**SEMESTER -1-**

**Course Outline**

**Chapter one**

1-1: Introduction to Mathematical statistics ( random variable, type of random variable )

1-2: Probability Mass and Density function

1- 3: The Joint probability distribution function ( J.P.D.F).

1- 4: The marginal probability distribution function (M.P.D.F).

1-5: **C**ovariance and Correlation Coefficient :-

1-6: Conditional Distribution function

**Chapter two**

2-1: Transformation of Discrete type

2- 2: Transformation of continuous type

2-3: Order statistics

2-4: T and F distribution

**References:-**

1- Richard J. Larsen, Morris L. Marx, Introduction to Mathmatical statistics and its Application 4th ed, 2006.

2-Hogg,R, andGraig A,(1978) , Introduction to mathematical statistics , collier macmillan 4th (Required) .

3- Lorson , R,and Mary .m(2009),Introduction to mathematical statistics and application , London , person education lid.(optional).

4-AL-Nasi, H. Rashid (1988) statistical inference.

5- امير حنا هرمز , الاحصاء الرياضى كلية الادارة والاقتصاد , جامعة الموصل

**Chapter One**

1-1 Mathematical statistics :- is the application of probability theory, a branch of mathematics, to statistics, as opposed to techniques for collecting statistical data. Specific mathematical techniques which are used for this include mathematical analysis, linear algebra, stochastic analysis, differential equations, and measure theory.

* Random Variable (r.v):

Definifion: A random variable X is a real-valued function defined on the sample space, which takes values from the sample space S to a space of real number R in a random experiment.

X:ƒn S R

i.e., the range of X is the set of real number of all possible values that can occur for X will be denoted by Rx.

Rx = { x: X(w) = x,wR

 Such that:

 X: random variable.

x :the value of this r.v.

 S: a sample space.

w: an events (elements) in space of X.

 R: the set of real number.

**Ex1//** Let a coin be tossed three times, and let X be the number of (heads) to be observed. Then

S = { HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

and; The space of the random variable X is; Rx = {x: x = 0, 1, 2,3}

**Ex2//**

 **a)** Let a dice is to be tossed one time, then; S = {1, 2, 3, 4, 5, 6} belongs to R.

Let; X: be odd numbers that occur, then the space of X is;

 Rx. = {x: x = 1, 3, 5}.

 **b)** Let a pair of fair dices are to be tossed one time, and then;

Let; X: be the sum of the two numbers that occur, then;

S={(x1,x2):x1=1,2,3,4,5,6;x2=1,2,3,4,5,6} belongs to R

and we have defined;

X(w) =x1+ x2 for w=(x1 , x2) S .

then; the range of X is;

Rx= {x: x = 2, 3,4, 5,...., 12} R

**Ex3**// Let the outcome of a random experiment be a point on an interval (1,3), then; Rx = {x; l < x < 3} R

* Types of Random Variable:-

There are two types of random variables ; A Random variable can be discrete or continuous depending on their range, as:

1. Discrete Random Variable.

If X be a r.v, then X is said to be discrete r.v. if the range of X (Rx) (sample space **S**) is countable set whether finite or infinite set.

**Examples:**

1- Number of students in Statistical Department. S = {1, 2,....; 65 }

2- A coin is tossed until a head occur. S = {1, 2, 3, ....}.

1. Continuous Random Variable.

X is called continuous r.v. if the runge of X (Rx) (sample space S) is uncountable set [i.e., is defined as on an interval or a set of intervals {that may take all the points (values) in an interval (a ≤ X ≤ b)}], whether finite Or infinite set.

**Examples:** Time, Length, Weight, Age (year, month, day ...).

**Ex4:** In experiment to choice the number from an interval [0, 1], then;

*S =* {x: *0* ≤ x ≤ *1*}

Uncountable set because there are infinite number defined in this interval; therefore, X is continuous r,v. Such sets as;

S = {x: 0 <X < 10}

S = {x:2 ≤ X ≤ 3 or 4 ≤ X ≤ 5} , S= {x:0<x<∞}

**1-2:- Probability Mass and Density function**

Defn 1: if x is a discrete r.v. with different values x1,x2,…,xn then the function

 Where :-

Is defined to be probability mass function of x (p.m.f) . It is a real - valued

function, and satisfies the following properties:-

1- 0 ≤ p(x) ≤ l , for all x

**2-**

3-) , where A is any subset of the space of .

Def"2: If X is a continuous r.v. defined on an interval then the function f(x) is defined to be probability density function of X (p.d.f) , it is a mathematical function which satisfies the following properties:-

 1- 0 ≤ f(x) ≤1 , Vx€R

2-

3- , where A is any subset of space of X .

Note: If the integral or the summation equal to one then the function be probability function, if not then the function not to be probability function.

 Ex5: Let a coin be tossed twice, then;

S = {HH, HT, TH, TT}

Let a r.v. X be the number of head that occur, then; **Rx=** {x:x = 0, **1,2}**

|  |  |  |  |
| --- | --- | --- | --- |
| x | 0 | 1 | 2 |
| P(x) | 1/4 | 2/4 | 1/4 |

EX6// let X be a r.v. with p.d.f. f(x)

Let

 .

Sol//

)=

)=

Some Remarks about How to Find the Probability of r.v.

a) - If X is a discrete r.v., then;

1-

2-

3-

4-

5-

b) - If X is a continuous r.v., then;

1-==dx

***2-***

**EX7//** let

 = 0 o.w

1- Check

2)- find:2)

Solution//

1-

2-

3- p(x=5)=

4-

**-Finding the Value (c) in Discrete and Continuous Function by:**

**EX8//** let p(x)=cx , x=1,2,3,4,5

1)-find the value of (c)

2)- 5)-

=1

 c(1+2+3+4+5)=1

c(15) = 1

c=1/15

3)-

=

=

4)-

 =

5)-

=

EX9// H .W

Let X be r.v. with p.d.f.

**1-3:-The Joint Probability Distribution Function (J.P.D.F.)**

IF (X1, X2,…, Xp) are r.v's defined on the sample probability space , then the (X1, X2,…,Xp) is called p-dimensional r.v's.

**The Joint probability Density Function**

Defh: IF (X1, X2,…, Xp) are a p-dimensions ; Then the Joint probability density function of (X1, X2, ..., Xp) is defined to be:

f(x1,x2,...,xp) = p[X1 =x1 , X2= x2 , ..., Xp=xp]

The Properties of the Joint Probability Density Function

A joint p.d.f. has the following properties:-

1)-

2)-

3)-

**Not//**IF p = 2 ;then p(a<xI <b,c<x2 <d)

=

4) – if (p= 1) Then the (j.p.d.f.) is called ( univariate p.d.f). if p=2 ; Then the( j.p.d.f.) is a fun. of two r.v's and it is called (Bivariate p.d.f.) , If(p>2); then the( j.p.d.f.) is called (Multivariate p.d.f).

5)- The joint p.d.f. is a function unique value at any value of variables distribution (X1,X2,…,XP)as;(x1,x2,…,xp) are two r.vs then :-

And f(x,y)is called the joint probability density fun. (j.p.d.f.) of X and Y , and has the following properties.

1-

2-



Example10//



**EX11//H.W**

Three coin are tossed , let X denoted the number of heads that occur on the first two coins , and , let Y denoted the number of tails that occur on the last two coins

1- Find the j.p.d.f. of X and Y .(i.e. ,find p(x,y)

2- show that p(x,y) is a j.p.d.f. of X and Y .

1-4:The Marginal probability Distribution Function (M.P.D.F.)

Defn: let X & Y are two r.v's with (j.p.d.f) **f(x,** y), then **f(x)** & **f(y)** are called the marginal p.d.f of X & Y respectively, which can be defined as follows:-

X and Y are independent iff ;

j.p.d.f. of X and Y =(m.p.d.f of X ) . (m.p.d.f of Y)

**Example 12//** let the j.p.d.f. of X and Y is

Find : 1)- The marginal p.d.f. of X. 2)- The marginal p.d.f. of Y

3)-Are X and Y independent .

**Sol:**

**1- ,** x= 1,2,3is m.p.d.f. of x.

**2-**

**3-**

 **, X and Y are indep.**

**Exampl 13//H.W**

Let X and Y are two cont. r.v.having a j.p.d.f.

Find: 1)The marginal p.d.f. of X?

2)- The marginal p.d.f. of Y?

3)- Are X and Y independent.

4)-show that f(x,y) is a j.p.d.f. of X and Y ?

**1-5:**Covariance **and** Correla**tion** **Coefficient:-**

**Dcfn:** Let X and Y are two r.v.'s defined on the same probability space, then the covariance of X and Y denoted by [cov(X, Y) or бxy] is defined as;

бx,y = cov(x,y) = E[(X - MX)(Y-MY)]= E(XY) - MXMY

where ; MX=E(X) , MY=E(Y)

**proof:**

COV(X,Y) = E[(X-MX)(Y-MY)=E[XY-MXY-MYX+MXMY]

=E(XY)-MX E(Y) – MY E(X) +MX MY

=E(XY)-MX MY – MY MX +MX MY

COV(X,Y)=E(XY) – MXMY

**Defn:** Let X and Y are two r.v.'s with the marginal p.d.f. of X and Y,f(x) , f(y) , and the j.p.d.f.f(x,y), then the correlation coefficient between X and Y denoted by Px,y is defined to be:

Where:-

**Example14 //** let the r.v. X and Y have the j.p.d.f. ;

Calculate the correlation coefficient between X and Y .

Sol//

**Example15//**let X and Y have j.p.d.f. describeas:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| (x,y) | (0,0) | (1,0) | (1,1) | (2,1) | o.w |
| F(x,y) | 1/6 | 2/6 | 2/6 | 1/6 | 0 |

**Calculate:** The correlation coefficient between X and Y .

Sol//

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Y X | 0 | 1 | 2 | Sum : f(y) |
| 0 | 1/6 | 2/6 |  | 3/6 |
| 1 |  | 2/6 | 1/6 | 3/6 |
| Sum: f(x) | 1/6 | 4/6 | 1/6 | 1 |

**1-6:Conditional Distribution function :-**

If X and Y are two r.v. with j.p.d.f. f(x,y) , then the con. Dist. Fun. Of X given that Y=y is defined as

And the con.Dist. Fun. Of Y given X=x defined as :-

Where are the marginal p.d.f. of X and Y respectively

If X and Y are two r.v. then the conditional probability of

**The** **conditional mean of *Y* given *X* = *x*** is defined as:

**The** **conditional variance of *Y* given *X* = *x*** is defined as:

**Remark :** If X and Y are two indep r.v. then

**Since :**

**Properties of the Conditional p.d.f.**

**Example 16**//let X and Y be two r.v.with j.p.d.f.

Find:

**Sol//**

=2y