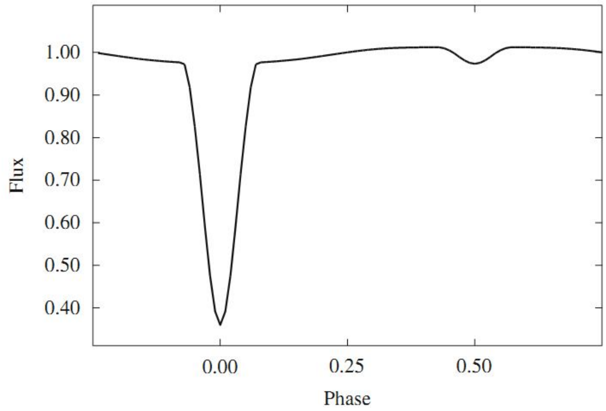


Figure (1-7): Synthetic " Algol "type light curve (V band) [19]

1.4.2 β Lyrae Stars

In the β Lyrae-type binaries, the total magnitude varies continuously. The stars are so close to each other that one of them has been pulled into ellipsoidal shape. Thus the brightness varies also outside the eclipses. The β Lyrae variables can be described as eclipsing ellipsoidal variables. In the β Lyrae system itself, one star has overfilled its Roche lobe and is steadily losing mass to its companion. The mass transfer causes additional features in the light curve [22]. Figure (1-8) shows synthetic β Lyrae type light curve [21 and 23]. The orbital period is in general greater than one day .

In addition, β Lyrae type stars has many features which can be summarized as follows:

1-It is continuously variable.

2-It has quiet difference in surface brightness.

3-It has large difference in minima depths [24].

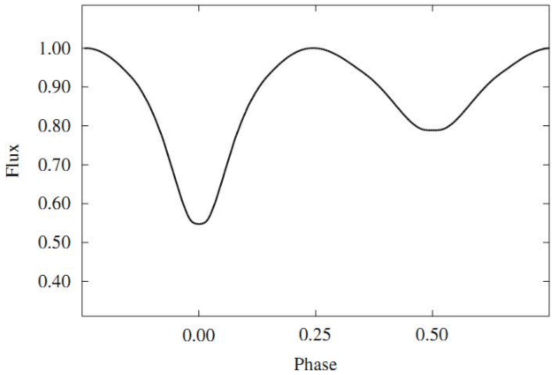


Figure (1-8): Synthetic β Lyrae type light curve (V band) [21 and 23]

2. Methods

Our method of detection and characterization of the sample bi- naries uses a classical approach assuming only one body on a quite distant orbit, meaning there is no dynamical interaction, and only the LITE term was used for ETV analysis. With this method, we would be able to find preferably the bodies with pe- riods of a few years to decades, and with LITE semiamplitudes of the order of 0.01 day (quite similarly to our former analysis, Zasche et al. 2014). This limitation mainly comes from the fact that the data cadence is typically limited, and the precision of the individual times of eclipses critically depends on the quality of the light curve (LC) and depth of a particular eclipse. The times of eclipses for the ETV analysis were collected from published papers. Additionally to the published eclipses, we also computed the new ones from the available data. These were our new dedicated observations of these targets (from ground-based observatories, using typically small amateur tele- scopes), as well as the rich databases of photometric observa- tions of these binaries from existing archives. The PDR code (Zejda et al. 2019) was used to obtain some of these data. These datasets were the following:

* HIP: the Hipparcos satellite, observing in special Hp filter, between 1989 and 1993 (Perryman et al. 1997).

*NSVS: the ROTSE-I experiment, unfiltered photometry, with a time span from 1999 to 2000 (Woz´niak et al. 2004).

*OMC: five-cm camera onboard the INTEGRAL satellite, ob- serving in V filter since 2002 (Mas-Hesse et al. 2003).

* ASAS: All Sky Automated Survey, observing since 1997 in V and I filters (Pojmanski 2002).

*ASAS-SN: ASAS for supernovae, 24 telescopes, V and g filters (Shappee et al. 2014; Kochanek et al. 2017).

– SuperWASP: 20-cm telescopes, using special filters, observ- ing since 2004 (Pollacco et al. 2006).

– Pi of the sky: small robotic cameras, observing since 2004, unfiltered (Burd et al. 2005).

– CRTS: Catalina Survey, 70-cm telescope, observing since

2007, unfiltered (Drake et al. 2009).

– Kepler: Kepler satellite, 95-cm telescope, observed from

2009 to 2018 in special filter (Borucki et al. 2010).

– TESS: TESS satellite, 10-cm diameter, observing since 2018 in special filter (Ricker et al. 2015).

– KWS: Kamogata/Kiso/Kyoto wide-field survey, observing in BV Ic filters (Maehara 2014).

– MASCARA: small 17-mm wide-field cameras, observing since 2017, unfiltered (Burggraaff et al. 2018).

– ZTF: the Zwicky Transient Facility, 120-cm telescope, g, and r filters (Masci et al. 2019).

For the derivation of times of mid-eclipse, our automatic fit- ting procedure (AFP, Zasche et al. 2014) was used. It uses an LC template and phased LCs at particular time intervals when the phase coverage of both minima is sufficient. Therefore, the cadency of these derived minima highly depends on the particular survey, its cadency, and the precision of individual photometric data points. For the systems where no previous LC solution exists, we also performed an LC analysis. This was typically based on the best available LC for that particular system, meaning from the source (from the aforementioned list) that provides the best phase coverage as well as the lowest scatter of the individual observations. The individual cases are discussed in more detail below.

Chapter Two

Modeling light Curves Theory of Algol Binary Stars

2.12 Photographic Photometry :

This depends on the production of permanent images by the action of light on certain sensitive materials. The astronomer Herschel played an important part in the early development of photography, in that he discovered the use of sodium thiosulphate as affixing agent for exposed photographic materials. This process was soon applied to astronomy and brought about great advances in the subject [21]. Photography has made it possible to obtain a permanent record of surface features of members of the solar system and images of galaxies, nebulae and other remote objects invisible to the naked eye. Spectroscopic observations still mainly carried out by photographic means, thought other forms of data recording are slowly replacing the plate. Photography provides, however, along time base line which has sometimes been useful in tracing long term variability of stars [22]. One difficulty with the photographic photometry is that time- exposures are need, amounting often to many hours for faint objects such as remote galaxies. It is quite practicable to guide the telescope accurately while the exposure is being made, but the Earth's atmosphere is always unsteady , as well as being opaque to a large range of wavelengths of the electromagnetic spectrum. However, there have been remarkable improvements in photographic techniques, and excellent results can now be obtained [20]. The principal advantages of photographic plate in spectrophotometry are that it is panoramic because it covers a large range of wavelengths at one time; cumulative because it integrates the light received over a given exposure; stable because it is permanent and can be studied at any time. The principal disadvantage of the photographic plate is that its response is not linear, and therefore, for accurate work, a suitable calibration must be placed on each plate to be studied [23].

2.13 Photoelectric Photometry:

Photoelectric photometry is used for precise stellar photometry and for observation of rapid fluctuation in the brightness of variable stars. The detecting device, in this case, may be a photoelectric cell or photomultiplier tube. The arrangement of the apparatus is such that the star light is allowed to strike the surface of the photocathode and consequently the photoelectrons are emitted. These electrons form the output current which is directly proportional to the intensity of the stellar light. This output current can be amplified and measured using a D.C or an A.C. pulse counting technique [23].Photometric techniques have been extensively studied by many authors. Work of different investigators in different fears a great contribution towards the development of stellar photometric techniques. Nowadays, photoelectric photometry is widely used for accurate studies of stellar photometry; but, even then its accuracy is limited due to the atmospheric condition and the conditions in the photometric instrument itself. It has a disadvantage too in most normal arrangements. One can often only observe one star at a time and with one filter only, especially for faint stars. Although, obviously, this could be a time consuming process as compared with photography, yet it still offers more internal precision, about 1 %, as compared with that for careful photography, 5%. This is basically because of high quantum efficiency for photomultipliers compared to that of 0.001 for photographic plates [59].The photoelectric photometer and photon counting techniques provide a very accurate method for measuring the colour of a star, rather than the conventional photographic methods. Also, the magnitudes and colors of much fainter stars are measured more easily with this method than the spectroscopic technique. The combination of photoelectric photometry provide a much more reliable and extensive observing method.

2.14 An Over View of Light Curve

Modeling

In the present work author makes use of this type of photometrical observation which have been sent to her by private communication.

Significant progress of computational astrophysics was made in the early 1970s.Models and programs were developed to compute (synthetic) light and velocity curves directly. Such models and programs were based on spherical stars, treated in the Eclipsing Binary Orbit Program (EBOP) [Nelson & Davis 1972; ellipsoidal geometry, treated in WINK program developed by Wood 1971. Lucy 1968, Wilson& Devinney 1971, produced models and programs based on Roche geometry's". FORTRAN program EBOP is based on the Nelson & Davis 1972 spheroidal model called the NDE model. **It** is an efficient software for the analysis of detached binary systems with minimal shape distortion due to proximity effects. The NDE model and its assumptions are close to those in the rectification model by Russell & Merrill [64]. EBOP makes use of spheroidal stars moving in circular or eccentric orbits The Wood 1972 WINK program is based on the Wood 1971 model. The model assumes the components of the binary system to be tri-axial ellipsoids. Physic_&_ models based on equipotential and Roche geometry are implemented in the Wilson-Devinney program, used in present work for W UMa type of eclipsing binary star. A breakthrough occurred with the introduction of Roche geometry. An example of improved astrophysical

understanding through eclipsing binary light curve analysis is the successful modeling of W UMa stars as over-contact systems[30]. The history of the study of eclipsing systems of later spectral classes such as W UMa goes back to the beginning of the Lucy (1960's). This class of contact systems of the close binary stars is the most numerous and has been most observed photometrically and spectroscopically. This is because of their short periods and the procedure for identifying them as eclipsing variables.

2. 15Wilson and Devinney Program:

The Wilson-Devinney program is the most widely used of all the synthetic and analytic light curve modeling codes in photometry of spectroscopy studies . It is appropriate to describe its features, capabilities, and continuing developments in some detail. It has seen continual improvements and the current version 2006 with its powerful features provides the opportunity to extract a maximum of information from a variety of observational data [25].The stand-alone Fortran77 program WD 2006 enables the user to make use of the Wilson- Devinney (WD) program to compute the parameters of eclipsing binary light and radial velocity curves and to analyze observational data, i.e., to fit light curves or merely to compute a synthetic light curve[64] . The overall program consists of two parts of FORTRAN program designed especially for the analysis of binary star data: one is LC (Light Curve generation) in which observers input data the star parameters and the program then calculates the light curves and/or radial velocity (RV) curves that observers should observe. The other is DC (Differential Corrections) in which the program takes observers data and calculates the corrections that astronomers need to make to the basic parameters in order to improve the fit [24]. The WD model provides several modes to specify the geometry of the binary system or to add constraints or relations between parameters. The

WD modes are summarized as follows:

Mode-1: This mode is for X-ray binaries for which the eclipse duration of a compact object is known from X-ray observations.

Mode0: No constraints are applied in mode 0, and the component luminosity ratio is not even required to be consistent with the surface temperatures. This mode is the closest analogue to the Russell model.

Mode1: This is a mode for over-contact binaries, such as WUMa system (T_1=T_2 common envelope)

Mode2: This is for detached binaries with no constraints' on the

Roche potential (surface potentials).

Mode3: This is for over-contact binaries and is the same as mode 1, except that the constraint on T2 is not applied. In the present work, only this mode have been used for the present investigation.

Mode4: This is for semi- detached binaries with star 1 accurately filling its limiting lobe, which is the classical Roche lobe for synchronous rotation and a circular orbit, but it is different from the Roche lobe for non- synchronous rotation and eccentric orbits.

Mode5: The same as mode4, except that it is star2 that fills its limiting lobe .This is the usual mode for Algol type binaries.

Mode6: This is for double contact binaries .

LD: This sets the limb darkening law. LD = 1 for the linear cosine law, LD = 2 for a logarithmic law. The complete logarithmic law is: [21]

$$\mathbf{I / I_o = 1- x+x\cos\beta - y \cos \beta \ln(\cos\beta)} \qquad (2\text{-}1)$$

If LD = 3, the bolometric square root law is used in this work. The complete square root law is: [21]

$$\mathbf{I / I_o = 1- x+x\cos\beta - y(1-\sqrt{\cos\beta}} \qquad (2\text{-}2)$$

JDPHS: This is 1, if the independent variable is time and 2 if it is phase. Where β **:** is the angle measured between observer line of sight and the surface normal of a particular surface area element. x: limb darkening coefficient, y: bolometric limb darkening coefficient, I: intensity. The input parameters are given in appendix [26].

2.16Atmospheric Parameters.

Generally, stellar atmospheres have non-uniform surface brightness, this fact is exaggerated when a star fills its Roche lobe and takes on a distorted shape. The two main phenomena that act to alter the surface brightness of a star are limb darkening and gravity darkening. If star spots are present, they will affect the appearance of the star and light curve too. Finally, if there is another source of light in the system which shines onto a portion of the star, the bolometric albedo of the star will determine how much of the light is reflected back and seen by the observer. All of these phenomena work together to give the Roche lobe-filling star a non- uniform brightness, and hence, affect the shape of the observed light curves [29]. Therefore, the computation of the flux emitted by the stellar photosphere based on several assumptions about the underlying photosphere physics. These include the choice of a model atmosphere and several effects: gravity darkening, limb-darkening, reflection effect and the Stellar spot [27].

2.17 Absolute Parameters:

The most of astronomers knowledge of absolute parameters of stars beyond the sun have been derived from the study of binary star system, and most especially of individual eclipsing photometric. A mathematical analysis of the light curves enables one not only to determine the physical and geometrical elements of selected binary systems but also to find their absolute parameters. In order to calculate the absolute parameters of the binary system the following formulae have been used [28,29].

$$\mathbf{A^3 = 74.5\, p^2\, (M_1+M_2} \quad \text{-------------------------------------} \quad \mathbf{(2\text{-}17)}$$

$$\mathbf{(M_2/M) = q\, (M_1/M)} \quad \text{--} \quad \mathbf{(2\text{-}18)}$$

$$\mathbf{R_1 = Ar_1,\ R_2 = Ar_2} \quad \text{---} \quad \mathbf{(2\text{-}19)}$$

$$L_1 = R_1^2\, T_1^4, \quad L_2 = R_2^2\, T_2^4 \quad \text{------------------------------------} \quad (2\text{-}20)$$

$$M_1 = \frac{1}{1+q} M\,, \quad M_2 = \frac{q}{1+q} M \quad \text{------------------------------------} \quad (2\text{-}21)$$

Where equation (2-17) is Kepler's third law, A is the separation between the two components expressed in solar radii, P is the orbital period in day, M_1 and M_2 are the masses of the components in solar mass. Equation (2-18) is the relation between masses for the primary components to the secondary one, q is the mass ratio. Equation (2-19) is the relation between absolute R_1 and R_2 , and relative r_1 and r_2 are radii of the components. The relative radii have been taken equal to the geometrical mean of the polar, side, and back radii, that were computed in this work for all selected binaries with (W-D) program. Equation (2-20) is the Stefan-Boltzmann law, L_1 and L_2 are bolometric luminosities expressed in solar units, T_1 and T_2 are the effective temperature in units of solar effective temperature. The Table (3-7) represent the present work comparison with other workers.

2.17.1 Mass Ratio (Q)

The mass ratio q is usually defined as the mass of the less massive star (M_2) divided by that of the more massive star (M_1) [20].

$$Q = M_2/M_1 \quad \ldots (2\text{-}8)$$

2.17.2 The Radii - R :

The following formula was used for determining the absolute radii determinations of both components in each system [21]

$$R_{1,2}/R_{sun} = ar_{1,2} \qquad (2\text{-}9)$$

Where a is the separation of the two components in solar units R_{sun}.

2.17.3 Bolometric Magnitudes-$M_{bol(1,2)}$:

These parameters can be evaluated using the relation given by [22,23].

$$M_{bol(1,2)} = 42.36\text{-}\log T_{1,2}\text{-}5\log(R_{1,2}/R_{o}) \qquad (2\text{-}10)$$

Where all parameters have the same mentioned meanings.

2.17.4 Age- Mass relationship for CA binaries of stars:

More recently relationship between stellar masses and ages have been used of the form given by.[24]

$$\mathrm{Log(Age)} = 9.883\text{-}2.965 \log \left[\frac{M}{M_{o}}\right] \qquad \ldots(2\text{-}11)$$

This relation is very close to that for stars on the TAMS, which was fitted for different luminosity classes. There is a good agreement between the estimates of the ages for the two components of a given system with literature, which proves that the more reliability of this relation. Table (3-9) represent the stars and some relevant stellar parameters, these new analyses serve the basis for new calculation values for the components ages.

2.17.5 The Life Time.

This life time has been for the short period binary systems from the famous mass-luminosity relation and the results are presented in the Table (3-10).

$$\frac{t_{1,2}}{t_o} = \frac{1}{(M_{1,2}/M_o)^{2.5}} \quad \dots (2\text{-}12)$$

2.17.6 First Contact Angle:

The first contact angle (θ_1) can be derived from the following equation [25].

$$\theta_1 = \sin^{-1} \sqrt{(r_1 + r_2)^2 \csc^2 i - \cot^2 i} \quad \dots (2\text{-}13)$$

$$a = r1 / (r1+r2) \quad \dots (2\text{-}14)$$

$$c = \cos i / (r1+r2) \quad \dots (2\text{-}15)$$

r_1 and r_2 denote the fraction radii and (i) the inclination of the orbital plane to the celestial sphere . The eclipse is bound to be partial if:1> c_o > /2a-1/, While it becomes total if c_o <1-2a , by applying the above formula to all stars under investigation we found that for about some values of them is not correct and give different type of the eclipse. [26] has mentioned that the theoretical value of the first contact angle is less than 55° for all type of eclipse. [27] found that G_1, for Algol vary from 23° to 30° while its value forW Uma-type system vary from 40° to 49° Jabbar (1983) .The results for the systems are given in Table (3-12).

2.17.7 Empirical Relationship between CI ,M_{bol} and T_{eff}:

Specifically bolometric correction is defined to be the number of magnitudes that must be nodded to absolute visual magnitude (M_v) to give absolute bolometric magnitude (M_{bol}), $M_{bol} = M_v + BC\ (T)$. The definition above was used since solar-like stars have their radiation maximum in the visual region of the spectrum, and due to the definition of the zero point of B.C. , most stars have negative bolometric correction. Some care is necessary in the use of B.C. from tables because sometimes the definition, $B.C. = M_v - M_{bol}$ is used as given by [28]:

Where B.C.: is the bolometric correction has been tabulated as a function of spectral type and (B-V) index.

$$\mathbf{M_{sun} = 4.83}$$

$$\mathbf{B.C._{\ sun} = -0.07}$$

$$\mathbf{B\text{-}V = -3.684 \log (T) + 14.551} \qquad \mathbf{(2\text{-}16)}$$

For log (T)<3.961

$$\mathbf{B\text{-}V = 0.344[\log (T)]^2 - 3.402 \log(T) + 8.037} \qquad \mathbf{(2\text{-}17)}$$

For log (T)>3.961

$$\mathbf{BC = -8.499[\log(T)-4]^4 + 13.421[\log(T)-4]^3 - 8.131[\log (T)-4]^2 - 3.901[\log(T)-4] - 0.438} \qquad \mathbf{(2\text{-}18)}$$

Where BC represents bolometric correction and CI denotes color index. While the Table (3-13) represents the empirical values for the present work.

2.17.8 Surface Potential (Ω)

The Surface Potential Ω, along with a specified mass ratio, completely describes the surface structure for synchronously rotating, circular orbit binary stars. It takes into account both the gravitational and centrifugal forces in the binary. As the value of the surface Potential is increased, the size of the star decreases. This makes sense since the gravitational potential increases as one approach the mass center. Ω is the value of the outer critical Roche equipotential and represents the limit to stability to any over contact system since the outer potential has a "hole" in it (gravitational acceleration = 0) and gas will leave the system [20].

2.17.9 Equipotential Surfaces (C)

Some workers (Mochnacki & Doughty 1972; Binnendijk 1977) use this parameter instead of **Ω** to parameterize the equipotential surfaces. The following relations relate it to **Ω is** [20]:

$$\mathbf{C=2B1\Omega \pm 2B_2^2} \qquad \mathbf{(2\text{-}19)}$$

Where B=1/1+q

$$\mathbf{B_2=q/1+q} \qquad \mathbf{(2\text{-}20)}$$

2.8.13 Gravity Brightening (Darkening) Exponent

[22] proved that for totally radiative stars the surface flux was directly proportional to the value of the gravitational acceleration (g) at the stellar surface. (The gravitational acceleration is ∇Ω) [20].The gravity-darkening coefficients were determined by black body radiation spectrum as:

$$g_{1,2} = \frac{c_2}{4\lambda T_{1,2}[1-\exp(-c^2/\lambda T_{1,2}} \qquad \mathbf{(2\text{-}23)}$$

Where c_2 is equal to $\frac{1.43883}{(\lambda T_{1,2})eff}$ and λ_{eff} is effective wavelength for used filter.

2.17.14 Reflection coefficients:

The reflection coefficients were evaluated the following equations[23]

$$E_{eff1} = t_1[\frac{T_2}{T_1}]^4 \frac{e(c_2/\lambda T_2 - 1)}{e(c_2/\lambda T_1 - 1)} \qquad (2\text{-}24)$$

$$E_{eff2} = t_2[\frac{T_1}{T_2}]^4 \frac{e(c_2/\lambda T_1 - 1)}{e(c_2/\lambda T_2 - 1)} \qquad (2\text{-}25)$$

2.17.17 Luminosity (wavelength dependent)

The luminosity of a star is a measure of its total output energy. Luminosities L_1 and L_2 that indicate the percentage of the total luminosity each star emits. The total luminosity is normalized to 1.00, so that $L_1 + L_2 = 1$. These quantities are wavelength dependent [27]. The parameters were evaluated for each component by using the well-known relation [28]:

$$L_{1,2} = (\frac{R_{1,2}}{R_o})^2 (\frac{T_{1,2}}{T_o})^4 \qquad \ldots(2\text{-}26)$$

Where T_o =5770 K is the Suns effective temperature [23], and R_1, R_2 are radius of primary and secondary stars respectively, L_1 and L_2, the luminosities of the components in solar units L_o . While T1 and T2 are the temperatures of the components.

Chapter Three

Results and Discussion

3. CV BOO

Modern astronomy pays close attention to the physics and evolution of triple and multiple stars. The stability of orbits in such systems, see, e.g. (Holman & Wiegert 1999; Zhuchkov et al. 2010), the influence of additional bodies on the central binary, and the dynamical evolution of orbits in multiple systems, see, e.g. (Saito et al. 2013; Naoz et al. 2013; Li et al. 2014, 2015) are particularly interesting. Some important astrophysical phenomena can be associated with the evolution and dynamical interactions of triple stars. Type supernovae may originate in triples due to the evolution of the central binary under the influence of a third star (Iben & Tutukov 1999; Hamers et al. 2013) or due to the dynamical interaction and subsequent direct collision of white dwarfs in triple systems (Kushnir et al. 2013). The giant eruption of Eta Carinae in 19th century could be the result of a merger of a massive close binary, triggered by the gravitational interaction with a massive tertiary companion, which is the current member of the binary (Portegies Zwart & van den Heuvel 2016). Investigations of multiplicity of stars are important

for understanding of star formation processes, see, e.g., (Tokovinin 1999). This makes searches for multiplicity in binary systems of high importance.

According to the General Catalogue of Variable Stars (Samus et al. 2014), the orbital period of CV Boo is P = 0.8469938 d and the spectral type of its primary component is G0. CV Boo was found as an eclipsing binary by Peniche et al. (1985), and it was studied by many observers during several decades using photographic, visual, PMT (photomultiplier tube), and CCD

(charge-coupled device) methods. In a detailed study of CV Boo, Torres et al. (2008) presented the results of V -band photometric studies and spectroscopic radial velocity (RV) measurements of CV Boo. Using their own data and information from literature, Torres et al. (2008) determined the masses, radii and effective temperatures of the primary and secondary components to be M1 = 1.032±0.013M⊙, R1 = 1.262±0.023R⊙ and 5760±150K, andM2 = 0.968±0.012M⊙, R2 = 1.173± 0.023R⊙ and 5670 ± 150K, respectively. Torres et al. (2008) found that CV Boo has a light curve variability with a period shorter than the orbital period. They connected this variability with the axial rotation of a spotted component (or both components) which rotates faster than the synchronous rate. Torres et al. (2008) noted that this fact disagrees with predictions from the tidal evolution theory according to which both companions should be synchronized. Also, they came to the conclusion that both components are near the end of the main sequence stage and that the primary component even probably entered into the shell hydrogen burning stage. Torres et al. (2008) estimated the age of the system as 9 Myr, and concluded that the radii of the companion stars in CV Boo are about 10% larger than what is expected from stellar evolutionary models. Torres et al. (2008) also claimed that CV Boo has a stable orbital period, based on available 98 primary and 50 secondary minima. Nevertheless, the existence of a large number of high precision CCD photometry.

2 Observations and data reduction

CV Boo was observed with the 50 cm AMT-1 telescope with the Apogee Alta-U16M 4Kx4K CCD camera at the Maidanak observatory of the Ulugh Beg Astronomical Institute (Uzbek Academy of Sciences) during 41 nights in spring–summer of 2014. We obtained more than 32 500 images in the Bessell R filter with exposures ranging from 8 to 20 s, with a typical exposure of 10 s. Continuos monitoring time intervals were 5-7 hours per night. Bias and dark frames with appropriate exposures were made every night before and after observations. Flat field frames were

Parameter	Star1	Star2
Mass Ratio (M2/ M1)	0.937600	
Surface Potential Ω	4.675200	4.901400
Temperature	5760.00	5670.00
Gravity Darkening	0.320	0.320
Limb Darkening	0.428	0.437
Reflection	1.262	1.173
Inclination	87.651	

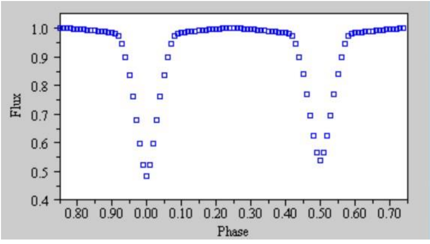
recorded for the twilight sky. To process the data we use the aperture photometry method with a specific program1. The optimum aperture corresponds to the minimum of the standard deviation for differential magnitudes. We used 2MASS J15270686+3659270 and 2MASS J15272880+3647225 as the reference stars. The aperture was constant during each night and didn’t change significantly from night to night. Precision values for a single exposure were in the range 0.0024m–0.004m for different nights. Standard dark and flat field corrections were made. Original fits files were converted to text files containing the CV Boo brightness with respect to the reference stars depending on the Heliocentric Julian Date (HJD). Light curves of CV Boo were created on the basis of these data. Figure 1 plots a sample light curve of CV Boo. In order to achieve the highest possible precision of minima moments we use only the best light curves with both branches around the light curve minimum.

Table (1) : The light curve fit parameters for CV Boo

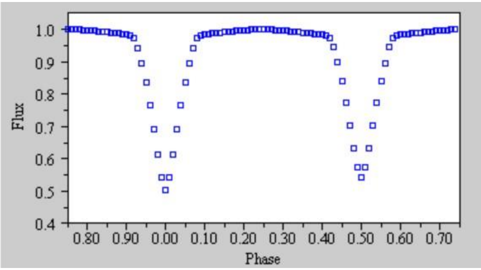
Where $\Omega_1$andΩ_2=surface potential of the primary and the secondary components ,C_1 and C_2= potential of the primary and the secondary components,f_1,f_2= Fillout of the primary and the secondary components,L_1,L_2=Absolute parameter of components in units of solar Luminosity.

Table (2): The output parameters for CV Boo

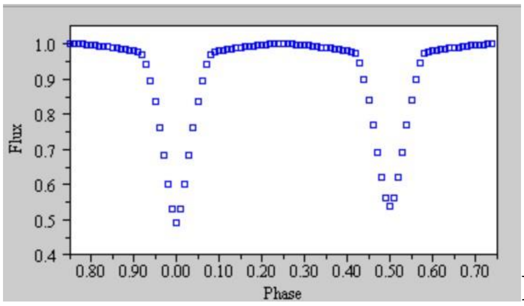
Ω1= 4.675200 Ω2 = 4.901400
Ωinner =3.647876 Ωouter = 3.132983
C1 = 5.059921   C2= 5.293405
C inner = 3.999512 C outer= 3.468037
f1= 0.209570   f2= 0.244435
Lagrangian L1 = 0.506632 Lagrangian L2= 1.687838
r1(back)= 0.277262 r2(back) = 0.249348
r1(side) = 0.270322 r2(side) = 0.244103
r1(pole)= 0.265325 r2 (pole) = 0.240538
Surface area 1 = 0.924640 Surface area 2 = 0.753523
Mean radius 1 = 0.270970 Mean radius 2 = 0.244663



B



V



R

Figure (1-1): Light curve fitting for B, V and R Filter for CV Boo **.**

Also we can calculate absolute parameters for stars by using equation(2.17 t0 2.22)

Table (3): Absolute parameter (in solar units)of the systems for short period binary star

Star	R1	R2	M1	M2	L1	L2	Type	Refrence
CV Boo	1.26	1.173	1.032	0.968	43	46	EA	(Torres et al., 2008)
CV Boo	1.2	1.13	1.07	0.823	42	43	EA	Present work

5.2 Bolometric Magnitudes-$M_{bol1,2}$:

By using equation

Table (4): Adapted auxiliary parameters for active short period binary system.

Star	x_1	x_2	g_1	g_2	E_1	E_2
CV Boo B	0.392	0.393	0.3019	0.3022	3.5244	3.5204
V	0.318	0.319	0.2933	0.2935	3.6577	3.6535
R	0.353	0.354	0.2860	0.2863	3.7762	3.7718

5.3 Age- Mass relationship for CA binaries of stars:

The Age- Mass can be determined using Equation (12), for this purpose programs handled using Mat lab program. The Table (5) represents the present work parameters regarding the age-mass relation.

Table (5): The age-mass relation for other investigation:

Star	Primary log Age	Secondary log Age
CV Boo	110.0367	111.2542

.4 The Life Time:

The Life time can be determined using Equation (13), for this purpose Mat lab programs handled. The table (6) represents the present results relating life time of short period binary systems.

Table (6): Lifetime of short period binary systems

Star	$t_1/t_{sun}*10^{83}$	$t_2/t_{sun}*10^{83}$

CV Boo	3.0133	3.0133

6.Geometry of Roche Loop:

The unique form of the Roche lobe relies upon at the mass ratio q = M1/M2, and should be evaluated numerically. However, for plenty functions it's miles beneficial to approximate the Roche lobe as a sphere of the identical volume. An approximate system for the radius of this sphere is

$r_1/A = \max [f_1, f_2]$,for $0 < q < 20$…………………………………………..(18)

Where $f_1 = 0.38 + 0.2 \log (q)$ and $f_2 = 0.46224 (q/1+q)^{1/3}$. Function f1 is extra than f2 for q > 0.5228 . The duration A is the orbital separation of the machine and r1 is the radius of the sector whose extent approximates the Roche lobe of mass M1. This components is correct inside to approximately 2%. (Paczynski, B. (1971) Another approximate components became possible Eggleton's method and reads as follows:

$r_1/A = 0.49 q^{2/3} / 0.6 q^{2/3} + \ln (1+q^{1/3})$……………………………………….(19)

This formula gives results up to 1% accuracy over the entire range of the mass ratio q (Eggleton, P. P. (1 May 1983) . The fiq (6) show The shape of the spotted model of the AD And binary system at different phases.

Figure (6): Shows the shape of the spotted model of CV Boo binary star at different phases.

7 First Contact Angle:

The First contact angle can be determined using Eqs. (14) to (16), for this purpose programs handled using Mat lab program. The Table (8) represents the present first contact angle and type of eclipse.

Table (8): First contact angle and type of eclipse.

Star	θ_1	a	C_o	Type of eclipse
CV Boo	1.2271	3.5175	1.7454	G3V

1.8 Empirical Relationship between CI ,M_{bol} and T_{eff}:

The Empirical Relationship between CI ,M_{bol} and T_{eff} can be determined using Eqs. (16) to (22), for this purpose programs handled using Matlab program. The Table (9) represents the present empirical values for primary and secondary stars of the binary system.

Table (9): Empirical values for present work.

Star	Component	BC= M_{bol}-M_v	CI= B-V	M_{bol}	M_v
CV Boo	Primary	-0.08	20.6169	4.24	4.32
	Secondary	-0.11	20.0349	4.46	4.57

DISCUSSION AND CONCLUSIONS

Despite the system's intrinsic variability, the absolute dimensions for the components of CV Boo have now been established quite precisely. The relative errors are better than 1.3% in the masses and 2% in the radii. The object can now be counted among the group of eclipsing binaries with well-known parameters. Under different circumstances the large number and high quality of the photometric observations we have collected might have permitted a more detailed study of the limb darkening laws and a comparison with theoretically predicted coefficients, but this possibility was thwarted here by the intrinsic variability. This phenomenon is not itself with out interest. Both stars in the binary systems CV Boo are similar to our sun, having comparable masses, luminosities, surface temperatures, etc. However, based on theoretical models of stellar abundances, CV Boo is much older than our solar system, at roughly 10.3 billion years old. Yet, the galaxy to which CV Boo belongs must be younger than ours, based on the small abundance of heavy elements – only 0.1% in each star versus approximately 1% in our sun .

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