Salahaddin University-Erbil	**QUESTION BANK**	**2021-2022**
Education College	**Calculus**	**First Year**
Physics Dept.		**Diyar Ali Rasool**

Lecture 1

1. Find the function values?

1. $\cos^2\frac{\pi}{8}$ 2. $\cos^2\frac{\pi}{12}$ 3. $\sin^2\frac{\pi}{12}$ 4. $\sin^2\frac{\pi}{8}$

2. Express the given quantity in terms of $\sin x$ and $\cos x$.

1. $\cos(\pi + x)$
2. $\sin(2\pi - x)$
3. $\sin\left(\frac{3\pi}{2} - x\right)$
4. $\cos\left(\frac{3\pi}{2} + x\right)$

3. Verify the following identity:

$a.\ tanh^2\, x + sech^2\, x = 1$ $b.\ \sinh 2x = 2\, sinhx \cosh x$

$c.\ sinh3x = 3\sinh x + 4\ sinh^3 x$ $d.\ \cos\left(x - \frac{\pi}{2}\right) = sinx$

$e.\ \cos\left(x + \frac{\pi}{2}\right) = -\sin x$ $f.\ \sin\left(x + \frac{\pi}{2}\right)$

$g.\ \sin\left(x - \frac{\pi}{2}\right) = -\cos x$

4. Derive or prove the following identities:

$a.\ \tan(A + B) = \frac{\tan A + \tan C}{1 - \tan A \tan B}$ $b.\ \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$

5. Use the definition of $sinhx$ and $cosh$ x to show that $cosh^2 x - sinh^2 x = 1$.

Lecture 2

1. Use the definition

$$f'(a) = \lim_{h\to 0}\frac{f(a+h) - f(a)}{h}$$

Find the derivative of the given function at the indicated point.

a) $f(x) = 3 - x^2$, $a = -1$. **b)** $f(x) = x^2 + 4$, $a = 2$

2. Find the dy/dx

a) $y = 1 - x + x^2 - x^3$ **b)** $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + x$ **c)** $y = \frac{\sqrt{x}-1}{\sqrt{x}+1}$

d) $y = \frac{x^{-4}}{4} - \frac{x^{-3}}{3} + \frac{x^{-2}}{2} - x^{-1} + 3$

3. Find the horizontal tangent for the following functions.

a) $y = x^3 - 2x^2 + x + 1$ **b)** $y = 4x^3 - 6x^2 - 1$

c) $y = 5x^3 - 3x^5$

4. Find an equation for the line tangent to the curve at the given point.

a) $y = x^3 - 3x + 1,\ at\ points\ (2,3)$ **b)** $y = \frac{x^4+2}{x^2}$ $, x = -1$

5. Derivative of Trigonometric functions.

Find dy/dx ?

a) $y = -10\,x + 3\cos x$. *b*) $y = (sin^2 x + \cos x)\sec\ 2x$.

c) $y = x^3 \cot\ \ x - \frac{1}{x^3}$ **d)** $y = x^2 - \sec x + 1$

e) $y = \dfrac{1 + \csc x}{1 - \csc x}$ *f*) $y = sec^3\, 2x \csc x$

g) $y = (1 + \sec\csc x)\sin x$ *h*)$y = \sec(\tan x)$

i) $y = (\sec x + \tan x^2) + (\sec x - \tan x)$

6. Find ds/dt

a) $s = 5 + \frac{1}{\cot t}$ **b)** $s = (1 + \csc t)\cos t$ **c)** $s = \sec t \csc t$

7. Find y'' for **a)** $y = \csc x$. **b)** $y = \sec x$

8. Find $y^4 = \frac{d^4y}{dx^4}$ for **a)** $y = -4\,x^2 sinx$. **b)** $y = 9x \cos\ x$.

Lecture 3

1. Find the derivatives of the following functions?

a) $s = (2x^2 + 1)^{-5/2}$ **b)** $y = \csc(\cot x)$ **c)** $y = sin^2(\pi t - 2)$

d) $y = \left(\frac{sinx}{1+\cos x}\right)^{-1}$ **e)** $f(x) = -(sec\ x + tan\ x)^{-1}$ **f)** $y = sec(\sqrt{x})\ tan\,(1/x)$

2. Use implicit differentiation to find dy/dx.

a) $x^3 + y^3 = 18\,xy$ **b)** $2xy + y^2 = x + y$ **c)** $x + tan\ xy = 0$

3. Find the derivatives?

a) $y = sin^{-1}(7x)$ **b)** $y = csc^{-1}(2x^2)$ **c)** $y = e^{cos^{-1}x}$

d) $y = x^3\ tan^{-1}\ 2x$ **e)** $y = \frac{cos^{-1}x}{sin^{-1}x}$ **f)** $y = \frac{1}{x^2}\ (tan^{-1}x)^3$

g) $y = sec^{-1}(-2x + 4)$ **h)** $y = csc^{-1}(x)\ cot^{-1}(\sqrt{x})$ **i)** $y = cot^{-1}\frac{1}{x} - tan^{-1}x$

4. Find $\frac{dy}{dx}$?

a) $y = e^{-7x^{-4}}$ **b)** $y = \frac{x^3}{e^x} - x^2e^{2x}$ **c)** $y = e^{-2x}\ 8^{\,x}$

d) $y = 5^{\cot x \csc x}$ **e)** $y = 4^{\sec x^2}$ **f)** $y = (\ln x)^4$ **g)** $y = \ln(ln\ 2x^2)$

h) $y = \ln\left(\frac{1}{x^{-2}}\right)$ **i)** $y = log_6 x^3$ **g)** $y = \ln(10).\log_4 x^2$ **k)** $y = \log_4(1 + \ln 5x)$

m) $y = \ln(2 - \cos x)$ ***n***) $y = x^{e^{2x^2}}$ ***O***) $y = x^{\cos x}$

5. Find an equation of the tangent line at the point indicated.

a) $f(x) = 6^x, x = 2$ ***b***) $y = (\sqrt{2})^x, x = 8$

6. Find the derivative?

a) $y = \sinh(2x^4)$ ***b***) $y = tanh(t^{1/2} - 2)$ ***c***) $y = (\ln(\cosh x))^{1/5}$

d) $y = \dfrac{\operatorname{sech} t}{1 + \tanh t}$ ***e***) $y = e^{\operatorname{csch} 2x}$ ***f***) $y = \sinh(cosh^{-3}(2x))$

g) $y = \operatorname{sech}(2x)\coth(1/x)$ ***h***) $y = tanh^{-1}(e^{3x} + x^4)$ ***i***) $y = (sinh^{-1}(x^2))^3$

j) $y = (csch^{-1} 3x)^4$ ***k***) $y = sinh^{-1}\left(\sqrt{x^2 + 1}\right)$ ***l***) $y = e^{cosh^{-1} x}$

m) $y = ln(tanh^{-1} x)$.

7. Prove the following formula?

a) $\dfrac{d}{dx}(\coth x) = -csch^2 x$ ***b***) $\dfrac{d}{dt}\, sinh^{-1} t = \dfrac{1}{\sqrt{t^2+1}}$

c) $\dfrac{d}{dt}\, cosh^{-1} = \dfrac{1}{\sqrt{t^2 - 1}}$

Lecture 4

1. In the following Exercises, find the most general anti-derivative or indefinite integral.

a. $\int x^{-4}(x + 2x)\, dx$ b. $\int (\sqrt{x} + \sqrt[3]{x}) dx$ c. $\int -\left(\dfrac{sec^2 x}{3}\right) dx$

d. $\int \dfrac{\csc\theta \cot\theta}{2}\, d\theta$ e. $\int (4\ \sec x \tan x - 2\, sec^2 x)\, dx$ f. $\int \dfrac{1}{2}(csc^2 x - csc\, x\, cot\, x) dx$

g. $\int cos\, x\,(tan\, x + sec\, x) dx$ h. $\int \dfrac{csc\, x}{csc\, x - sin\, x}\, dx$ i. $\int (1 + tan^2 x) dx$ $(Hint\ (1 + tan^2 x) = sec^2 x)$

j. $\int (2 + tan^2 x)\, dx$ k. $\int \dfrac{1}{t^2} cos\left(\dfrac{1}{t} - 1\right) dt$

2. Find the indefinite integral?

a. $\int 4x^3 \sin x^4\, dx$ b. $\int \frac{1}{\theta^2} \sin\frac{1}{\theta} \cos\frac{1}{\theta}\, d\theta$ c. $\int \sqrt{\cot y}\, csc^2 y\, dy$

d. $\int \frac{\sec z \tan z\, dz}{\sqrt{\sec z}}$ e. $\int csc^2 2x \cot 2x\, dx$ f. $\int \frac{\sin x}{\cos^3 x}\, dx$

g. $\int \cot^2 x\, dx$ $Hint(1 + \cot^2) = csc^2 x$

3. Evaluate the integral of the following integrations indicted below

a. $\int (e^{2x} + 5e^{-3x})dx$ b. $\int 8\, e^{(x+1)}\, dx$ c. $\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\, dr$ d. $\int t^3\, e^{t^4} dt$

e. $\int \frac{e^{1/x}}{x^2}\, dx$ f. $\int e^{\sec \pi t} \sec \pi t\, \tan \pi t\ \ dt$ g. $\int \frac{dx}{1 + e^x}$

4. Evaluate the following integrations?

a. $\int (1.3)^x dx$ b. $\int 4\, x^{\sqrt{4}}\, dx$ c. $\int \frac{dx}{x \log_{10} x}$

L 5

1. Evaluate the following integrals?

a) $\int \frac{dx}{\sqrt{9-x^2}}$ b) $\int \frac{dx}{9+3x^2}$ c) $\int \frac{dx}{x\sqrt{5x^2-4}}$

d) $\int \frac{e^{\sin^{-1}x} dx}{\sqrt{1-x^2}}$ e) $\int \frac{(\sin^{-1}x)^2\, dx}{\sqrt{1-x^2}}$ f) $\int \frac{dy}{(\tan^{-1}x)\,(1+y^2)}$

2. Evaluate the integrals?

a) $\int \sinh\frac{4x}{9}\, dx$ b) $\int \tanh\frac{2x}{7}\, dx$ c) $\int sech^2\left(1 - \frac{1}{2}\right) dx$

d) $\int \coth\frac{\theta}{\sqrt{3}} d\theta$ e) $\int e^x\ sec^2(e^x 7)dx$ f) $\int (sec^2 x)\, e^{\tan x}\, dx$

g) $\int \frac{\operatorname{sech}\sqrt{t}\, \tanh\sqrt{t}\ dt}{\sqrt{t}}$ H) $\int \frac{1}{y\sqrt{4y^2-1}}\, dy$ I) $\frac{24}{y\sqrt{y^2-16}} dy$

3. Solve the following Integrations?

$a) \int \frac{dx}{x\sqrt{1+(\ln x)^2}}$ $b) \int \frac{\cos x}{\sqrt{1+sin^2 x}} dx$ $c) \int \frac{csc^2 x\, dx}{1+(cotx)^2}$

Lecture 6

1. Evaluate each integral by using a substitution to reduce it to standard form:

$a) \int cot^3\, y\ \ csc^2 y\, dy$ $b) \int \frac{dx}{\sqrt{2x-x^2}}$ $c) \int (\csc x - \tan x)^2\ dx$

$d) \int \frac{x^2}{x^2+1} dx$ $e) \int \frac{1-x}{\sqrt{1-x^2}}\ dx$ $f) \int \frac{1}{\csc\theta + cot\theta} d\theta$

2. Integration by part:

$a) \int x\, sec^2 x\ dx$ $b) \int (x+1)^2 e^x dx$ $c) \int e^{\theta} \cos 2\theta\ \ d\theta$

$d) \int x^2\ lnx\ dx$

3. Trigonometric Integrals:

a) $\int_0^{\frac{\pi}{2}} sin^5\ x\ \ dx$ $b) \int_0^{\frac{\pi}{2}} 35\ sin^4 x\ \ cos^3 x\ dx$ $c) \int_{-\pi}^{0} \sin 3x \cos 2x\ dx$

$d) \int_0^{\frac{\pi}{4}} sec^4\ \theta\ \ d\theta$

4. Basic Trigonometric Substitutions:

$a) \int \sqrt{1-9t^2}\ \ dt$ $b) \int \frac{dx}{x^2\sqrt{x^2+1}}$ $c) \int sin^5 x\ cos^5 x\ dx$

$d) \int tan^4 x\ \ sec^2 x\ dx$ $e) \int sin\, 5\theta \cos 6\theta\ \ d\theta$ $f) \int \sqrt{1+\cos\left(\frac{t}{2}\right)}\ \ dt$